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Color confinement, abelian dominance and the dynamics of magnetic monopoles in SU(3) gauge theory

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Extending earlier work, we investigate the vacuum structure of the pure SU(3) gauge theory in the maximally abelian gauge. In this gauge the long-distance physics is carried essentially by the abelian degrees of freedom. Monopoles show up as particle-like singularities in the gauge fixing procedure. We find evidence that these monopoles condense in the confined phase of the vacuum, in accordance with the picture of a dual superconductor, whereas they are dilute and increasingly static as the temperature increases in the deconfined phase.

1. Introduction

It is important to understand the mechanism of quark confinement and, finally, to derive an effective action that describes the long-distance properties of QCD. This requires, first of all, to identify the dynamical variables that are relevant at the confinement scale and beyond.

It has been conjectured [1,2] that the QCD vacuum is a coherent state of color magnetic monopoles. In this picture color electric charges, i.e. quarks and gluons, are confined by the dual Meissner effect. If this idea is right, the long-distance physics must be carried entirely by the abelian components of the theory, because monopoles are constructed within the maximally abelian (Cartan) subgroup of SU(3), i.e. $U(1)\times U(1)$.

The abelian degrees of freedom are singled out by fixing to a gauge, such that a $U(1)\times U(1)$ gauge freedom (or a $U(1)^{N-1}$ gauge freedom in the general case of gauge group SU(N)) remains, and by subsequent abelian projection [3]. In ref. [4] we have provided the framework for quantitative analysis by constructing the abelian projection on the lattice.

In order that only the abelian components of the theory are relevant at long distances, the gauge must not be accompanied by ghosts that can propagate over macroscopic length scales [3]. Furthermore, the gauge must be manifestly renormalizable. This does not leave much freedom for choosing a gauge. A gauge which satisfies these criteria at least approximately is the abelian gauge [3]. According to an argument by Mandelstam [5], this gauge resembles the Lorentz gauge at short distances and the unitary gauge at large distances. It has ghosts, but they will only propagate over those short distances described by the Lorentz gauge. In ref. [6] we have introduced and employed the maximally abelian gauge for gauge group SU(2) on the lattice. In the classical continuum limit the maximally abelian gauge reduces, by construction, to the abelian gauge. It has been shown that the abelian gauge is renormalizable [7].

In the pure SU(2) gauge theory we have found evidence for the presence of monopoles in the underlying vacuum [6]. The qualitative agreement of the results with those of the pure compact U(1) gauge theory, which is well understood [8], suggests that the color magnetic monopoles condense in the confined phase in accord with the picture of a dual superconductor, whereas they form a Coulomb gas in

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the high temperature, deconfined phase.

This work has led Suzuki and Yotsuyanagi [9] to examine the question of abelian dominance and the choice of gauge from an independent point of view. They simulated the pure SU(2) gauge theory on the lattice, transformed the gauge field configurations to the maximally abelian gauge and performed the abelian projection as described in refs. [4,6]. Then they investigated large Wilson loops composed of the residual U(1) gauge fields and computed the associated string tension K. They found not only that K scales according to the gauge group SU(2), but also that $\sqrt{K} \approx 58\Lambda_L$, which is in agreement with the SU(2) result [10]. Furthermore, they found a much improved signal-to-noise ratio. This is what one expects if the picture outlined above is correct. Other, non-renormalizable gauges they have investigated did not show this effect.

Motivated by this success, we shall extend our earlier work [6] to the more realistic case of gauge group SU(3). Here we have two different types of "photons" and monopoles, which require special attention. Because in SU(3) the deconfining phase transition is of first order, we also expect to find a more unambiguous signal of monopole condensation in this case.

2. Gauge fixing and abelian projection

We consider a hypercubic lattice and take the standard Wilson action

$$S=\beta \sum_{\square} [1-\tfrac{1}{3}\,\mathrm{Re\,Tr}\,U(\square)]\,, \tag{2.1}$$

where $U(\square)$ is the product of parallel transporters $U(s,\hat{\mu})$ around a plaquette $\square$. The boundary conditions are chosen to be periodic.

Under gauge transformations the parallel transporters transform as

$$U'(s,\hat{\mu})=g(s)U(s,\hat{\mu})g(s+\hat{\mu})^{-1}\,. \tag{2.2}$$

The maximally abelian gauge is obtained by performing a local gauge transformation,

$$\tilde{U}(s,\hat{\mu})=V(s)U(s,\hat{\mu})V(s+\hat{\mu})^{-1}\,, \tag{2.3}$$

such that the quantity

$$R=\sum_{s,\mu} [\,|\tilde{U}_{11}(s,\hat{\mu})|^2+|\tilde{U}_{22}(s,\hat{\mu})|^2+|\tilde{U}_{33}(s,\hat{\mu})|^2] \tag{2.4}$$

is maximized. In the classical continuum limit this condition reduces to the local gauge condition

$$\partial_\mu A_\mu^{ij}-\mathrm{i}(A_\mu^{ii}A_\mu^{ij}-A_\mu^{ij}A_\mu^{jj})=0\,, \quad i\neq j\,, \tag{2.5}$$

where $A_\mu^{ij}=\lambda_a^{ij}A_\mu^a$ are the gauge fields. Eq. (2.5) specifies the abelian gauge introduced by 't Hooft [3]. This determines $V(s)$ only up to left multiplication by

$$\begin{aligned} d(s)&\\ &=\mathrm{diag}\{\exp[\mathrm{i}\alpha_1(s)],\exp[\mathrm{i}\alpha_2(s)],\exp[\mathrm{i}\alpha_3(s)]\}\\ &\in U(1)\times U(1)\,, \end{aligned} \tag{2.6}$$

where

$$\alpha_1(s)+\alpha_2(s)+\alpha_3(s)=0\,. \tag{2.7}$$

Thus, under general SU(3) gauge transformations $g(s)$ the parallel transporters in the maximally abelian gauge, $\tilde{U}(s,\hat{\mu})$, transform as

$$\tilde{U}'(s,\hat{\mu})=d(s)\tilde{U}(s,\hat{\mu})d(s+\hat{\mu})^{-1}\,, \tag{2.8}$$

i.e. they transform under the residual U(1)×U(1) gauge group. To extract matter fields $c(s,\hat{\mu})$ and abelian parallel transporters $u(s,\hat{\mu})$ [4] from $\tilde{U}(s,\hat{\mu})$, we perform a coset decomposition with respect to the subgroup U(1)×U(1),

$$\tilde{U}(s,\hat{\mu})=c(s,\hat{\mu})u(s,\hat{\mu})\,, \tag{2.9}$$

such that •

$$\begin{aligned} c'(s,\hat{\mu})&=d(s)c(s,\hat{\mu})d(s)^{-1}\,,\\ u'(s,\hat{\mu})&=d(s)u(s,\hat{\mu})d(s+\hat{\mu})^{-1}\\ &\in \mathrm{U}(1)\times\mathrm{U}(1)\,. \end{aligned} \tag{2.10}$$

This is achieved by defining

$$\begin{aligned} u(s,\hat{\mu})&=\mathrm{diag}[u_1(s,\hat{\mu}),u_2(s,\hat{\mu}),u_3(s,\hat{\mu})]\,,\\ u_i(s,\hat{\mu})&=\exp\{\mathrm{i}\arg[\tilde{U}_{ii}(s,\hat{\mu})]-\tfrac{1}{3}\mathrm{i}\varphi(s,\hat{\mu})\}\,,\\ \varphi(s,\hat{\mu})&=\sum_i \arg[\tilde{U}_{ii}(s,\hat{\mu})]\,|_{\mathrm{mod}\,2\pi}\in(-\pi,\pi]\,. \end{aligned} \tag{2.11}$$

In components eq. (2.8) reads

$$u'_i(s,\hat{\mu}) = \exp[\mathrm{i}\alpha_i(s)]\, u_i(s,\hat{\mu})\, \exp[-\mathrm{i}\alpha_i(s+\hat{\mu})]\,,$$
$$c'_{ij}(s,\hat{\mu}) = \exp[\mathrm{i}\alpha_i(s) - \mathrm{i}\alpha_j(s)]\, c_{ij}(s,\hat{\mu})\,. \qquad (2.12)$$

The $u_i(s,\hat{\mu})$ represent two "photons" (to be exact, three "photons" with one constraint) and the $c_{ij}(s,\hat{\mu})$ describe six charged "gluons". In total there are eight degrees of freedom as it should be for gauge group SU(3).

As the maximally abelian subgroup U(1)×U(1) is compact, there are, in addition, topological excitations. These are the color magnetic monopoles, which manifest themselves as half-integer valued magnetic currents on the links of the dual lattice [4]:

$$m_i({}^*s,\hat{\mu}) = \frac{1}{4\pi} \sum_{\square \in \partial f(s+\hat{\mu},\hat{\mu})} \arg u_i(\square)$$
$$= 0,\ \pm\tfrac{1}{2},\ \pm 1\,, \qquad (2.13)$$

where $u_i(\square)$ is the product of abelian parallel transporters $u_i(s,\hat{\mu})$ around a plaquette $\square$, and $f(s+\hat{\mu},\hat{\mu})$ is an elementary cube perpendicular to the μ-direction with origin $s+\hat{\mu}$. The phases $\arg u_i(\square)$ must be chosen such that

$$\sum_i \arg u_i(\square) = 0\,,$$
$$|\arg u_i(\square) - \arg u_j(\square)| \leqslant 2\pi\,. \qquad (2.14)$$

Because of this constraint

$$\sum_i m_i({}^*s,\hat{\mu}) = 0\,. \qquad (2.15)$$

That means there are only two independent types of monopoles. Each magnetic current is conserved at $^*s$:

$$\sum_\mu [m_i({}^*s,\hat{\mu}) - m_i({}^*s-\hat{\mu},\hat{\mu})] = 0\,. \qquad (2.16)$$

As a consequence, the magnetic currents of each type form closed loops on the dual lattice. The Dirac quantization condition is topologically expressed by

$$\Pi_2\{\mathrm{SU}(3)/[\mathrm{U}(1)\times\mathrm{U}(1)]\} = \Pi_1[\mathrm{U}(1)\times\mathrm{U}(1)]$$
$$= \mathbb{Z}^2\,. \qquad (2.17)$$

3. Monopole density below and above the phase transition

We have performed simulations on $8^3\times4$ and $12^3\times4$ lattices at various values of β ranging from $\beta=5.4$ to $\beta=6.2$. We use a Metropolis algorithm for updating the system. On these lattices the deconfining phase transition takes place at $\beta\approx5.67$ (see also below). The gauge fixing is done by an iterative procedure. We maximize R in eq. (2.4) by letting V run through the various SU(2) subgroups of SU(3). After each iteration we perform a microcanonical overrelaxation step, which leaves R unchanged. We need about 1000 iterations altogether until the gauge fixing procedure has converged. This restricts our investigations to smaller lattices at the moment.

In figs. 1a–1c we show a two-dimensional perspective projection of the monopole currents for typical gauge field configurations at $\beta=6.1$, 5.8 and 5.5, respectively. Because of the constraint (2.15) each monopole is accompanied by an antimonopole of a different type. In fig. 1a we have displayed the different types of monopoles. This allows, unlike SU(2), for monopole contact interactions as indicated by the eight-link monopole loop. In figs. 1b, 1c the different types of monopoles are not distinguished. In the deconfined phase (figs. 1a, 1b) the monopole loops are small. At higher temperatures (fig. 1a) the fraction of approximately static loops (that close over the temporal boundary) increases. In the confined phase (fig. 1c) we find a dense state of long entangled monopole loops. We have verified that most of these currents belong to a single connected loop, which suggests that the monopoles condense in this phase.

To substantiate these result, we have computed the perimeter density of the monopole loops,

$$l_i = \frac{1}{4V} \sum_{{}^*s,\mu} |m_i({}^*s,\mu)|\,. \qquad (3.1)$$

Note that $\langle l_i\rangle$ is the same for all $i=1$, 2 and 3. In the following we shall therefore average over the different monopole types and drop the index i. Before we attribute any significance to this quantity now, let us briefly discuss the effect of dislocations [11,12]. The minimal action for an elementary monopole loop is known to be [12]

$$S_{\min} = 4.52\beta \qquad (3.2)$$

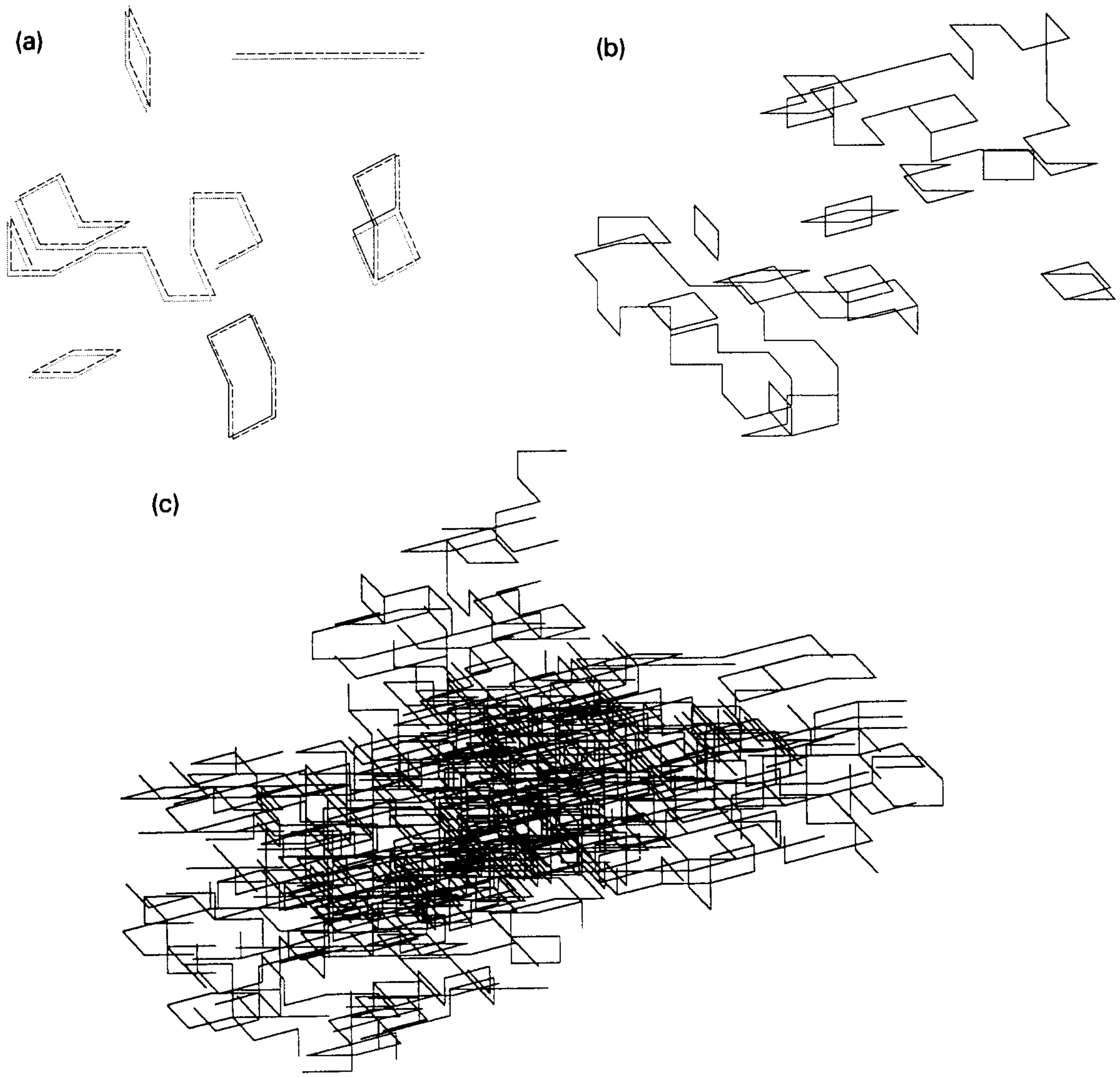

Fig. 1. Two-dimensional perspective projection of the monopole currents on the $8^3\times 4$ lattice. Apparently open loops are in fact closed due to the periodic boundary conditions. (a) $\beta=6.1$. The different types of monopoles are marked by solid, dashed and dotted lines, respectively. The time direction is in the horizontal direction. (b) $\beta=5.8$. In this and the next figure it is not distinguished between the various types of monopoles, and we try to show long loops in their entirety and thereby occasionally leave the lattice. (c) $\beta=5.5$.

for the Wilson action. In ref. [13] it has been argued that in the confined phase $\langle l\rangle$ scales like a^3, where a is the lattice spacing, which is possibly correct [#1]. Thus, the minimal action is to be compared with the exponent of the asymptotic decay of a^3, i.e.

$$a^3 \propto \exp(-\tfrac{4}{11}\pi^2\beta)\,. \tag{3.3}$$

Because $4.52>\frac{4}{11}\pi^2$, we conclude that in SU(3) $\langle l\rangle$ is not affected by dislocations in the continuum limit. In the deconfined phase there are several competing mechanisms with different scaling laws, which we will discuss in more detail below. The dominant contribution to $\langle l\rangle$ at large values of β is also expected to scale like a^3, and there are indications that this is indeed the case.

[#1] Note that $\langle l\rangle$ is a dimensionless quantity, which can be interpreted as the number of monopoles per three-dimensional lattice volume. Unfortunately, we cannot test this statement here because the temporal extent of our lattices is too small for that.

In fig. 2 we show $\langle l \rangle$ as a function of β. Each entry is based on 100 (well equilibrated) gauge field configurations separated by 15 Monte Carlo sweeps, except for the data within $\Delta\beta = 0.015$ of the critical value of β. The latter is obtained from either the next lower or higher value of β after applying 100 Monte Carlo sweeps for equilibration. This data is based on 50 gauge field configurations separated by 15 Monte Carlo sweeps. It is drawn on a larger scale in fig. 3, where the triangles (squares) are found by stepping up (down) in β. The curves are freehand drawings through the data points and are meant to guide the eye only. The data show a clear hysteresis effect, which suggests a first order phase transition of the monopole state of matter from a Coulomb gas phase at high temperatures to a monopole condensate (liquid) at zero temperature. It also shows that color magnetic monopoles are a genuine feature of the Yang–Mills vacuum and not a lattice artifact. From fig. 3 we read off the critical coupling of $\beta_c \approx 5.67$. This is in agreement with the value found from the investigation of Polyakov loops on lattices of the same temporal extent [14].

Let us go back to fig. 2 now. It appears that $\langle l \rangle$ decreases exponentially with β for $\beta \geq 5.8$. In ref. [6],

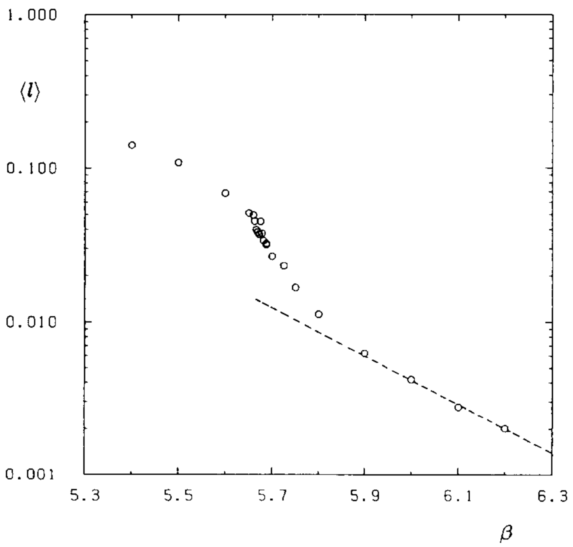

Fig. 2. The perimeter density $\langle l \rangle$ as a function of β. The data for $\beta \leq 5.9$ are taken on $8^3 \times 4$ lattices, whereas the data for $\beta \geq 6.0$ are taken on $12^3 \times 4$ lattices. The errors are smaller than the symbols. The dashed line shows the β dependence of a^3.

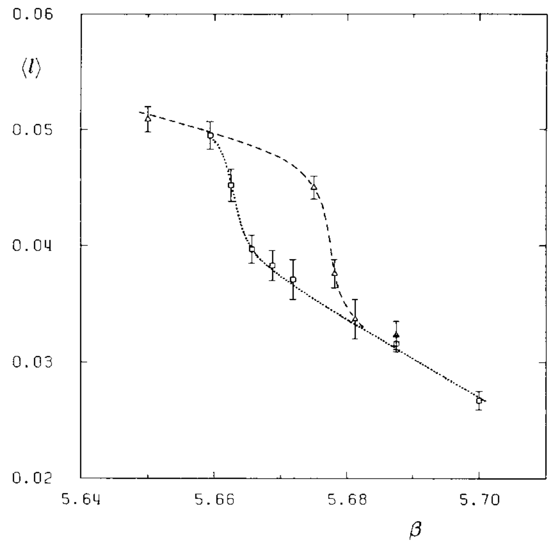

Fig. 3. The perimeter density $\langle l \rangle$ as a function of β in the vicinity of the deconfining phase transition. The triangles (squares) denote the values of $\langle l \rangle$ found while increasing (decreasing) the temperature. The curves are freehand drawings through the data points.

i.e. in the case of SU(2), we have attributed some significance to the slope of the exponential decrease. We found that it was compatible with $-\pi^2$, which is the result of a dilute monopole gas in the U(1) theory. In the U(1)×U(1) theory the slope would be $-\frac{2}{3}\pi^2$. This is also what a study of the SU(3) vacuum at finite temperature [15] would suggest, which yielded a monopole action of $S = \frac{2}{3}\pi^2\beta$. But this is not what we observe at large values of β. We have found evidence already that the monopoles become static at high temperatures. In fig. 4 we show the ratio of spatial to temporal monopole currents,

$$Q = \frac{1}{3} \frac{\langle \sum_{*s,\mu=1,2,3} |m_i(^*s, \hat{\mu})| \rangle}{\langle \sum_{*s} |m_i(^*s, \hat{4})| \rangle}, \tag{3.4}$$

as a function of β. This quantity will be zero for static monopoles. We find that Q starts to decrease rapidly above the deconfining phase transition. Approximately static monopole loops that wind around the lattice will give rise to the scaling behavior $\langle l \rangle \propto a^3$. Note that these monopoles are responsible for the observed area-law behavior of spacelike Wilson loops at high temperatures [16]. Hence, we expect the perim-

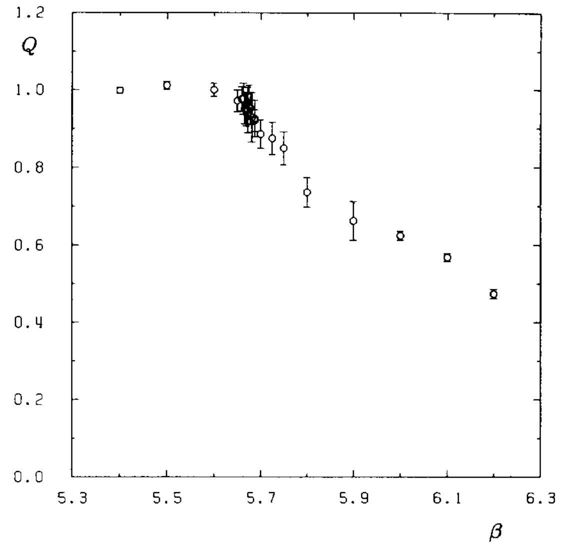

Fig. 4. The ratio Q of spatial to temporal monopole currents as a function of β.

eter density $\langle l \rangle$ to fall off eventually like [#2] $\exp(-\frac{4}{11}\pi^2\beta)$ (cf. eq. (3.3)). Such a behavior, which is indicated by the dashed line in fig. 2, is supported by our data.

We have also considered partial abelian gauge fixing such that a $SU(2)\times U(1)$ gauge freedom remains. In this case there is only one type of monopole associated with the (one) $U(1)$ subgroup. We found quantitatively the same results for the perimeter density as in the $U(1)\times U(1)$ case. This shows that $U(1)\times U(1)$ is in fact the subgroup relevant for confinement.

4. Conclusions

We have extended our earlier work to the case of gauge group SU(3) and found, also in this case, support for the dual superconductor picture of confinement. In particular the hysteresis effect found in fig. 3 may be considered as evidence for the condensation of magnetic monopoles in the confined phase. These results pave the way for the construction of an infrared-effective action. The main building blocks will be the abelian gauge fields and perhaps their dual counterparts. First promising attempts in this direction can already be found in the literature [17,18].

At high temperatures the only significant contribution to the functional integral comes from the neighborhood of configurations, which are static modulo gauge transformations [19], so that the spatial and temporal components of the gauge fields decouple. The gauge conditions then lead to $\tilde{U}(s,4)=$diagonal and $A_4=\lambda_3 A_4^3+\lambda_8 A_4^8$ ($\neq 0$), respectively. This means that the monopoles become (semiclassical) 't Hooft–Polyakov monopoles in this limit. As 't Hooft–Polyakov monopoles are experimentally observable, it would be interesting to work out their signatures and to search for them in forthcoming heavy-ion experiments.

[#2] This was also assumed in ref. [17].

References

[1] G. 't Hooft, in: High energy physics, in: Proc. EPS Intern. Conf. (Palermo, 1975), ed. A. Zichichi (Editrice Compositori, Bologna, 1976).
[2] S. Mandelstam, Phys. Rep. 23 (1976) 245.
[3] G. 't Hooft, Nucl. Phys. B 190 (1981) 455.
[4] A.S. Kronfeld, G. Schierholz and U.-J. Wiese, Nucl. Phys. B 129 (1987) 493.
[5] S. Mandelstam, in: Monopoles in quantum field theory, Proc. Monopole Meeting (Trieste, 1981), eds. N.S. Craigie, P. Goddard and W. Nahm (World Scientific, Singapore, 1982).
[6] A.S. Kronfeld, M.L. Laursen, G. Schierholz and U.-J. Wiese, Phys. Lett. B 198 (1987) 516.
[7] H. Min, T. Lee and P.Y. Pac, Phys. Rev. D 32 (1985) 440.
[8] T. Banks, R. Myerson and J. Kogut, Nucl. Phys. B 129 (1977) 493;
T.A. DeGrand and D. Toussaint, Phys. Rev. D 22 (1980) 2478;
J.L. Cardy, Nucl. Phys. B 170 (1980) 369;
L. Polley and U.-J. Wiese, HLRZ preprint HLRZ 90-68 (1990).
[9] T. Suzuki and I. Yotsuyanagi, Phys. Rev. D 42 (1990) 4257.
[10] C. Michael and M. Teper, Phys. Lett. B 199 (1987) 95.
[11] D.J.R. Pugh and M. Teper, Phys. Lett. B 224 (1989) 159.
[12] M. Göckeler, A.S. Kronfeld, M.L. Laursen, G. Schierholz and U.-J. Wiese, Phys. Lett. B 233 (1989) 192.
[13] V.G. Bornyakov, E.-M. Ilgenfritz, M.L. Laursen, V.K. Mitrjushkin, M. Müller-Preussker, A.J. van der Sijs and A. Zadorozhny, Phys. Lett. B 261 (1991) 116.
[14] N.H. Christ and A.E. Terrano, Phys. Rev. Lett. 56 (1986) 111;
P. Bacilieri et al., Phys. Rev. Lett. 61 (1988) 1545;
F. Brown, N.H. Christ, Y. Deng, M. Gao and T.J. Woch, Phys. Rev. Lett. 61 (1988) 2058;

M. Fukugita, M. Okawa and A. Ukawa, Phys. Rev. Lett. 63 (1989) 1768; Nucl. Phys. B 337 (1990) 181.
[15] M.L. Laursen and G. Schierholz, Z. Phys. C 38 (1988) 501.
[16] E. Manousakis and J. Polonyi, Phys. Rev. Lett. 58 (1987) 847.
[17] J. Smit and A.J. van der Sijs, Amsterdam preprint ITFA-89-17 (1989).
[18] S. Maedan and T. Suzuki, Prog. Theor. Phys. 81 (1989) 229;
T. Suzuki, Prog. Theor. Phys. 81 (1989) 752.
[19] L.G. Yaffe and B. Svetitsky, Phys. Rev. D 26 (1982) 963.