Neutrino masses, neutral vector bosons and the scale of $B-L$ breaking

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Extensions of the standard model with extra neutral vector bosons predict additional fermions whose allowed masses are constrained by the masses of the new vector bosons. For the minimal case with one extra $Z'$ boson and an extended gauge group contained in the unified group $SO(10)$, we derive upper and lower bounds on the masses of heavy Majorana neutrinos. We also discuss implications for the standard model Higgs boson.

Despite its extraordinary success the standard model of strong and electroweak interactions is generally believed to be only the low energy approximation of a more fundamental, unified theory. A possible remnant of unification at energies much below the unification scale are extra neutral vector bosons, which in recent years have been discussed in particular in connection with superstring theories [1]. Extended gauge theories, which contain $U(1)$ factors in addition to the standard model gauge group, also predict fermions in addition to the quarks and leptons of the standard model. They are contained in the possible anomaly free fermion representations of the unified theory. If one or more $U(1)$ factors of the unified group are spontaneously broken far below the unification scale, also some of these fermions will in general acquire masses much smaller than the unification mass.

In the following we shall consider the minimal extension of the standard model of this type, which is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Y'}$, contained in the unified group $SO(10)$. The smallest complex $SO(10)$ representation, the 16plet, contains a “right-handed neutrino” $\nu_R$ in addition to the 15 Weyl fermions of one quark-lepton family of the standard model. $\nu_R$ is a singlet with respect to the standard model gauge group. Hence, in the sequential breaking

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Y'} \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{em},$$

$\nu_R$ can acquire a Majorana mass of order $\nu'$ and thereby explain the smallness of the $\nu_e$, $\nu_\mu$ and $\nu_\tau$ masses via the see-saw mechanism [2]. Here $\nu'$ and $\nu$ are the vacuum expectation values which break the subgroups $(U)_Y$ and $SU(2)_L \times U(1)_Y$, and $U(1)_{em}$ is the unbroken group of electromagnetic interactions. As we shall see, the Majorana masses of the heavy neutrinos are bounded from above essentially by $\nu'$, and they are also bounded from below by the experimental upper bounds on the light neutrino masses.

Heavy Majorana neutrinos with masses up to a few hundred GeV can be produced at present and future...
ep and e\(^+\)e\(^-\) colliders, and detailed discovery limits have recently been worked out [3]. In order to obtain the precise relation between the Z' vector boson mass and the masses of the heavy Majorana neutrinos we study in the following the spontaneous breaking of the extended gauge symmetry. Contrary to the fermion content, which is constrained by the requirement of anomaly freedom, there are no restrictions on the Higgs fields which remain light compared to the unification scale. We shall therefore begin with the minimal Higgs sector necessary to break the extended gauge symmetry and comment later on the effects of additional Higgs fields.

The spontaneous breaking of U(1)\(r\), requires a complex scalar field \(X\) in addition to the doublet \(q\) of Higgs fields, and the lagrangian, which also yields Majorana masses for the right-handed neutrinos, reads

\[
\mathcal{L} = -\frac{1}{4} W_{\mu\nu}^2 W^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} C_{\mu\nu} C^{\mu\nu} + i \bar{\psi} D \psi + i \bar{\epsilon}_R D \epsilon_R + (D_{\mu} \phi)^*(D^\mu \phi) - V(\phi, \chi) - \bar{\chi} Q_R \phi_R - \bar{\epsilon}_R h \epsilon_R - \frac{1}{2} \chi^2 h_R + \frac{1}{2} \epsilon^2 \phi_R^2 .
\]

\(\phi = (\phi^0, \phi^-), \bar{\phi} = (\phi^-, \phi^0)\) and \(\chi\) are the SU(2) doublet and singlet Higgs fields, \(\xi = (\nu_L, e_L^\nu)\) and \(\nu_R\) are the lepton doublet and the right-handed neutrino. \(g_\chi\) and \(h\) are 3x3 complex Yukawa matrices with \(h = h^\dagger\). The covariant derivatives differ from the standard model expressions by the term \(-ix/\sqrt{2} g' \times Y'\), with \(Y'(\xi) = \frac{1}{2}, Y'(\bar{\psi}_R) = \frac{1}{2}, Y'(\epsilon_R) = \frac{1}{2}, Y'(\phi) = -\frac{1}{2}, Y'(\chi) = -\frac{1}{2}\). All terms involving quark fields have been omitted. \(B_{\mu\nu}, C_{\mu\nu}\) and \(W_{\mu\nu}\) are the U(1)\(Y\), U(1)\(y\), and SU(2)\(_L\) gauge fields, and \(B_{\mu\nu}, C_{\mu\nu}\) and \(W_{\mu\nu}\) are the corresponding field strengths. The couplings of the vector field \(C_{\mu}\) are determined by the orthonormality conditions for the U(1) charges \([Y' = a Y + b (B - L)]]\),

\[
\text{tr} [Y Y''] = 0, \quad \text{tr} [Y^2] = \frac{1}{3} \text{tr} [Y''],
\]

where the trace extends over all fermions, which yields

\(Y' = Y - \frac{1}{3} (B - L)\).

The gauge coupling constants of \(B_{\mu}\) and \(C_{\mu}\) are equal as the \(\beta\)-functions of both U(1) groups are the same. The most general Higgs potential is given by

\[
V(\phi, \chi) = \mu_1 \phi^* \phi + \mu_2 \chi^* \chi + \frac{1}{2} \lambda_1 (\phi^* \phi)^2 + \frac{1}{2} \lambda_2 (\chi^* \chi)^2 + \lambda_3 (\phi^* \chi) (\phi^* \chi),
\]

where \(\lambda_3 > (\lambda_1 \lambda_2)^{1/2}\), so that the potential is bounded from below. This Higgs sector is precisely the well known “singlet majoron” model [4] for spontaneous breaking of lepton number.

It is now straightforward to minimize the Higgs potential (5), to determine the vacuum expectation values \(v\) and \(v'\) of the Higgs fields \(\phi\) and \(\chi\), and to diagonalize the vector boson mass matrix. In addition to the photon one obtains two massive neutral vector bosons Z and Z'.

\[
Z_{\mu} = \cos \xi Z_{\mu}^0 - \sin \xi C_{\mu},
\]

\[
Z'_{\mu} = \sin \xi Z_{\mu}^0 + \cos \xi C_{\mu},
\]

where \(Z_{\mu}^0 = -\sin \Theta B_{\mu} + \cos \Theta W_{\mu}^3\) corresponds to the neutral vector boson of the standard model. Neglecting fermion masses, predictions of the standard model can be expressed in terms of three independent parameters which one may choose as the fine structure constant \(\alpha\), the Fermi constant \(G_F\) and the Z boson mass \(m_Z\). Since in our model the Higgs sector is specified, there is only one additional parameter which we choose to be the Z' boson mass \(m_{Z'}\). The mixing angles \(\Theta\) and \(\xi\) can now be expressed in terms of \(\alpha, G_F, m_z\) and \(m_{Z'}\). One then obtains the following exact relations (\(\mu^2 = \pi \alpha / \sqrt{2} G_F\)):

\[
\sin^2 \Theta = \frac{1}{2} - \left( 1 - \frac{\mu^2}{pm_{Z'}^2} \right)^{1/2},
\]

\[
\sin 2\xi = -2\sqrt{\frac{3}{2}} \sin \Theta \frac{pm_{Z}}{m_{Z'}^2 - m_{Z'}^2},
\]

\[
\Delta \rho = \rho - 1 = \sin^2 \xi \frac{m_{Z'}^2 - m_{Z}^2}{m_{Z}^2},
\]

Here we have introduced as auxiliary quantity the parameter \(\rho\), which is related to the ratio of neutral to charged current amplitudes at zero momentum transfer. In the relevant case \(m_{Z'}^2 \ll m_{Z}^2\) the quantities \(\sin \Theta, \sin 2\xi, \Delta \rho\) may be expanded around the standard model limit \(m_{Z} = \infty\), and one obtains to first order in \(m_{Z'}^2 / m_{Z}^2\):

\[
\xi = -\sqrt{3} \sin \Theta \frac{m_{Z}}{m_{Z'}},
\]

\([\text{396}]\)
\[ \Delta \rho = \frac{1}{2} \sin^2 \Theta \frac{m_Z^2}{2 m_Z}, \]
\[ \sin \Theta = \sin \left( 1 - \frac{1}{12} \sin^2 \Theta \frac{m_Z^2}{2 m_Z} \right), \]
where \( \Theta = \Theta \left|_{\rho=1} \right. \).

The mixing angles \( \Theta \) and \( \xi \) determine the couplings of the vector bosons \( Z \) and \( Z' \) to quarks and leptons. The interaction lagrangian is given by
\[ \mathcal{L} = J_{\mu} Z_{\mu} + J'_{\mu} Z'_{\mu}, \]
where
\[ J_{\mu} = \frac{g \cos \xi}{2 \cos \Theta} \sum_i \bar{\psi}_i \gamma^\mu (V'_{i} - A'_{i} \gamma_5) \psi_i, \]
\[ V'_{i} = T_{3L}(i) - 2 \sin^2 \Theta \cdot Q_i, \]
\[ A'_{i} = (1 + \sqrt{3} \tan \xi \sin \Theta) \cdot T_{3L}(i), \]
\[ J'_{\mu} = \frac{g \sin \xi}{2 \cos \Theta} \sum_i \bar{\psi}_i \gamma^\mu (V''_{i} - A''_{i} \gamma_5) \psi_i, \]
\[ V''_{i} = T_{3L}(i) - 2 \sin^2 \Theta \cdot Q_i, \]
\[ A''_{i} = (1 - \sqrt{3} \cot \xi \sin \Theta) \cdot T_{3L}(i). \]

Here the sum is over all quarks and leptons, and \( T_{3L}(i), Q_i \), and \( (B-L) \), denote weak isospin, electric charge and \( B-L \) of the \( i \)th fermion.

The \( Z-Z' \) mixing affects various observables which have been measured with high precision at LEP [5]. Two particularly sensitive quantities are the \( \rho \)-parameter (cf. eqs. (9) and (11)) and the leptonic width of the \( Z \) boson, given by
\[ \Gamma_{ee} = \frac{G_F m_Z^3 \rho}{6 \pi} \left[ \frac{1}{2} - \sin^2 \Theta + 4 \sin^4 \Theta \right. \]
\[ + \left. \xi \frac{1}{3} \sin \Theta \left( \frac{1}{3} - 4 \sin^2 \Theta \right) \right]. \]

The three variables \( \Theta, \rho \) and \( \xi \) on the right hand side of eq. (20) can be expressed in terms of the independent parameters \( \alpha, G_F, m_Z \) and \( m_Z \) according to eqs. (10)–(12). Here we neglect the effect of the mixing between heavy and light neutrinos on the Fermi constant \( G_F \). A detailed comparison of LEP data with the standard model predictions shows that the mixing angle \( |\xi| \) has to be smaller than about \( 10^{-2} \) (cf. refs. [6–8]). From eqs. (11) and (20) one finds that this corresponds to an accuracy of about 0.4% in \( \rho \) and \( \Gamma_{ee} \). Eq. (10) then yields the lower bound on the \( Z' \) boson mass (\( |\xi| < 10^{-2} \))
\[ m_{Z'} = \sqrt{\frac{3}{8} \sin \Theta \frac{m_Z}{|\xi|}} > 570 \text{ GeV}, \]
where we have used \( \sin^2 \Theta = 0.23 \) and \( m_Z = 91 \text{ GeV} \).

In eqs. (14)–(19) the neutral currents \( J_{\mu} \) and \( J'_{\mu} \) are expressed in terms of weak eigenstates which are related to mass eigenstates by unitary transformations. For the neutrinos these transformations are determined by requiring that the two Majorana mass matrices
\[ m = h v', \]
\[ m_{\nu} = -m_D \frac{1}{m} m_D^T, \]
with
\[ m_D = g_e v, \]
are diagonal and real (cf., e.g., ref. [3]). \( g_e \) is the 3 × 3 matrix of neutrino Yukawa couplings. The eigenvalues of \( m \) are the masses of three heavy Majorana neutrinos, the eigenvalues of \( m_{\nu} \) correspond to the masses of \( v_e, v_\mu \) and \( v_\tau \).

Using eqs. (22), (23) and the relation (cf. eq. (2))
\[ m_{Z'}^2 = \frac{25}{9} G_F^2 v'^2 [1 + O(v^2/v'^2)], \]
we can now estimate the mass range, in which heavy Majorana neutrinos are to be expected. We assume that the Yukawa couplings \( g_e \) and \( v \) vary between the smallest and largest known Yukawa couplings \( g_e = m_e/v = 3 \times 10^{-6} \) and \( g_e \approx 1 \), which is about the largest value allowed by “triviality” considerations [9]. From eqs. (22) and (25) we then obtain an upper bound on the Majorana masses \( m_{\nu} \). Furthermore eq. (23) and the experimental upper limits \( m_{\nu} < 17 \text{ eV}, m_{\nu} < 0.27 \text{ MeV}, m_{\nu} < 35 \text{ MeV} \) imply the lower bound \( \det (m) = m_1 m_2 m_3 > m_e^2 m_{\nu} m_{\nu} m_{\nu} m_{\nu} \). Upper and lower bound together yield the allowed mass range
\[ 5 \text{ MeV} < (m_1 m_2 m_3)^{1/3}, \]
If, instead of \( m_3 \), we take \( \det(m) \equiv m_\mu m_\tau m_\nu \) as lower bound on the determinant of the Dirac neutrino mass matrix \( m_{\nu\nu} \), then the lower bound on \( (m_1 m_2 m_3)^{1/3} \) increases to \( \sim 40 \text{ GeV} \). The lower bound crucially depends on the assumption about the Yukawa couplings \( g_\nu \); it disappears for \( g_\nu = 0 \).

From eqs. (26) and (27) we conclude that Majorana neutrinos with masses of a few tens of GeV or a few hundred GeV are likely to exist if an SO(10) \( Z' \) boson with mass of order 1 TeV is found. If the mixings of these heavy neutrinos with \( \nu_e, \nu_\mu \) and \( \nu_\tau \) are not too small, they can be produced in charged and neutral current processes at HERA, LEP or LEP\( \otimes \) LHC [3].

The upper bound on the heavy neutrino masses stems from the fact that the vacuum expectation value, which generates the Majorana mass term, unavoidably also contributes to the \( Z' \) vector boson mass. Hence, the upper bound (27) on \( m_1 \), which we obtained for the minimal Higgs sector, will become even more stringent if more Higgs scalars are introduced which contribute to the \( Z' \) boson mass, but do not couple to \( v_\mu v_\tau \). In the case of more than one singlet scalar field with the quantum numbers of \( \chi \) our assumption on the maximal Yukawa coupling refers to the linear combination of scalar fields which acquires a vacuum expectation value.

One may worry that a low mass scale of \( B-L \) breaking of order 1 TeV is in conflict with constraints on lepton number violating processes from low energy experiments [11]. This is not the case although some bounds on mixing angles in charged and neutral currents can be derived. As an example, let us briefly mention neutrinoless double \( \beta \) decay. The present experimental bound on light Majorana neutrino masses reads [12]

\[
\langle m_{\nu\nu} \rangle = \sum_{\nu=e,\mu,\tau} U_{\nu e}^2 m_{\nu e} < (1-10) \text{ eV},
\]

where \( U_{\nu e} \) denotes elements of the leptonic Kobayashi–Maskawa (MK) matrix. For instance, with \( m_{\nu e} = m_e^2/m \) and \( m \sim 500 \text{ GeV} \) one has \( m_{\nu e} \sim 0.5 \text{ eV} \), which satisfies the bound (28). Furthermore, if \( U_{\nu e} = O(\sqrt{m_{\nu e}/m_{\nu\nu}}) \), which approximately holds for quark KM matrix elements, also the contributions from \( v_\mu \) and \( v_\tau \) satisfy the bound (28).

A mass scale of \( B-L \) breaking of order 1 TeV is also cosmologically acceptable [13,14].

Let us finally consider the Higgs sector. The fields \( \phi \) and \( \chi \) contain the neutral bosons \( H^0 \) and \( H'^0 \):

\[
\phi^+ \phi = \left( v + \frac{1}{\sqrt{2}} H^0 \right)^2,
\]

\[
\chi^+ \chi = \left( v' + \frac{1}{\sqrt{2}} H'^0 \right)^2.
\]

The mass eigenstates \( H \) and \( H' \) are linear combinations of \( H^0 \) and \( H'^0 \), i.e., \( H^0 = \cos \beta H + \sin \beta H' \) and \( H'^0 = -\sin \beta H + \cos \beta H' \). For the mixing angle one finds

\[
\tan \beta = \frac{2 \lambda_3 v' v}{\lambda_2 v^2 - \lambda_1 v'}. \tag{30}
\]

The corresponding Higgs boson masses are

\[
m_H^2 = \lambda_1 v^2 + \lambda_2 v'^2
= \sqrt{(\lambda_1 v^2 - \lambda_2 v'^2)^2 + 4\lambda_3^2 v^2 v'^2}, \tag{31}
\]

\[
m_{H^0}^2 = \lambda_1 v^2 + \lambda_2 v'^2
+ \sqrt{(\lambda_1 v^2 - \lambda_2 v'^2)^2 + 4\lambda_3^2 v^2 v'^2}. \tag{32}
\]

For \( v' = O(1 \text{ TeV}) \) the mixing angle \( \beta \) and the splitting between the two Higgs boson masses may be large. Furthermore the couplings of one Higgs boson to ordinary quarks and leptons could be substantially reduced. Since \( H'^0 \) interacts only with \( C_{\mu\nu} v_\mu \) and \( H^0 \), the couplings of the mass eigenstates \( H \) and \( H' \) to \( W \) and \( Z \) bosons, quarks and charged leptons have essentially the same structure as the corresponding Higgs couplings in the standard model, however, their size is reduced by \( \cos \beta \) and \( \sin \beta \), respectively. In the extreme case \( \beta = \frac{1}{2} \pi \) the couplings of the lighter Higgs boson to quarks and leptons would even vanish!

To summarize, we have shown that the existence of a neutral \( Z' \) vector boson with mass of order 1 TeV has important implications for other particles in the low energy theory. For an extended gauge group contained in SO(10) we have estimated upper and lower bounds on the masses of heavy Majorana neutrinos, and we have studied implications for the standard model Higgs boson. For \( Z' \) vector bosons contained in \( E_6 \) similar bounds on masses of the additional neu-

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tral and charged leptons and quarks can be derived.

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