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# QCD corrections to final state photon bremsstrahlung in $e^+e^-$ annihilation

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We calculate the partial widths of the decay Z into one photon plus $n$ jets ($n=1, 2, 3$) as a function of the invariant mass cut parameter $y=m_{ij}^2/m_Z^2$ where $m_{ij}$ is the minimum jet invariant mass. Our calculation uses fixed order QCD perturbation theory up to $O(\alpha_S)$. Results in two schemes for defining jet variables are presented.

## 1. Introduction

It is well known that final state photon bremsstrahlung off quarks produced in $e^+e^-$ annihilation at high energies is suited to disentangle the couplings of down and up type quarks [1]. This is particularly promising for $e^+e^-$ annihilation at the Z resonance at LEP where the collection of a large number of hadronic Z decays has given a fair amount of multi-jet events with a final state photon of sufficient energy and transverse momentum with respect to the final jet axis [2]. First results of the OPAL Collaboration at LEP have become available just recently [3]. In this and earlier work [1,2] the selection of final state photons into photons emitted from initial state leptons and final state quarks and the elimination of background is considered in detail.

In the standard model the coupling of the Z resonance to quarks of flavour f is given by the vector and axial couplings $v_f$ and $a_f$ which are

$$v_f = 2\, I_{3,f} - 4 e_f \sin^2\theta_w , \tag{1.1}$$

$$a_f = 2\, I_{3,f} , \tag{1.2}$$

where $I_{3,f}$ is the weak isospin, $e_f$ is the quark charge and $\theta_w$ is the weak mixing angle. With $\sin^2\theta_w = 0.23$ and the well-known isospin assignments this leads to $v_d = 0.69$, $v_u = 0.39$ and $a_{d,u} = \mp 1$ where d and u stand for any down and up type quark. We denote the overall coupling $c_f = v_f^2 + a_f^2$. Then the total hadronic width of the Z is given up to second order QCD by

$$\Gamma(Z \to X) = \frac{G_\mu M_Z^3}{24\sqrt{2}\pi} N_c \left[ 1 + \frac{\alpha_s}{\pi} + 1.42 \left( \frac{\alpha_s}{\pi} \right)^2 \right] \times (2c_u + 3c_d) , \tag{1.3}$$

where $N_c$ is the number of colours, $G_\mu$ the muon decay constant, $M_Z = 91.18$ GeV the mass of the Z and $\alpha_s$ the strong $q\bar{q}g$ coupling constant at $\sqrt{s} = M_Z$. The factor in front of $c_u$ ($c_d$) counts the number of u- and d-type quarks.

In zeroth order QCD the radiative Z width $\Gamma_{q\bar{q}\gamma}$ is given by the simple formula

$$\Gamma_{q\bar{q}\gamma} = \frac{G_\mu M_Z^3}{24\sqrt{2}\pi} N_c \cdot \frac{3}{2} \frac{\alpha}{2\pi} \left( \tfrac{4}{9} \cdot 2c_u + \tfrac{1}{9} \cdot 3c_d \right) , \tag{1.4}$$

where the new factors $\frac{4}{9}$ and $\frac{1}{9}$ in front of $c_u$ and $c_d$ originate from the square of charges $e_u^2$ and $e_d^2$ of u- and d-type quarks, respectively. This formula includes all photons in the final state, i.e. also photons which are collinear with final state quarks and/or those which are soft. To disentangle these final state photons in Z decay from background and photons produced from initial state leptons selection cuts have

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to be used. Eq. (1.4) is modified correspondingly, but the yield remains proportional to $\frac{8}{9}c_u+\frac{1}{3}c_d$. Then the combination of (1.3), which has been measured already, and (1.4) or its modification including selection cuts allows the determination of $c_u$ and $c_d$ separately [1-3].

It is the purpose of this note to report the results of a calculation of $\Gamma_{q\bar{q}\gamma}$ including first order QCD corrections ($O(\alpha_S)$) for the case that the photon is separated from the q and $\bar{q}$ jet by an invariant mass squared cut $s_{q\gamma}$, $s_{\bar{q}\gamma}\geqslant y\, M_Z^2$. This procedure has been applied with good results to the selection of multihadronic jets in $e^+e^-$ annihilation at low energies and at the Z resonance [4] and to the definition of jets in higher order calculations [5]. For this selection procedure the theoretical cross section can be obtained using methods and results of our calculation of jet cross sections for $e^+e^-$ annihilation up to $O(\alpha_S^2)$ [5]. If we replace there one of the gluons by a photon [5] with the appropriate replacement of colour factors we can recover the partial widths for the decay of the Z into the $q\bar{q}g\gamma$ final state and the $O(\alpha_S)$ corrections to the $q\bar{q}\gamma$ final state. In the following we shall present the results for the Z decay into the $q\bar{q}\gamma$ final state, which experimentally is the 2-jet+$\gamma$ final state, including $O(\alpha_S)$ corrections, the $q\bar{q}g\gamma$ final state which experimentally corresponds to the 3-jet+$\gamma$ final state and the $(q\bar{q})\gamma$ and $(q\bar{q}g)\gamma$ final state, respectively, in which $(q\bar{q})$ and $(q\bar{q}g)$ are unresolved, which experimentally corresponds to the class 1-jet+$\gamma$.

We shall represent our results as ratios to the hadronic Z width $\Gamma_{had}=\Gamma(Z\to X)$. So we get for the ratio of the total inclusive one-photon emission width $\Gamma(Z\to\gamma+X)$, where X stands for all hadronic final states to $\Gamma_{had}$

$$\frac{\Gamma(Z\to\gamma+X)}{\Gamma_{had}}=\frac{(\frac{8}{9}c_u+\frac{1}{3}c_d)(\alpha/2\pi)\,g_{tot}}{(2c_u+3c_d)[1+\alpha_s/\pi+1.42(\alpha_s/\pi)^2]}, \quad (1.5)$$

where

$$g_{tot}=\frac{3}{2}\left(1-\frac{4}{9}\frac{\alpha_s}{2\pi}\right). \quad (1.6)$$

The second term in (1.6) represents the $O(\alpha_s)$ correction to $\Gamma(Z\to\gamma+X)$. In the following we shall report results for the partial width of the Z into a photon and $n$ jets ($n=1$, 2, 3) which depend on the resolution cut $y$. These partial widths $\Gamma(Z\to\gamma+n\,\text{jets})$ are written as

$$\frac{\Gamma(Z\to\gamma+n\,\text{jets})}{\Gamma_{had}}=\frac{(\frac{8}{9}c_u+\frac{1}{3}c_d)(\alpha/2\pi)g_n(y)}{(2c_u+3c_d)[1+\alpha_s/\pi+1.42(\alpha_s/\pi)^2]}. \quad (1.7)$$

In the next section we shall present results for $g_1(y)$, $g_2(y)$ and $g_3(y)$ in form of tables for various values of $y$.

## 2. Decay rates into one photon plus jets

First we consider the rate for the decay of the Z into a photon plus 2 jets. The photon is isolated from the two jets by requiring a minimum invariant mass squared $ys$, where $\sqrt{s}=M_Z$ is the total energy in the decay, between the photon and either of the two jets. In lowest order QCD the hadronic final state is $q\bar{q}$. If we denote in the decay $Z\to q+\bar{q}+\gamma$ the momenta of q, $\bar{q}$ and $\gamma$ by $p_1$, $p_2$ and $p_3$ respectively, we demand

$$y_{ij}=(p_i+p_j)^2/s\geqslant y \quad (2.1)$$

for $i,j=1,2,3$. The $\gamma+q+\bar{q}$ rate as a function of $y$ is proportional to $g_2^{(0)}(y)$. The relation to $\Gamma(Z\to\gamma+2$ jets) follows from (1.7) with $n=2$. $g_2^{(0)}(y)$ is easily calculated numerically or from the corresponding formula for the $O(\alpha_s)$ 3-jet rate in $e^+e^-$ annihilation which can be found in ref. [5]. For convenience we have calculated $g_2^{(0)}(y)$ for various $y$ values between $y=0.005$ and $y=0.20$. The result is tabulated in table 1. By restricting the phase space to the region $y_{13}$, $y_{23}\geqslant y$ and $y_{12}\leqslant y$ we obtain the decay rate for $n=1$, i.e. the final state: photon+1 jet. The $q\bar{q}$ is combined to one jet which recoils against the photon. This decay rate is proportional to $g_1^{(0)}(y)$ which is tabulated in table 1. It is clear that the ($\gamma$+1-jet) rate is much smaller except for very large $y$ cuts where the ($\gamma$+2-jet)- and the ($\gamma$+1-jet) rate become comparable. The results in table 1 are useful for a first orientation what magnitude for the ($\gamma+n$-jet) rate is to be expected in zeroth order of QCD.

Next we consider the $O(\alpha_s)$ corrections to $g_2(y)$ and $g_1(y)$. They are denoted by $g_2^{(1)}(y)$ and $g_1^{(1)}(y)$

Table 1
Reduced rates for $Z\to\gamma+1$ jet ($g_1^{(0)}(y)$) and $Z\to\gamma+2$ jets ($g_2^{(0)}(y)$) in zeroth order QCD as a function of $y$.

| $y$ | $g_2^{(0)}(y)$ | $g_1^{(0)}(y)$ |
|---|---|---|
| 0.005 | 39.53 | 0.043 |
| 0.01 | 27.94 | 0.073 |
| 0.02 | 18.31 | 0.119 |
| 0.03 | 13.60 | 0.155 |
| 0.04 | 10.68 | 0.185 |
| 0.05 | 8.648 | 0.212 |
| 0.06 | 7.145 | 0.235 |
| 0.08 | 5.062 | 0.273 |
| 0.10 | 3.688 | 0.302 |
| 0.12 | 2.716 | 0.325 |
| 0.14 | 2.015 | 0.342 |
| 0.16 | 1.484 | 0.354 |
| 0.18 | 1.079 | 0.361 |
| 0.20 | 0.7669 | 0.363 |

so that the total $\gamma+n$-jet rates ($n=1$, 2, 3) are obtained from

$$g_1(y)=g_1^{(0)}(y)+\frac{\alpha_s}{2\pi}g_1^{(1)}(y)\,,$$

$$g_2(y)=g_2^{(0)}(y)+\frac{\alpha_s}{2\pi}g_2^{(1)}(y)\,. \qquad (2.2)$$

In $O(\alpha_s)$ we have also the decay $Z\to q\bar{q}g\gamma$. Part of this 3-parton final state contributes to $g_1(y)$ and $g_2(y)$ namely when all three partons are combined into one jet or when two partons are combined into one jet, i.e. into qg, $\bar{q}$g or q$\bar{q}$. The case that all three partons q, $\bar{q}$ and g obey the constraint of being resolved $y_{ij}\geqslant y$, $i,j=1$, 2, 3, 4 where $y_{ij}$ is defined in (2.1) (the momenta of q, $\bar{q}$, g and $\gamma$ are $p_1$, $p_2$, $p_3$ and $p_4$) gives us the decay rate for $Z\to\gamma+3$ jets (i.e. $n=3$ in (1.7)). The reduced rate is $g_3(y)$. This is proportional to $\alpha_s$ and will be given in the form that we factor out $\alpha_s/2\pi$ as in (2.2) and write

$$g_3(y)=\frac{\alpha_s}{2\pi}g_3^{(1)}(y)\,, \qquad (2.3)$$

$g_3^{(1)}(y)$ has been calculated essentially in our earlier work [5]. It can be obtained from the $C_F^2$-part of the 4-jet cross section $\sigma_{4\text{-jet}}$ given there by dividing the cross section by $C_{F/2}=\frac{2}{3}$. The factor $C_F$ must be divided out since one factor $C_F\alpha_s$ is replaced by $\alpha$, the additional factor 2 comes from the fact that now g and $\gamma$ are distinct particles, so that the statistical factor $\frac{1}{2}$ present in $\sigma_{4\text{-jet}}$ must be removed. The result for $g_3^{(1)}(y)$ is presented in table 2 again for eleven $y$ values between 0.005 and 0.14. For the larger $y$ values $g_3^{(1)}(y)$ is very small and therefore the decay rate for $Z\to\gamma+3$ jets is negligible there.

The calculation of $g_2(y)$ proceeds along the lines of our earlier work in which results for $\sigma_{3\text{-jet}}(y)$ have been presented. Compared to ref. [5] one of the gluons in the final state q$\bar{q}$gg is replaced by a photon. For the final state q$\bar{q}$g$\gamma$ only one gluon is present which is combined with the q or $\bar{q}$ whereas the photon is always outside the cut region. Therefore compared to the $\sigma_{3\text{-jet}}$ calculation [5] the region where g or $\gamma$ are combined does not contribute. This and other details, in particular which contributions are taken into account in the $O(\alpha_s)$ correction to $g_1(y)$, are reported in ref. [6].

Actually in ref. [5] two different predictions for the $O(\alpha_s^2)$ correction to the 3-jet cross section in $e^+e^-$ annihilation have been presented. In both versions all subleading 4-parton contributions were included. It was found, however, that the 3-jet cross section depends on the way the variables describing 3 jets were formed out of the momenta of the 4 partons. In ref. [5] these two versions are called KL and KL′. The second order corrections in the KL scheme are larger, i.e. more positive than in the KL′ scheme. The same non-uniqueness appears for the $O(\alpha_s)$ corrections to the decay $Z\to q\bar{q}\gamma$. Here the result depends on the way

Table 2
$O(\alpha_s)$ correction to reduced rates for $Z\to\gamma+1$ jet ($g_1^{(1)}(y)$) and $Z\to\gamma+2$ jets ($g_2^{(1)}(y)$) for the KL2 scheme, together with reduced rate for $Z\to\gamma+3$ jets ($g_3^{(1)}(y)$) as a function of $y$.

| $y$ | $g_1^{(1)}(y)$ | $g_2^{(1)}(y)$ | $g_3^{(1)}(y)$ |
|---|---|---|---|
| 0.005 | −0.886 | −1836.3 | 976.20 |
| 0.01 | −0.883 | −894.9 | 425.55 |
| 0.02 | −0.664 | −347.2 | 147.78 |
| 0.03 | −0.378 | −182.3 | 65.76 |
| 0.04 | −0.095 | −106.6 | 32.42 |
| 0.05 | 0.173 | −66.71 | 16.82 |
| 0.06 | 0.413 | −43.34 | 8.622 |
| 0.08 | 0.792 | −19.90 | 2.198 |
| 0.10 | 1.127 | −9.920 | 0.4563 |
| 0.12 | 1.359 | −5.120 | 0.04512 |
| 0.14 | 1.518 | −2.665 | 0.001718 |
| 0.16 | 1.618 | −1.286 | |
| 0.18 | 1.667 | −0.5263 | |
| 0.20 | 1.673 | −0.1084 | |

how the variables of q, q̄ and γ are formed out of the momenta of q, q̄, g and γ, i.e. it depends how the variables of the jets (qg) and (q̄), respectively, are defined. This phenomenon is known as recombination dependence. The results for $g_1^{(1)}(y)$ and $g_2^{(1)}(y)$, $g_3^{(1)}(y)$ is independent of the choice of jet variables, as presented in table 2 correspond to the KL′ scheme as defined in ref. [5]. It yields smaller values for $g_2^{(1)}(y)$, i.e. $g_2^{(1)}(y)$ is more negative, than the corresponding $O(\alpha_s)$ correction in the KL scheme. In order to obtain information on the recombination dependence of the (γ+2-jet)-decay rate we have calculated $g_1^{(1)}(y)$ and $g_2^{(1)}(y)$ also for the KL scheme. The results are shown in table 3. We see that for very small $y$ values $g_2^{(1)}(y)$ in tables 2 and 3 tend to agree, as we would expect, but for larger $y$ the $g_2^{(1)}(y)$ in the KL scheme is larger and even becomes positive above $y>0.05$. Thus for the larger $y$ values $g_2^{(1)}(y)$ in the KL scheme leads to a positive correction in $g_2(y)$ and in the KL′ scheme it produces a small negative correction.

Of course, the QCD corrections and the decay width for Z→γ+3 jets depend on the value of $\alpha_s$. Since these are first order QCD results they depend strongly on the scale of $\alpha_s$. To illustrate the magnitude of the three partial widths for γ+$n$ jets ($n$=1, 2, 3) and their dependence we have plotted in fig. 1 the quantities $10^3\,\Gamma(Z\to\gamma+n\,\text{jets})/\Gamma_{had}$, where $\Gamma_{had}$ is the total decay width of the Z into hadrons, for $n$=1, 2, 3 as a function of $y$ and for $\alpha_s(M_Z^2)=0.118$ which

Table 3
$O(\alpha_s)$ correction to reduced rate for Z→γ+1 jet ($g_1^{(1)}(y)$) and Z→γ+2 jets ($g_2^{(1)}(y)$) for the KL1 scheme as a function of $y$.

| $y$ | $g_1^{(1)}(y)$ | $g_2^{(1)}(y)$ |
|---|---|---|
| 0.005 | 0.434 | −1581.0 |
| 0.01 | 0.933 | −683.1 |
| 0.02 | 1.723 | −219.3 |
| 0.03 | 2.139 | −87.46 |
| 0.04 | 2.803 | −33.00 |
| 0.05 | 3.183 | −6.847 |
| 0.06 | 3.511 | 6.313 |
| 0.08 | 3.982 | 15.97 |
| 0.10 | 4.281 | 16.84 |
| 0.12 | 4.453 | 14.80 |
| 0.14 | 4.527 | 12.25 |
| 0.16 | 4.515 | 9.937 |
| 0.18 | 4.431 | 7.785 |
| 0.20 | 4.281 | 5.830 |

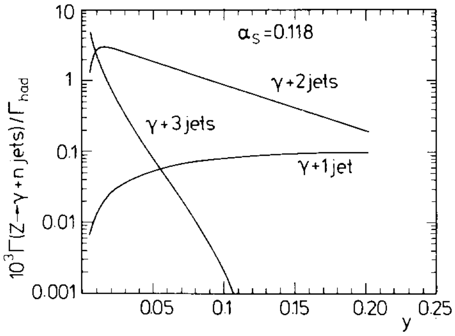


Fig. 1. Partial widths $\Gamma(Z\to\gamma+n\,\text{jets})$ ($n$=1, 2, 3) as a function of $y$ for $\alpha_s$=0.118 and normalized with $10^{-3}\,\Gamma_{had}$.

was obtained from fits to jet rates in multihadronic events [7]. We see that $\Gamma(Z\to\gamma+2\,\text{jets})$ dominates and decreases with increasing $y$. The width for Z→γ+3 jets is strongly $y$ dependent and is appreciable only for very small $y$ cuts. $\Gamma(Z\to\gamma+1\,\text{jet})$ is small and approaches zero for $y\to 0$ as one would expect. Since $\Gamma(Z\to\gamma+1\,\text{jet})$ depends on contributions which are non-singular the prediction depends very much on the special definition of the cuts.

Since the calculation of the γ rates is only $O(\alpha_s)$ one should choose a smaller scale than $M_Z^2$ in $\alpha_s$ to compensate for unknown higher order effects. This would increase $\alpha_s$ and has the effect that the width for γ+3 jets will be larger, the width for γ+2 jets will be slightly smaller and $\Gamma(\gamma+1\,\text{jet})$ will decrease for small $y$ and increase slightly for large $y$. Since the higher order corrections in $\Gamma(\gamma+1\,\text{jet})$ and $\Gamma(\gamma+2\,\text{jet})$ are rather moderate the dependence on the choice of $\alpha_s$ is not very strong.

The rates for Z→γ+3 jets and Z→γ+2 jets shown in fig. 1 agree with the experimental data in ref. [3]. However, our result for Z→γ+1 jet is smaller than the experimental data. Experimentally the rate for γ+1 jet at large $y$ values is approximately a factor of four larger than our prediction in fig. 1. Only for $y\leqslant 0.05$ there is agreement with the data. The reason for this discrepancy is unclear. It may have to do with particular strong hadronization corrections and/or a mismatch of the theoretical versus the experimental definition of resolution cuts for this particular channel. To study this problem in detail a full Monte Carlo including hadronization is needed.

Concerning the (γ+2-jet)- and (γ+3-jet) rate not only the absolute rates in fig. 1 but also the $y$-dependence agrees with the data quite well. The $y$-dependence of the ($\gamma$+2-jet) rate is of interest for the following reason. Since $g_2^{(1)}(y)$ is fairly large and negative (see table 2) the $O(\alpha_s)$ corrected rate for $Z\to\gamma+2$ jets increases less strongly with decreasing $y$ than the lowest order prediction $g_2^{(0)}(y)$ (see table 1). For example the $O(\alpha_s)$ corrected rate in fig. 1 increases by a factor 15 if we compare the rate at $y=0.2$ with the rate at $y=0.02$. The corresponding increase $g_2^{(0)}(y)$, i.e. the lowest order prediction, is a factor 24. On the other hand, the multihadronic 3-jet rate increases by a factor 22 in the same interval. In the 3-jet cross section the $O(\alpha_s^2)$ correction is positive since the contributions originating from the non-abelian nature of QCD, i.e. the terms proportional to the colour factor $C_F N_c$ coming from diagrams with the triple-gluon coupling, etc., give a large positive contribution. This contribution is much larger than the negative terms from the colour factors $C_F^2$ and $C_F T_R$ (see ref. [5] for further details). The reduced rate for $\Gamma(Z\to\gamma+2$ jets) in lowest order is identical to the reduced 3-jet rate, i.e. $g_2^{(0)}(y)$ gives up to known factors involving $\alpha_s$, etc., the hadronic 3-jet rate in $O(\alpha_s)$. We expect that the ratio of the hadronic width $\Gamma(Z\to 3$ jets) and $\Gamma(Z\to\gamma+2$ jets) increases with decreasing $y$ and that this increase is a measure of the $C_F N_c$ term in the hadronic 3-jet rate. The $C_F^2$ terms in $\Gamma(Z\to 3$ jets) and $\Gamma(Z\to\gamma+2$ jets) are almost equal (they differ only by the extra term where the two gluons are combined inside the resolution cut) and therefore cancels in the ratio up to $O(\alpha_s^2)$. We write for the reduced hadronic 3-jet width

$$\frac{\Gamma(Z\to 3\,\text{jets})}{\Gamma_{\text{had}}} = \frac{1}{1+\alpha_s/\pi+1.42(\alpha_s/\pi)^2}\frac{\alpha_s}{2\pi}C_F \times\left(B+\frac{\alpha_s}{2\pi}(C_F h_C+N_c h_N+T_R h_T)\right), \tag{2.4}$$

where $B$ is the lowest order contribution $B=g_2^{(0)}(y)$. For the ratio

$$r=\frac{\Gamma(Z\to 3\,\text{jets})}{(\alpha_s/2\pi)C_F(2c_u+3c_d)} \times\left(\frac{\Gamma(Z\to\gamma+2\,\text{jets})}{(\alpha/2\pi)(\frac{8}{9}c_u+\frac{1}{3}c_d)}\right)^{-1} \tag{2.5}$$

we obtain

$$r=1+\frac{\alpha_s}{2\pi}\frac{N_c h_N+T_R h_T}{B+(\alpha_s/2\pi)\,C_F h_C}\,. \tag{2.6}$$

In (2.6) we assumed that the $h_C$ contribution is the same for $Z\to 3$ jets and $Z\to\gamma+2$ jets. The ratio $r$ as given by (2.6) is plotted in fig. 2 as a function of $y$ and taking $\alpha_s=0.118$ as before (dashed curve). We see that $r$ as a function of $y$ increases as $y$ decreases. Its value above 1 is a measure of the second term in (2.6) which is dominated by the $N_c$-term. This term is a measure of the non-abelian nature of QCD. When we put the $N_c h_C$-term in (2.6) to zero, this way simulating an "abelian QCD", we obtain the dash-dotted curve in fig. 2. Then $r$ is below 1 and decreases with decreasing $y$ as expected since $T_R h_T$ is negative. Therefore the value of $r$ as a function of $y$ constitutes an interesting check on the non-abelian nature of QCD. In fig. 2 we also show the exact result for the ratio (2.5) (full curve), where the denominator is $g_2(y)$ and the numerator is obtained from our results in ref. [5] ($h_C$ and $h_T$ in (2.6) are also taken from this reference). Both $g_2(y)$ and the numerator are calculated in the KL2 scheme. The deviation from the dashed curve is small. It is significant only for $y\leqslant 0.05$, where the difference of the $h_C$-term in $g_2(y)$ and the 3-jet width mentioned above is not negligible anymore.

When $r$ as defined in (2.5) is compared to experimental data the value of $\alpha_s$ enters. Therefore $r$ depends on the value of $\alpha_s$ chosen. We have taken

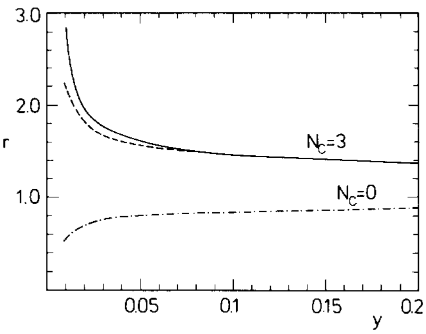


Fig. 2. The ratio $r$, the reduced hadronic 3-jet width divided by the reduced $\gamma$+2-jet width as defined in (2.5) as a function of $y$ in the approximation (2.6) (dashed curve), for "abelian QCD" (dash-dotted curve) and calculated from the theoretical 3-jet width and the $\gamma$+2-jet width (full curve).

$\alpha_s = 0.118$ which is obtained form fitting hadronic jet rates. It corresponds to the coupling at the scale $M_Z^2$. For $\alpha_s$ at smaller scales, where $\alpha_s$ is larger, $r$ diminishes somewhat. In the same way $h_N$ and $h_T$ in (2.6) decrease which partly compensates the increase of $\alpha_s$.

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