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Dirac versus Wigner. Towards a universal particle concept in local quantum field theory

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Based on the concept of particle weight a unified description of stable particle-like systems in local quantum field theory is given which applies also to so called infraparticles. Particle weights carry definite charge, four momentum and spin and give rise to characteristic mass shell singularities. A collision theory for these weights is proposed which allows to calculate transition probabilities directly from local gauge invariant operators.

1. Particle concepts

The customary particle interpretation in local quantum field theory is based on the hypothesis, put forward by Wigner [1] more than 50 years ago, that the states of stable elementary particles can be described by vectors in some irreducible representation of the Poincaré group, respectively its covering group. As is well known, this idea does not work for particles carrying an electric charge. The states of such particles always contain clouds of low energy photons and therefore cannot be described by eigenvectors of the mass operator or vectors in some Lorentz invariant superselection sector of the physical Hilbert space, cf. refs. [2,3].

The formulation of mathematical concepts which are adequate for the description of such "infraparticles" [4] is a longstanding problem. In spite of some progress [5,6] many basic questions, such as the definition of spin and statistics of infraparticles, the general construction of collision states etc. remain open to date.

We present in this letter a framework which seems to be suitable for a solution of these problems. Our approach is based on the observation that infraparticles can be viewed as elementary systems (like particles) if one proceeds to Dirac's idealization of *improper* states of sharp momentum [7]. The standard treatment of improper states requires, however, some modifications.

Standard (rigged Hilbert space) approach. Improper momentum eigenstates $|\boldsymbol{p}\rangle$ of a particle are regarded as Hilbert space valued distributions,

$$f \in L^2(\mathbb{R}^3) \to \int d^3p\, f(\boldsymbol{p})\, |\boldsymbol{p}\rangle \in \mathscr{H}, \qquad (1)$$

i.e. as linear mappings from the space of wave functions into the physical Hilbert space. This definition is based on the implicit assumption that the superposition principle holds unrestrictedly for the improper states $|\boldsymbol{p}\rangle$, a condition that is not satisfied in the case of infraparticles. Improper states of infraparticles with different momenta cannot coherently be superimposed since they are affiliated with different superselection sectors which can be distinguished by the shape of the asymptotic electromagnetic field [2,6].

Present (algebraic) approach. Improper momentum eigenstates $|\boldsymbol{p}\rangle$ are regarded as linear mappings from some left ideal $\mathscr{L}$ of localization operators, contained in the algebra $\mathscr{A}$ of local observables of the theory, into the physical Hilbert space,

$$L \in \mathscr{L} \to L|\boldsymbol{p}\rangle \in \mathscr{H}. \qquad (2)$$

Such mappings are called weights. In local QFT $\mathscr{L}$ consists of almost local observables which annihilate the vacuum. Examples of such operators are ob-

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tained by smearing any local field $A(x)$ with a test function $f(x)$ whose Fourier transform has compact support in the complement of the forward light-cone V_+,

$$L=\int \mathrm{d}x\, f(x)\, A(x)\ . \tag{3}$$

(In non-relativistic quantum mechanics $\mathscr{L}$ is generated by rapidly decreasing functions of the position operator.) The present characterization complies with the intuitive idea that an improper momentum eigenstate (a plane wave) should become normalizable after localization.

In completely massive theories there always exist localization operators L such that the vector $L|\boldsymbol{p}\rangle$ describes the state of a single particle. Our approach is then equivalent to the standard treatment of improper states. In the presence of long range forces the process of localization of a particle may, however, inevitably be accompanied by the production of massless particles. Then the vector $L|\boldsymbol{p}\rangle$ describes a multiparticle state for any choice of L, although the pre-image $|\boldsymbol{p}\rangle$ can be interpreted in terms of a single particle. It is this phenomenon which one faces in the case of infraparticles and which is taken into account in our definition.

2. Properties of particle weights

It has been noticed in ref. [8] that, irrespective of the type of interaction in the underlying QFT, the time-like asymptotic structure of physical states can be described in terms of *particle weights* characterized by the following definition.

Definition. A particle weight $|\ \rangle$ is a continuous (in a suitable topology) linear mapping from a Poincaré invariant left ideal $\mathscr{L}\subset\mathscr{A}$ of almost local vacuum annihilation operators into a Hilbert space $\mathscr{H}$, such that (i) the family of operators $\pi(A)$, $A\in\mathscr{A}$ fixed by

$$\pi(A)\,|L\rangle \doteq |A\cdot L\rangle\ ,\quad L\in\mathscr{L}\ , \tag{4}$$

defines a self-adjoint representation $(\pi, \mathscr{H})$ of the algebra $\mathscr{A}$ on the Hilbert space $\mathscr{H}$, and (ii)

$$U(x)\,|L\rangle \doteq |L(x)\rangle\ ,\quad L\in\mathscr{L}, \tag{5}$$

defines a continuous unitary representation of the space–time translations x with spectrum in some light-cone $V_+ + q$.

Detailed properties of particle weights, which justify their interpretation as improper momentum eigenstates of a particle, have recently been established in ref. [9]. We list the most interesting ones.

(i) *Charge and momentum.* Particle weights can be decomposed into direct integrals

$$|\ \rangle=\int \mathrm{d}\mu(\lambda)\ |\ \rangle_\lambda \tag{6}$$

of *pure* particle weights $|\ \rangle_\lambda$ inducing irreducible representations of $\mathscr{A}$. The label λ characterizes different particle weights, i.e. it subsumes all properties of the corresponding particles. In particular, there exists for each weight $|\ \rangle_\lambda$ a unique four vector $p_\lambda\in V_+$ such that the continuous unitary representation of the translations given by

$$U_\lambda(x)\,|L\rangle_\lambda \doteq \exp(\mathrm{i}p_\lambda\cdot x)\,|L(x)\rangle_\lambda\ ,\quad L\in\mathscr{L}, \tag{7}$$

has Lorentz invariant spectrum. Thus pure particle weights $|\ \rangle_\lambda$ have fixed superselection quantum numbers, sharp four momentum p_λ, and mass $m_\lambda=\sqrt{p_\lambda^2}$.

(ii) *Spin.* Here we assume that the space of pure particle weights $|\ \rangle_\lambda$ corresponding to given four momentum p is finite dimensional (finite particle multiplets). With this input we can show that in the representation $(\pi_\lambda, \mathscr{H}_\lambda)$ of $\mathscr{A}$, induced by $|\ \rangle_\lambda$, there exists a continuous unitary projective representation of the "little group" of p, i.e. of SO(3), respectively E(2). (Lorentz boosts may not be represented on $\mathscr{H}_\lambda$.) This implies that all particle weights of mass $m_\lambda>0$ can be decomposed into weights of definite (half-) integer spin, as in the familiar case of particles of Wigner type. If $m_\lambda=0$ the helicity need not be quantized, however. This leaves open the interesting possibility of massless infraparticles with fractional helicity in four space–time dimensions.

(iii) *Spectral properties.* Pure particle weights $|\ \rangle_\lambda$ induce representations $(\pi_\lambda, \mathscr{H}_\lambda)$ of $\mathscr{A}$ with characteristic spectral properties of energy and momentum. There exist vectors $\Phi_\lambda\in\mathscr{H}_\lambda$ such that the functions (for given $\boldsymbol{p}$)

$$x_0\to\int \mathrm{d}^3x\,\exp(-\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{x})\,(\Phi_\lambda,\ U_\lambda(x_0,\boldsymbol{x})\Phi_\lambda) \tag{8}$$

are the Fourier transforms of certain specific measures. These measures have an atomic (δ-function) part at $p_0=\sqrt{\boldsymbol{p}^2+m_\lambda^2}$ if $\boldsymbol{p}$ is put equal to the spatial momentum $\boldsymbol{p}_\lambda$ of the underlying particle weight $|\ \rangle_\lambda$. If $\boldsymbol{p}\neq\boldsymbol{p}_\lambda$ this atomic part may, however, be absent [#1]. This is the case for infraparticles (in contrast to particles of Wigner type, where the measures have an atomic part for any $\boldsymbol{p}$, i.e. on the whole mass shell). Lorentz boosts are then spontaneously broken on $\mathscr{H}_\lambda$ and particle weights corresponding to different momenta induce inequivalent representations of $\mathscr{A}$.

(iv) *Local normality.* Assuming decent phase space properties of the theory (which can be expressed in terms of compactness conditions [10]) the representations induced by particle weights are locally unitarily equivalent to the vacuum representation. This result shows that particle weights are affiliated with states of physical interest.

3. Collision theory

Within the present setting the task of collision theory consists in the construction of states on $\mathscr{A}$ which describe asymptotic configurations of (infra) particles with prescribed charge, spin, mass, and momentum. Contributions from low energy massless particles need (and can) not be specified. The resulting states suffice for the calculation of cross sections for collision processes of "hard" particles in which an unspecified number of "soft" particles is present.

We have developed a method for the construction of such states which does not require the knowledge of charged fields [11]. The charged (infra) particle content of the theory is determined in a first step from the observables $\mathscr{A}$ on the vacuum Hilbert space $\mathscr{H}_0$.

Following ideas of Araki and Haag [12] we consider for any $L\in\mathscr{L}$ and suitable, sufficiently regular functions $h(\boldsymbol{v})$ the space–time averages

$$(L^*L)(t;h)=t^{-1}\int_t^{2t}\mathrm{d}t'\int\mathrm{d}^3x\,h(\boldsymbol{x}/t')(L^*L)(t',\boldsymbol{x})\,. \qquad (9)$$

These operators are uniformly bounded in t on all subspaces of $\mathscr{H}_0$ of finite energy [13]. The proposed construction of collision states is then accomplished by the following steps:

(i) *Determination of particle content.* Given $\Psi\in\mathscr{H}_0$ one calculates the limit

$$^{\mathrm{as}}\langle L|L\rangle^{\mathrm{as}}\doteq\lim_{t\to t^{\mathrm{as}}}(\Psi,(L^*L)(t;h)\Psi)\,, \qquad (10)$$

where $t^{\mathrm{as}}=\pm\infty$ and as stands for in, respectively out. This expression defines, for positive h, a positive sesquilinear form $^{\mathrm{as}}\langle\ |\ \rangle^{\mathrm{as}}$ on $\mathscr{L}$ which can be decomposed into a mixture of pure particle weights [9],

$$^{\mathrm{as}}\langle\ |\ \rangle^{\mathrm{as}}=\int\mathrm{d}\mu^{\mathrm{as}}(\lambda)\,{}_\lambda\langle\ |\ \rangle_\lambda\,. \qquad (11)$$

The pure weights describe the (infra) particle content appearing asymptotically in state Ψ in the space–time cone fixed by the support of h and the sign of t.

(ii) *Construction of selective counters.* Having determined the particle weights appearing in the theory one constructs for each weight $|\ \rangle_\lambda$ a linear combination of operators $\sum_\mu(L_\mu^*L_\mu)(t;h_{\mu,\lambda})$ such that the range of the operator S_λ^{as} defined by

$$S_\lambda^{\mathrm{as}}\Psi\doteq s-\lim_{t\to t^{\mathrm{as}}}\sum_\mu(L_\mu^*L_\mu)(t;h_{\mu,\lambda})\Psi \qquad (12)$$

consists of states in which a particle weight with properties specified by λ appears at asymptotic times t^{as}.

In the case of massive (infra) particles this construction is performed as follows: one first complements the weight $|\ \rangle_\lambda$ to a basis of weights $|\ \rangle_{\lambda'}$ with the same velocity, $\boldsymbol{p}_{\lambda'}/(p_{\lambda'})_0=\boldsymbol{p}_\lambda/(p_\lambda)_0=\boldsymbol{v}_\lambda$. Then one picks sufficiently many operators $L_\mu\in\mathscr{L}$ and determines functions $h_{\mu,\lambda}$ with support about $\boldsymbol{v}_\lambda$ such that there holds the relation

$$\sum_\mu{}_{\lambda'}\langle L_\mu|L_\mu\rangle_{\lambda''}h_{\mu,\lambda}(\boldsymbol{v}_\lambda)=\delta_{\lambda\lambda'}\delta_{\lambda\lambda''}\,. \qquad (13)$$

This construction, which amounts to the solution of a linear equation, always works if the respective basis consists of a finite number of elements. In that case one even can find functions $h_{\mu,\lambda}$ such that relation (13) holds for velocities in some neighbourhood of

[#1] Such "isolated singularities" appear for example in some unitary gauges of quantum electrodynamics in the fermion propagators. This fact was pointed out to one of the present authors (D.B.) by K. Symanzik many years ago. Yet its significance was unclear at that time.

v_λ. (For massless particles the construction is somewhat different.) The operators S_λ^{as} obtained this way may be compared with selective particle counters which are only sensitive to particles of type λ.

(iii) *Fixing of configurations.* For most purposes it suffices to consider asymptotic configurations of particles, fixed by labels $\lambda_1, ..., \lambda_n$, which have different velocities. Then the corresponding selective counters $S_{\lambda_i}^{as}$, $i=1, ..., n$ commute, i.e. they refer to compatible properties of quantum states.

Each configuration of particles has total four momentum $p=p_{\lambda_1}+...+p_{\lambda_n}$. It is said to be (essentially) neutral if for any open neighbourhood Δ_p of p the operator $E(\Delta_p)S_{\lambda_n}^{as}...S_{\lambda_1}^{as}$, where $E(\ )$ is the spectral projection of energy and momentum on $\mathscr{H}_0$, is different from 0. This terminology is suggested by the fact that the range of this operator consists of states in the vacuum sector $\mathscr{H}_0$ describing (apart from – possibly charged – massless particles of arbitrarily low energy) the asymptotic configuration $\lambda_1, ..., \lambda_n$.

Now given any (not necessarily neutral) configuration $\lambda_1, ..., \lambda_m$ one first extends it to a neutral one [#2], $\lambda_1, ..., \lambda_m, \bar\lambda_{m+1}, ..., \bar\lambda_n$. The operator $E(\Delta_p)\times S_{\bar\lambda_n}^{as}...S_{\bar\lambda_{m+1}}^{as} S_{\lambda_m}^{as}...S_{\lambda_1}^{as}$ is then different from 0. Picking any vector Ψ in the set theoretic complement of its kernel (which is an open set in $\mathscr{H}_0$, hence almost all vectors Ψ will do) one proceeds to the vector

$$\Psi^{as}=E(\Delta_p)S_{\bar\lambda_n}^{as}...S_{\bar\lambda_{m+1}}^{as}S_{\lambda_m}^{as}...S_{\lambda_1}^{as}\Psi, \tag{14}$$

which describes the desired asymptotic configuration $\lambda_1, ..., \lambda_m$, additional particles of type $\bar\lambda_{m+1}, ..., \bar\lambda_n$ carrying compensating charges, and possibly also an unspecified number of low energy massless particles.

(iv) *Removal of compensating charges.* In order to remove the compensating charges from Ψ^{as} one replaces the functions $h_{\mu,\bar\lambda}(v)$, appearing in the definition of the selective counters $S_{\bar\lambda}^{as}$ for the compensating charges, by

$$\exp(\mathrm{i}\boldsymbol{r}\cdot m_{\bar\lambda}\boldsymbol{v}/\sqrt{1-\boldsymbol{v}^2})\,h_{\mu,\bar\lambda}(\boldsymbol{v}) . \tag{15}$$

The phase factor has the effect that the asymptotic localization centers (impact parameters) of the compensating charges in Ψ^{as} are shifted by $\boldsymbol{r}$. The resulting vector is denoted by Ψ_r^{as}. One then proceeds to the limit state

$$\omega^{as}(A) \doteq \lim_{|\boldsymbol{r}|\to\infty} (\Psi_r^{as}, A\Psi_r^{as}) , \quad A\in\mathscr{A} . \tag{16}$$

By macroscopic causality the contributions of all compensating charges factor out in this limit and the collision state ω^{as} describes the desired asymptotic particle configuration $\lambda_1, ..., \lambda_m$. A corresponding Hilbert space representative can be recovered from the state ω^{as} by the GNS reconstruction theorem.

This construction of collision states mimics a kind of filtering experiment. The ambiguities left in the definition of these states with regard to the energy and momentum content (given by the size of Δ_p) and the contributions from low energy massless particles correspond precisely to the experimental ambiguities in the preparation of such states.

(v) *Cross sections.* Having constructed the incoming states ω^{in}, the cross sections for (inclusive) collision processes can be determined from the expectation values

$$\omega^{in}(S_{\lambda_1'}^{out}...S_{\lambda_n'}^{out}) , \tag{17}$$

where the selective counters are defined as in eq. (12) as limits of observables $A\in\mathscr{A}$. These expectation values are directly related to the probability that the incoming configuration $\lambda_1, ..., \lambda_m$ turns into an outgoing configuration $\lambda_1', ..., \lambda_n'$ (plus undetected particles).

A general proof that this method works in the framework of local QFT has so far only been given for asymptotically complete massive theories [14]. Even there the method is of interest since it has the character of an algorithm which, in contrast to the reduction formulas in standard collision theory, avoids the use of charged fields. Such fields are frequently hard to construct in gauge theories. Since the method relies on quantities which are always given in local QFT, it can be applied to any model. Its effectiveness can then be checked directly. For example, the method has been tested [15] in the Schroer model of infraparticles [4].

The problem of particle statistics, the status of the *PCT* theorem and the formulation of conditions for

[#2] The assumption that this is possible amounts to the hypothesis that for each charged particle there exists a particle carrying a compensating charge. Since massless particles can have arbitrarily small energy it suffices to consider supplementary particles $\bar\lambda_{m+1}, ..., \bar\lambda_n$ which are massive.

asymptotic completeness in this setting are presently under investigation.

References

[1] E.P. Wigner, Ann. Math. 40 (1939) 149.
[2] J. Fröhlich, G. Morchio and F. Strocchi, Phys. Lett. B 89 (1979) 61.
[3] D. Buchholz, Phys. Lett. B 174 (1986) 331.
[4] B. Schroer, Fortschr. Phys. 11 (1963) 1.
[5] J. Fröhlich, G. Morchio and F. Strocchi, Ann. Phys. (NY) 119 (1979) 241.
[6] D. Buchholz, Commun. Math. Phys. 85 (1982) 49.
[7] P.A.M. Dirac, The principles of quantum mechanics (Clarendon Press, Oxford, 1930).
[8] D. Buchholz, in: VIII Intern. Congress on Mathematical physics (Marseille, 1986) (World Scientific, Singapore, 1986).
[9] D. Buchholz and M. Porrmann, in preparation.
[10] D. Buchholz and M. Porrmann, Ann. Inst. H. Poincaré 52 (1990) 237.
[11] D. Buchholz and U. Stein, in preparation.
[12] H. Araki and R. Haag, Commun. Math. Phys. 4 (1967) 77.
[13] D. Buchholz, Commun. Math. Phys. 129 (1990) 631.
[14] U. Stein, Thesis (Universität Hamburg, 1989).
[15] K. Johannsen, Diplomarbeit (Universität Hamburg, 1991).