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A mechanism generating axion hair for Kerr black holes

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Abstract. Recently Campbell *et al* considered Kalb-Ramond axion fields with a Lorentz Chern-Simons coupling to gravity. They find that in the background metric of a slowly rotating Kerr black hole the axion field strength develops a long range scalar 'hair'. We show that the same phenomenon also occurs with 'ordinary' axions, which are not related to form-valued fields. If these scalars couple to the divergence of some anomalous axial vector current, the Hirzebruch signature density acts as a source for the axion. In a Kerr background this leads to an asymptotic $1/r^2$ fall off of the axion field, i.e. to a scalar 'hair'. Because of the chiral anomaly of the photon (a local version of the signature index theorem), this can happen even in purely bosonic field theory models.

The 'no hair' conjecture of black hole physics states that the only long range fields present in exterior black hole solutions are the field strengths associated with local gauge invariances. In this way general coordinate invariance and electromagnetic gauge invariance, respectively, are related to the mass, angular momentum and charge of a black hole. These parameters (and, according to the conjecture, only these) can be measured at spatial infinity. The 'no hair' conjecture has been proven in a number of special cases, but there exists no general proof which would apply to an arbitrary matter field sector. For pure gravity it is known that the Kerr metric is the unique stationary exterior solution, and similarly for gravity coupled to a $U(1)$ gauge theory the Kerr-Newman solution is the only possibility. It describes a rotating black hole of definite mass, angular momentum and charge. In particular, for vanishing angular momentum one obtains the Schwarzschild and the Reissner-Nordström spacetimes, respectively. As for minimally coupled boson fields, it has been shown that exterior black hole solutions cannot contain 'hairs' due to scalars or to massive spin-1 and spin-2 bosons. Also minimally coupled spin-$\frac{1}{2}$ fermions have been ruled out. Furthermore, models involving non-Abelian gauge fields or non-minimally coupled scalars were shown to possess some novel solutions which, however, turned out to be unstable against small perturbations. They are assumed to decay into the Schwarzschild solution. (See [1] for a detailed list of references.)

Recently Campbell *et al* [1] considered Kalb-Ramond-type axion fields with a Lorentz Chern-Simons coupling to gravity. In the background of a Kerr black hole the Chern-Simons term provides a source for the axion field, which is found to give rise to a long range ($\propto r^{-2}$) axion hair for the field strength. It appears that this is the first known example of a stable non-gauge hair of a black hole. The kind of axion considered in [1] naturally appears in the graviton multiplet of string theory (for a review, see [2]). Coupling bosonic or superstrings to external Yang-Mills and gravitational fields, for instance, anomaly cancellation requires the presence of a 2-form field

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B whose (gauge invariant) field strength [3]

$$H = \mathrm{d}B + \omega_{\mathrm{L}} - \omega_{\mathrm{YM}} \tag{1}$$

contains the Lorentz and the Yang–Mills Chern–Simons 3-forms ω_{L} and ω_{YM}, respectively. Under a gauge transformation B transforms according to the Chapline–Manton transformation law. In this context, Chern–Simons terms were first encountered in coupling the super-Yang–Mills multiplet to ten-dimensional supergravity [4]. The pseudoscalar axion field $a(x)$ arises as follows. The action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{16\pi}R - H_{\mu\nu\rho}H^{\mu\nu\rho}\right) \tag{2}$$

for the coupled axion–graviton system implies the equation of motion $\mathrm{d}({}^*H) = 0$ for the field strength 3-form H. Thus, at least locally, ${}^*H$ can be represented by the exterior derivative of a scalar, the axion: ${}^*H = \mathrm{d}a$. Setting the Yang–Mills connection to zero, equation (1) implies the Bianchi identity $\mathrm{d}H = \mathrm{d}\omega_{\mathrm{L}} = \mathrm{Tr}(R \wedge R)$, where R is the curvature 2-form. Thus the equation of motion for a becomes

$$D_\mu D^\mu a(x) = -\frac{1}{4!}\frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}R_{\alpha\beta\mu\nu}R^{\alpha\beta}{}_{\rho\sigma}. \tag{3}$$

Campbell *et al* [1] have investigated black hole solutions of equation (3), together with Einstein's equation, in a perturbative manner. For slowly rotating black holes it is sufficient to solve equation (3) in a fixed Kerr background metric, and to ignore the backreaction of the axion on the metric. To first order in the angular momentum parameter A one finds that for $r \gg 2M$ the axion field falls off as $a \propto Ar^{-2}$, i.e. for $A \neq 0$ the black hole has a long range 'axion hair'.

The purpose of this paper is to point out that the phenomenon of long range axion fields surrounding Kerr black holes is much more general than one might expect from the above derivation. The scalar $a(x)$ was introduced in order to represent the 3-form field strength of a 2-form field; for the effect to occur it was crucial that the field strength contains a Chern–Simons term whose presence was motivated by anomaly cancellation in string theory. In the following we shall see that the same kind of scalar hair can also be obtained from a conventional pseudoscalar $\phi(x)$ which is not related to any form-valued field, but which is required to couple to the photon field A_μ via $\phi F_{\mu\nu}{}^*F^{\mu\nu}$. This coupling is inspired by the standard QCD axion [5], but for our purposes it is sufficient to consider the following toy model:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left\{-\frac{1}{16\pi}R + \tfrac{1}{2}D_\mu\phi D^\mu\phi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}\lambda\phi F_{\mu\nu}{}^*F^{\mu\nu}\right\}. \tag{4}$$

To obtain an axion hair, we may treat the metric and the scalar as classical fields, but we have to take the (1-loop) vacuum fluctuations of A_μ into account. (At this level the non-renormalizability of the model is inessential.) The essential ingredient of our argument is the bosonic chiral anomaly discovered by Dolgov, Khriplovich and Zakharov [6]. These authors have shown that when an Abelian gauge field is quantized in the background of a curved spacetime for which the Hirzebruch signature density $\varepsilon^{\mu\nu\rho\sigma}R_{\alpha\beta\mu\nu}R^{\alpha\beta}{}_{\rho\sigma}$ does not vanish, the vacuum expectation value of the chiral current

$$K^\mu = \frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}A_\nu D_\rho A_\sigma \tag{5}$$

is not conserved, and the pseudoscalar $F_{\mu\nu}{}^*F^{\mu\nu}$ acquires a vacuum expectation value:

$$D_\mu\langle K^\mu\rangle \equiv \langle \tfrac{1}{2}F_{\mu\nu}{}^*F^{\mu\nu}\rangle = \frac{1}{192\pi^2}\frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}R_{\alpha\beta\mu\nu}R^{\alpha\beta}{}_{\rho\sigma}. \tag{6}$$

Equation (6) expresses the fact that quantum effects spoil the invariance of the classical theory under duality rotations [7]. For a free electromagnetic field this symmetry causes the difference of the numbers of right and left circularly polarized photons to be conserved. Hence, if the RHS of equation (6) is non-zero, the gravitational field produces (chiral) photons from the vacuum. This is analogous to the anomalous fermion pair creation by Yang–Mills fields as expressed by the relation $\Delta Q_5 = (1/8\pi^2)\int d^4x\, \mathrm{tr}(F_{\mu\nu}{}^*F^{\mu\nu})$. Here Q_5 is the chiral charge. In the case of the photon the chiral charge density $\psi^+\gamma_5\psi$ is replaced by K^0. Its space integral equals +1 (−1) for right- (left-) handed photons. It can be shown [7] that equation (6) is a local version of the signature index theorem, and that similar anomalies also exist in higher dimensions.

From (4) we obtain the field equations

$$D_\mu D^\mu\phi = \tfrac{1}{2}\lambda F_{\mu\nu}{}^*F^{\mu\nu} \tag{7a}$$

$$D_\mu F^{\mu\nu} = \lambda\frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}D_\mu\phi F_{\rho\sigma} \tag{7b}$$

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = -8\pi\{T_{\mu\nu}(\phi) + T_{\mu\nu}(A)\}. \tag{7c}$$

We are interested in solutions for which the metric is close to the Kerr metric. We also make the assumption that the black hole is rotating very slowly, i.e. that it is sufficient to retain only terms linear in the angular momentum parameter A. In this limit the Kerr metric in Boyer–Lindquist coordinates becomes

$$ds^2 = \left(1-\frac{2M}{r}\right)dt^2 - \left(1-\frac{2M}{r}\right)^{-1}dr^2 - r^2\{d\theta^2 + \sin^2\theta\, d\phi^2\}$$
$$-\frac{4MA\sin^2\theta}{r}dt\,d\phi + \ldots. \tag{8}$$

It gives rise to the Hirzebruch signature density

$$\frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}R_{\alpha\beta\mu\nu}R^{\alpha\beta}{}_{\rho\sigma} = 1152\frac{M^2A\cos\theta}{r^7} + O(A^2) \tag{9}$$

which vanishes for the Schwarzschild solution ($A=0$). Since we quantize only A_μ but keep ϕ and $g_{\mu\nu}$ as classical fields we have to replace ($7a$) by

$$D_\mu D^\mu\phi = \lambda\langle\tfrac{1}{2}F_{\mu\nu}{}^*F^{\mu\nu}\rangle = \frac{\lambda}{192\pi^2}\frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}R_{\alpha\beta\mu\nu}R^{\alpha\beta}{}_{\rho\sigma}. \tag{10}$$

This is essentially the same as equation (3) for the Kalb–Ramond axion $a(x)$. Because of the bosonic chiral anomaly of A_μ the Hirzebruch signature density acts as a source for ϕ. To first order in A, a solution of equations ($7a$) and ($7c$) is given by the metric (8) together with a scalar field solving the equation

$$D_\mu D^\mu\phi = \frac{6\lambda}{\pi^2}\frac{M^2A\cos\theta}{r^7} \tag{11}$$

which follows from (10) with (9). This approximation is consistent because to lowest order the backreaction of ϕ and A_μ on the metric may be ignored. Similarly, the RHS of equation (7*b*) can be neglected so that the photon couples to gravity only. (This was assumed in the derivation of (6).) In order to solve equation (11) it is sufficient to construct the Green function $(D_\mu D^\mu)^{-1}$ for a Schwarzschild background since the source term is already of order $O(A)$. One finds [1]

$$\phi(r,\theta)=-\frac{5\lambda}{384\pi^2}\frac{A\cos\theta}{M^3}\left(\frac{4M^2}{r^2}+\frac{8M^3}{r^3}+\frac{72M^4}{5r^4}\right)=-\frac{5\lambda}{96\pi^2}\frac{A}{M}\frac{\cos\theta}{r^2}+O\left(\frac{1}{r^3}\right). \tag{12}$$

Apart from the prefactor on the RHS of equation (12) this is the same kind of scalar 'hair' as the one found in [1] for the Kalb–Ramond axion $a(x)$. The scalar field shows a long range (r^{-2}) fall off, and a dipole form which implies that the total axion charge of the black hole vanishes, however. For a further discussion we refer to [1].

Up to now we only considered the coupling of the scalar to the divergence of the photonic chiral current K^μ. Similarly we could use an interaction of the form $\lambda'\phi\partial_\mu j_5^\mu$ where $j_5^\mu\equiv\bar\psi\gamma^\mu\gamma_5\psi$ is the axial vector current of some massless fermion. Now $\partial_\mu j_5^\mu$ replaces $F_{\mu\nu}{}^*F^{\mu\nu}$ as the source for ϕ. In the presence of gravitational and electromagnetic fields we have the usual (fermionic) chiral anomaly so that the analogue of equation (10) reads

$$D_\mu D^\mu\phi=\lambda' D_\mu\langle j_5^\mu\rangle=\frac{\lambda'}{384\pi^2}\frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}R_{\alpha\beta\mu\nu}R^{\alpha\beta}{}_{\rho\sigma}-\frac{\lambda' e^2}{8\pi^2}\langle F_{\mu\nu}{}^*F^{\mu\nu}\rangle. \tag{13}$$

Even if the fermion does not couple to A_μ, $e=0$, equation (13) contains the Hirzebruch signature density. Therefore, in a Kerr background, the solution for $\phi(x)$ is again of the form (12). If $e\neq 0$, the expectation value $\langle F_{\mu\nu}{}^*F^{\mu\nu}\rangle$ has to be taken from equation (6). In this way the bosonic chiral anomaly leads to a finite renormalization of the coefficient in front of the signature density:

$$D_\mu D^\mu\phi=\frac{\lambda'}{384\pi^2}\left(1-\frac{e^2}{2\pi^2}\right)\frac{\varepsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}R_{\alpha\beta\mu\nu}R^{\alpha\beta}{}_{\rho\sigma}. \tag{14}$$

(See also reference [8] for a discussion of this 2-loop result which amounts to a 'reiteration of anomalies'.) Thus the solution for $\phi(x)$ changes by an overall factor $(1-e^2/2\pi^2)$.

This completes our demonstration that 'ordinary' scalars, unrelated to *p*-form fields, can also give rise to a scalar hair for Kerr black holes provided they have a coupling $\propto\phi\partial_\mu J_5^\mu$ to some axial vector current J_5^μ which develops an anomaly when coupled to gravity. As we have stressed, the standard fermionic anomaly of $J_5^\mu=j_5^\mu$ is not the only possibility here. Even a purely bosonic theory involving only axions and photons has this feature, because the axial vector current $J_5^\mu=K^\mu$ of the photon is also anomalous.

We close with a few remarks. In the model discussed in this paper, as well as in the string-inspired theory of [1], the axion hair is due to the ϕR^*R coupling of the scalar to the (background) geometry. In the former case this coupling was the result of the photonic chiral anomaly, in the latter it was due to the Chern–Simons term in the classical Lagrangian. In either case the Hirzebruch signature density R^*R acts as a spatially extended source for the axion: $D^2\phi\propto R^*R$. This means that the scalar 'charge' responsible for the axion field is not really localized on the black hole itself, but rather in the gravitational field surrounding it. In this respect the axionic charge is different from the more conventional electric charge, say. This difference makes it plausible that we can have axion hair even if the integrated axion charge density is zero. The Hirzebruch density is not the only possible axionic source term; the same role could also be played by $F_{\mu\nu}{}^*F^{\mu\nu}$: $D^2\phi \propto F^*F$. In fact, recently it has been shown [9, 10] that the field equations (7) of the model (4) possess solutions with a long range axion hair even at the classical level. (Recall that our mechanism is a quantum effect.) In order to make the RHS of equation (7*a*) non-zero, the black hole must carry both electric and magnetic charge so that $F^*F \sim \boldsymbol{E}\cdot\boldsymbol{B} \neq 0$. For such a 'dyonic' black hole the radial electric and magnetic fields surrounding it provide the source for the axion field. In this case it turns out that the integrated axion charge is non-zero. It is not a free parameter, however, but is uniquely determined by the electric and the magnetic charge [9, 10]. (For a recent discussion of magnetically charged black holes see [11] and the references therein.)

In our discussion, in a slight abuse of language, we used the term 'axion' generically for any pseudo-scalar with a ϕF^*F coupling to the photon. This includes the neutral pion, for example, whose effective πF^*F interaction is responsible for its 2γ-decay. In nature (and in many phenomenological models) these pseudoscalars are typically massive, however. In solving equation (11) we assumed the scalar to be exactly massless. For a massive particle the solution (12) would still be valid as long as r is much smaller than the Compton wavelength λ of the particle. On the other hand, for $r \gg \lambda$, the axion field would die out much faster, essentially as rapidly as the source term R^*R. Thus, for the pseudoscalar halo surrounding a rotating black hole to be of any phenomenological significance, the pseudo-scalars should be as light as possible, because only then can long range hair develop. For the standard QCD axion, say, λ is of the order of a few centimetres, so that only very small black holes could have a genuine 'halo'. But even if we think of some even lighter pseudoscalar particle, the experimental detection of the axion hair seems to be extremely difficult. In principle the axion halo manifests itself by the following effect, for instance. Assume there is some astrophysical synchrotron source of linearly polarized photons. Because of the ϕF^*F coupling, the plane of polarization of these photons will be rotated whenever they propagate through a region of spacetime where ϕ is non-constant. (This happens even in flat space. Cosmological implications have been considered in [12] recently.) Thus, on their way to the earth, the photons emitted by the synchrotron source could experience a rotation of their plane of polarization when they pass through the pseudoscalar halo of some very massive, rotating object. Unfortunately it would be very difficult to separate this effect from the one due to the direct gravitational interaction of the photon, however. Finally we mention that this effect would not occur for the type of axionic black holes discussed by Bowick *et al* [13]. They are characterized by a vanishing 3-form field strength, but a non-vanishing 2-form potential. (This is possible because of the non-trivial topology $R^2 \times S^2$ of the Schwarzschild spacetime.) The solutions have non-zero axion charge. It cannot be detected by any point particle, but can give rise to a topological phase in Aharonov–Bohm-type experiments with strings [13]. Contrary to the situation above this effect persists even if the axion has a (topological) mass term [14].

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