New Universality Class for Superconducting Order Parameter

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We present a model of superconductivity with the BCS-like pairing due to Aharonov-Bohm forces. The gap is proportional to the first power of the small parameter (in which the self-consistent perturbation scheme is developed), as opposed to the BCS class of models where the gap is exponentially suppressed with the small parameter.

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The challenge of high transition temperature in new superconductors has inspired searches for unconventional mechanisms of superconductivity. The exponential suppression of T_c in the BCS theory [1] is, in fact, related to the renormalization properties of the dimensionless coupling λ. Indeed, the BCS solution for the gap,

$$\Delta \sim \omega_D \exp(-1/\lambda)\,, \tag{1}$$

may be viewed as a conventional formula of dimensional transmutation, where Δ is a physical (renormalization invariant) parameter, and the Debye frequency ω_D plays the role of cutoff. To keep the physical parameter unchanged, λ should depend logarithmically on the cutoff, which is a kind of universal behavior for dimensionless couplings [2]. In this respect, the problem of high transition temperature may be regarded as that of finding a new universality class for the gap parameter. Another way to identify the universality class is to consider the scaling properties of the gap $\Delta(p)$ as a function of momentum. For the BCS models the gap is essentially independent of the momentum.

These considerations imply that to get solutions which would be parametrically larger than BCS, one has to consider a theory where, under quite general conditions, some dimensionless coupling does not acquire logarithmic renormalization. In this Letter we present an example of such a theory provided by the Aharonov-Bohm (statistical) interaction in (2+1) dimensions. This interaction may be cast into the field-theoretical language by introducing the gauge field with topological (Chern-Simons) action. The appearance of the effective topological gauge field may be related to a variant of the fractional quantum Hall (FQH) state [3], or the flux phase [4] in two-dimensional spin systems (the viability of these scenarios remains, however, still far from being established). On the other hand, the field-theoretical investigations [5] have shown, in many cases, the absence of ultraviolet divergences in the dimensionless Chern-Simons coupling.

When all particles have the same sign of charge with respect to the statistical gauge field, a scenario for superconductivity is already widely known under the name of anyon superconductivity [6]. Let us stress that this is *not* what we are going to consider. The superconducting solution we obtain in the present Letter results from the gap equation and corresponds to pairing due to statistical gauge forces in a system where the particles of opposite charges with respect to the statistical gauge field are present in equal amounts.

The pairing of oppositely charged anyons was previously considered in Refs. [7–10], motivated mostly by the properties of elementary excitations in the FQH state and the flux phase [11]. However, none of the previous work has found the unsuppressed gap solution which occurs, as will be shown here, in this problem *at weak coupling.* The gap solution found by Khveshchenko and Kogan [7] is exponentially small and, contrary to our results, becomes zero for the case of purely Aharonov-Bohm interaction [12]. Balatsky and Kalmeyer [10] addressed only the possibility of *s*-wave pairing. As we will see, for the *s* wave the most long-ranged part of the potential, the Aharonov-Bohm interaction, is not operative. Hence, the authors of Refs. [7] and [10] rely on the short-range interactions corresponding to the diagonal terms of our Eq. (12) below. For the weak coupling, this would lead to the exponentially suppressed gap solution, as indeed obtained by Khveshchenko and Kogan. The absence of the exponential suppression found by Balatsky and Kalmeyer is of no surprise since they were in the strong-coupling regime from the very beginning. The more general possibility of the pairing with arbitrary orbital momentum l due to Aharonov-Bohm forces was addressed previously in Refs. [8] and [9], and in Ref. [9] the enhancement of the pairing potential for nonzero l was noted.

The primary purpose of the present Letter is to show that the pairing solution in the case of long-range potential may differ substantially from the BCS form, Eq. (1). With this motivation, we will not be very conscious about the precise origin of our model. We will see that the Aharonov-Bohm interaction gives rise to the following asymptotics of the gap parameter:

$$\Delta \sim \epsilon_F/\kappa\,, \tag{2}$$

where κ is the Chern-Simons coupling. The weak coupling corresponds to large κ. Thus, the gap proves to be proportional not to the ultraviolet cutoff but to the intrinsic energy scale, and the suppression in Eq. (2) is only power law, as opposed to the exponential suppression in Eq. (1). This is in accord with the previous discussion of the renormalization properties of the coupling constant.

When long-range interactions are present in a problem one expects to be able to construct a consistent approximation scheme on the basis of the random-phase approximation (RPA). In the present Letter we construct such a scheme for the case of a two-component anyon gas. Our approximation is equivalent to summing up the ladder diagrams for the four-fermion vertex in the way familiar from the BCS theory, but with the important modification that the gauge propagator forming the step of the ladder is improved by including the repeated one-loop fermion bubble. The fermionic Green functions are taken to be modified by the gap which itself is determined self-consistently from the gap equation with the potential corresponding to the vertex built as above. The most important effect of the RPA bubbles, occurring for the pairing with nonzero orbital momentum, is the one-loop *finite* renormalization of the Chern-Simons coupling [9,13] given by

$$\kappa_{\rm ren}=\kappa-l/2\pi\,, \tag{3}$$

where l is the orbital momentum of the pair. Thus, for small momenta along the gauge line, and large l, close to $2\pi\kappa$, the system develops the strong pairing potential. It is this enhancement of the potential that leads to the drastic increase in the gap parameter, changing its weak-coupling asymptotics from the exponential to the power law, Eq. (2). For the most conventional case $2\pi\kappa=$integer, we find that the system prefers pairing with the orbital momentum $l=2\pi\kappa-1$. We prove that, nevertheless, our scheme is consistent by showing that all other corrections to the gap equation, except the bubbles, are suppressed at least by $1/\kappa$ (up to logarithms of κ) at $l\sim2\pi\kappa$. The key reason for such behavior is that the enhancement of coupling is restricted to the narrow region of small transfer momenta in the scattering of two (quasi)particles (see below), while for other momenta the coupling remains weak.

Thus, we start with the Lagrangian of Ref. [9],

$$L=\frac{\kappa}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda+\Psi^\dagger\left[i\partial_0+\sigma_zA_0-\epsilon\left(-i\frac{\partial}{\partial\mathbf{x}}-\sigma_z\mathbf{A}\right)\right]\Psi\,, \tag{4}$$

where the statistical coupling κ is taken positive and large, the components of the fermionic doublet $\Psi=(\psi_+,\psi_-)^T$ have opposite charges with respect to the statistical gauge field A, and $\epsilon(\mathbf{k})$ is the (quasi)particle dispersion law below taken to be $\epsilon(\mathbf{k})=\mathbf{k}^2/2m-\epsilon_F$. The statistical gauge interaction is attractive for (quasi)particles of opposite statistical charges with nonzero relative orbital momentum l having the same sign as κ (Refs. [8] and [9] and below). [The attraction exists also for equal charges, but in that case the most long-range part of the potential will be screened by the combined action of Debye and Meissner effects for the statistical field, as explained after Eq. (13) below, and the gap will be suppressed. So, we consider the former case in what follows.] The standard gap equation reads

$$\Delta_{\mathbf{p}}=-\tfrac{1}{2}\int\frac{d\mathbf{p}'}{(2\pi)^2}U_{\mathbf{pp}'}\frac{\Delta_{\mathbf{p}'}}{(\epsilon_{\mathbf{p}'}^2+|\Delta_{\mathbf{p}'}|^2)^{1/2}}\,. \tag{5}$$

Though we have already advocated the necessity to include the RPA bubble corrections in the pairing potential, it is useful to consider first the potential without these corrections. We refer to such a potential as a tree one (the tree gauge propagator is used), as opposed to the renormalized potential which is the one with the above corrections included. In the tree approximation the pairing potential is

$$U(\mathbf{p},\mathbf{p}')=\frac{2i}{\kappa m}\frac{\varepsilon_{ij}p_ip_j'}{(\mathbf{p}-\mathbf{p}')^2}=-\frac{2i}{\kappa m}\frac{\sin\theta}{p/p'+p'/p-2\cos\theta}\,, \tag{6}$$

where $\mathbf{p}=p(\cos\phi,\sin\phi)$, $\mathbf{p}'=p'(\cos\phi',\sin\phi')$, $\theta=\phi-\phi'$. Consider pairing with the orbital momentum l,

$$\Delta_{\mathbf{p}}=\Delta(p)e^{il\phi}\,, \tag{7}$$

where $\Delta(p)$ may be taken real. Then the angle integration in Eq. (5) may be carried out explicitly and yields, for $l\geq1$, the integral equation

$$\Delta(p)=\frac{1}{4\pi\kappa m}\int_0^\infty dp'p'\left[\theta(p'-p)\left(\frac{p}{p'}\right)^l+\theta(p-p')\left(\frac{p'}{p}\right)^l\right]\frac{\Delta(p')}{[\epsilon^2(p')+\Delta^2(p')]^{1/2}}\,. \tag{8}$$

For $l=0$ the potential vanishes due to the antisymmetry, and for $l<0$ the interaction is repulsive.

To find the scaling properties of the gap function with respect to the momentum, it is convenient to deduce the differential equation from Eq. (8),

$$p^2\Delta''+p\Delta'+\left(\frac{l}{2\pi\kappa}\frac{p^2}{m}\frac{1}{[\epsilon^2(p)+\Delta^2]^{1/2}}-l^2\right)\Delta=0\,. \tag{9}$$

At large κ one can neglect the first term in the brackets compared to the second one everywhere except for the nearest vicinity of the Fermi level. Thus, we find the asymptotics of the regular solution,

$$\Delta(p)\simeq\Delta_0(p/p_F)^l,\quad p<p_F\,;$$
$$\Delta(p)\simeq\Delta_0(p_F/p)^l,\quad p>p_F\,, \tag{10}$$

which show that the gap indeed scales nontrivially, and the integral in Eq. (8) converges. Nevertheless, in the tree approximation the magnitude of Δ is exponentially small in $1/\kappa$, by the standard argument. According to our general reasoning, this should imply that the tree approximation is insufficient in the pairing problem, and we have not accounted properly for renormalization of the statistical interaction. We now do so.

The enhancement of the pairing potential is due to the partial cancellation of the bare topological term κ by the induced one [9,13], see Eq. (3). Clearly, for large κ the system prefers pairing with large orbital momenta $l \sim 2\pi\kappa$ [14]. Note that, with the potential of the form Eq. (6), the integral in the gap equation (5) is saturated, at large l, in the narrow region of momenta $|\mathbf{p}'-\mathbf{p}| \lesssim p/l$. As verified *a posteriori* by the solution, see Eq. (2), for $p \approx p_F$ this corresponds to the region of small momenta $\mathbf{q}$ along the gauge line, $v_F|\mathbf{q}| \lesssim \Delta$, precisely where the renormalization of κ takes place. Hence, one can understand the effect of this renormalization by simply substituting κ_{ren} instead of κ in the potential Eq. (6). In this way we obtain

$$\Delta_0 \equiv \Delta(p_F) \sim \frac{\epsilon_F}{l \sinh(2\pi\kappa_{\text{ren}})}, \qquad (11)$$

while the asymptotics Eq. (10) remain unchanged. For small κ_{ren} (which the system chooses itself by adjusting l) the exponential suppression of the gap disappears. As follows from Eqs. (3) and (11), the preferred value of the orbital momentum is the integer part of $2\pi\kappa$, except for a somewhat subtle case $2\pi\kappa=$integer, when the preferred value is actually $l=2\pi\kappa-1$; see discussion after Eq. (13). Also, the calculation of the critical temperature from the equation for the four-fermion vertex in the ladder approximation [15], but now with the RPA improved gauge interaction, yields the essentially unsuppressed value of the same form as Eq. (11), $T_c \sim \Delta_0$. In particular, for $l=2\pi\kappa-1$ (possible when $2\pi\kappa=$integer), neglecting the temperature dependence of κ_{ren}, we have obtained numerically the estimates $\Delta_0 \lesssim \epsilon_F/l$ and $2\Delta_0/T_c \lesssim 2$. The latter ratio is smaller than the BCS value 3.52 [15], though our number (not the order of magnitude) may be changed by the temperature corrections to κ_{ren}.

Now let us see that the previous analysis is indeed consistent, in a sense that all other contributions to the pairing potential, besides the tree-RPA ones, are suppressed by $1/\kappa$. The omitted terms are of two types. First, there are nontree non-RPA contributions to the potential. Second, the RPA graphs in the gauge-field propagator induce other terms bilinear in the gauge field, in addition to the topological term. In fact, both types of contributions are suppressed for one and the same reason. Namely, all these new interactions appear to be short ranged as compared to the Aharonov-Bohm RPA improved one. Let us start with the second type of corrections since, if large, they would enter also non-RPA contributions of the first type. In the Coulomb gauge, the gauge-field polarization operator $\hat{\Pi}$ is a 2×2 matrix in the space (A_0,f) with $A_i=\varepsilon_{ij}\partial_j f$, and, as established previously [9], in the static long-range limit it reads

$$\hat{\Pi}(\mathbf{q}) = \begin{pmatrix} -A\mathbf{q}^2+O(\mathbf{q}^4) & B_1\mathbf{q}^2+B_2\mathbf{q}^4+O(\mathbf{q}^6) \\ B_1\mathbf{q}^2+B_2\mathbf{q}^4+O(\mathbf{q}^6) & C\mathbf{q}^4+O(\mathbf{q}^6) \end{pmatrix}, \qquad (12)$$

where the one-loop values of the coefficients are [9,16]

$$A=\frac{\epsilon_F}{6\pi}\frac{1}{\Delta^2}, \quad B_1=\frac{l}{2\pi}, \quad B_2=-\frac{l}{2\pi}\frac{v_F^2}{12\Delta^2}, \quad C=\frac{1}{4\pi m}\left(\frac{1}{3}+l^2\right). \qquad (13)$$

The coefficient B_1 is the induced topological term entering Eq. (3) [17]. It is seen from Eqs. (12) and (13) that at $l\neq2\pi\kappa$ the pairing interactions mediated by the (00) and (ij) components of the gauge field are both nonsingular in $\mathbf{q}=\mathbf{p}-\mathbf{p}'$, that is they are $1/r^2$ in the coordinate space, while the Aharonov-Bohm interaction is $1/q$, that is $1/r$ [18]. The case $l=2\pi\kappa$ (possible when $2\pi\kappa=$integer) is, however, exceptional. Then, the topological term in the effective action cancels completely, and both the (00) and (ij) components of the gauge propagator behave as $1/\mathbf{q}^2$ for small $\mathbf{q}$. But now their contribution to the gap equation (5) diverges at $\mathbf{p}=\mathbf{p}'$, leading to the absence of any nontrivial solutions. Thus, the system chooses pairing with l close but not equal to $2\pi\kappa$. Note that for the pairing of equal statistical charges the diagonal terms in Eq. (12) would be $O(1)$ (Debye screening) and $O(\mathbf{q}^2)$ (Meissner screening), respectively, so that neither of the propagator components (including the Aharonov-Bohm one) would be long ranged [19]. As for the first type of corrections, the argument essentially follows that of Gell-Mann and Brueckner [20] showing that the non-RPA graphs are at most logarithmic in $|\mathbf{p}-\mathbf{p}'|$.

We now should explain why it is instructive to study the singularity of the various contributions to the potential in Eq. (5) in the limit when $|\mathbf{p}-\mathbf{p}'|$ goes to zero. The point is that, as we have seen above, only the large angular momenta $l\sim\kappa$ are essential in the gap equation, and the Riemann-Lebesgue lemma [21] says that for any integrable function of angle θ between $\mathbf{p}$ and $\mathbf{p}'$, $\lim_{l\to\infty}\int V(\theta)e^{il\theta}d\theta=0$. Hence, all nonsingular (and logarithmic) contributions are suppressed. In fact, this suppression is at least $1/l$ up to logarithms. Note that the suppression takes place also for the tree-RPA result everywhere except the point $|\mathbf{p}|=|\mathbf{p}'|$. But at this point all angular harmonics of the potential indeed come with equal weight, as seen from Eq. (8), and this leads to the uniformly unsuppressed solution for the gap.

In conclusion, we have shown that the presence of long-ranged pairing interaction combined with the nonrenormalization property of certain couplings may lead to new patterns in weak-coupling asymptotics of the gap function and critical temperature. We have developed a self-consistent scheme for the particular case of Aharonov-Bohm ($1/r$) interaction in (2+1) dimensions, the role of expansion parameter being played by the inverse angular momentum of a pair. Though we do not claim any concrete applications in the present Letter, we hope that our results may form a new framework in studies of high-temperature superconductivity.

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