Class. Quantum Grav. 9 (1992) 2557–2571. Printed in the UK

A semiclassical approach to quantum gravity

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Received 8 August 1991, in final form 3 June 1992

Abstract. The interaction of a scalar quantum field with gravity is investigated in the semiclassical context where the spacetime is treated classically. It is essentially understood as a self-interaction of the quantum field, mediated by its own states. The relevant states here are not arbitrary but are selected by the principle of equivalence which is incorporated in the form of specific non-linear constraint equations. A (state-dependent) dynamics for the quantum field is proposed which is based on a suggested non-linear field equation.

1. Introduction

Since Hawking's original discovery of black hole radiation a great deal of work has been done on the foundation of 'the semiclassical model of self-consistent dynamics'‡ describing the interaction of linear quantum fields with gravity. The general framework adopted in this model may be indicated as follows. One starts by considering a quantum field obeying a linear covariant dynamical equation and the standard commutation relations on a fixed global spacetime, the latter understood classically in the sense of the general theory of relativity. The central assumption is that the back reaction of the quantum field to gravity can be described in a self-consistent manner via the Einstein equations coupled to the renormalized expectation value of the energy-momentum tensor operator of the quantum field in some appropriately chosen state, namely

$$G_{\mu\nu} = -\kappa \langle T_{\mu\nu} \rangle_{\text{ren}}. \tag{1}$$

In its underlying structure this model originates, of course, from striving for a semiclassical approach to quantum gravity. But, for this purpose its basic assumptions have turned out to be very restrictive. Looking, for example, at the technical side there is a complete lack of success in dealing with the problem of how to define the right-hand side of (1). Indeed, despite several attempts, e.g. [2]–[11], no truly satisfactory procedure for renormalization of $\langle T_{\mu\nu} \rangle$ has been developed§.

At the present time there is a feeling around that the conventional approach based on this model is not even consistent to serve as a basis for a semiclassical quantum gravity. But outside that model no attempts at a formulation of a self-consistent semiclassical scheme have been made.

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‡ This subject is, for example, described in [1].
§ Some details are given at the end of section 4.

0264-9381/92/122557+15$07.50

It may be, of course, that the incorporation of gravity into the quantum field theory could be accomplished only at the level of the fully quantized theory of gravity. At present one school of thought shares this conviction and maintains that the principles of semiclassical quantum gravity ultimately will not define a theory. There are, however, other aspects. Nobody knows today the principles of the fully quantized theory of gravity. Granted this ignorance the semiclassical approach remains the natural one towards the incorporation of gravity into the quantum field theory.

In any case, there is the desire to understand the inherent objective weaknesses of the conventional semiclassical approach. Concerning this task we have to take seriously the many conceptual difficulties surrounding the nature of its underlying assumptions. The history of science teaches us that such an investigation may help to establish the guiding line along which the future theory should be formulated. In this context it is important to realize, first, that the conventional framework indicated is based on the inadmissible notion of a rigid global background metric. This necessarily introduces, of course, a non-local element in the theory and degenerates the characteristic feature of the general theory of relativity, in which the spacetime becomes a dynamical object and all physical laws are strictly local. Conceptually, this feature of the general theory of relativity must be preserved in any theory incorporating the gravitational interaction. In order to have an example of the kind of difficulty one encounters consider the problem of the general covariance. It is obvious that the notion of a rigid global background metric implies the existence of *a priori* causal relations between observables of different spacetime regions. On the other hand, since the group of all local diffeomorphisms does not leave the causal relations unchanged, so the latter should not be given *a priori* if the former is regarded as the symmetry group.

Another unsatisfactory aspect of the conventional frame concerns the nature of the dynamical laws. It is by no means clear that a model based on linear dynamical equations for the quantum field could fit into the essentially non-linear gravitational interaction. On the contrary, we expect that the incorporation of gravity into the quantum field theory can only be provided by a non-linear theory.

The main goal of the present article is to propose an alternative approach. We shall study in particular how the semiclassical theory can be formulated without referring to any rigid global background metric. In arriving at dynamical laws our guiding principle will be the principle of equivalence. We demonstrate a possibility of incorporating that principle into the quantum field theory.

The scheme presented is by no means intended to be complete and final and many questions remain to be answered. But in the absence of any conclusive treatment of gravity in quantum field theory, we believe that the scheme presented is quite instructive because it stresses the significance of a non-linear (self-interacting) dynamics for the quantum field and therefore none of the restrictive features of the linear dynamics of the conventional approach can affect the conclusions.

Our discussion will be mainly based on the algebraic approach to generally covariant quantum field theory, presented by Fredenhagen and Haag [12], in which the principle of locality is advanced in its most stringent form dispensing with the existence of *a priori* causal relations between distant observables. Their work seems to clarify considerably the question of how the general covariance and the strict locality can be incorporated into the quantum field theory.

To begin with, let us present a description of the principal frame. At the most basic level we consider a four-dimensional manifold M, not yet equipped with a metric, and associate to each open set $\mathcal{O} \subset \mathcal{M}$ an involutive algebra $\mathcal{A}(\mathcal{O})$. The self-adjoint elements of $\mathscr{A}(\mathcal{O})$ are interpreted as observation procedures which are pure descriptions of laboratory measurements in $\mathcal{O}$. There should not be any *a priori* relations between observation procedures associated with different spacetime regions, in other words the algebra $\mathscr{A} = \bigcup \mathscr{A}(\mathcal{O})$ has to be flexible.

This interpretation allows us to implement the principle of the general covariance by considering the group of all local diffeomorphisms of the manifold as acting by automorphisms on $\mathscr{A}$, i.e. each local diffeomorphism χ is represented by an automorphism α_χ of $\mathscr{A}$ such that

$$\alpha_\chi(\mathscr{A}(\mathcal{O})) = \mathscr{A}(\chi(\mathcal{O})). \tag{2}$$

There are, of course, many observation procedures which are equivalent with respect to their action on a physical system, i.e. of measuring an observable. Thus, the question arises of how to construct the observables as an equivalence class of observation procedures. For this aim we note that the precise mathematical description of a physical system is given in terms of a (physical) state ω, i.e. a positive linear functional on $\mathscr{A}$. Given a state ω one gets via the GNS construction a representation π_ω of $\mathscr{A}$ by an operator algebra in a Hilbert space $\mathscr{H}_\omega$. In the representation π_ω one can select a family of related states on $\mathscr{A}$ (the so-called folium of ω), namely those represented by vectors and density matrices of $\mathscr{H}_\omega$†.

Once a physical state ω on $\mathscr{A}$, has been specified, one can consider in each subalgebra $\mathscr{A}(\mathcal{O})$ the equivalence relation

$$A \sim B \Leftrightarrow \omega'(A-B) = 0 \qquad \forall \omega' \in \mathscr{F}_\omega. \tag{3}$$

Here $\mathscr{F}_\omega$ denotes the folium of the state ω. The set of such equivalence relations generates a two-sided ideal $\mathscr{T}(\mathcal{O})$ in $\mathscr{A}(\mathcal{O})$. The construction of the algebra of observables $\mathscr{A}_{\text{obs}}(\mathcal{O})$ from the algebra of observation procedures is then accomplished by taking the quotient

$$\mathscr{A}_{\text{obs}}(\mathcal{O}) = \mathscr{A}(\mathcal{O})/\mathscr{T}(\mathcal{O}). \tag{4}$$

This standpoint in the treatment of local observables is essential for our approach to semiclassical quantum gravity. Clearly, in this setting the emphasis in the specification of the physical laws, i.e. the relations between local observables, is placed on the characterization of the admissible folia of physical states‡. If there are superselection rules there exist several folia (sectors) of physical states on $\mathscr{A}$ which correspond to different unitary inequivalent representations of $\mathscr{A}$.

To approach the problem of specification of the admissible folia of physical states we shall make the basic assumption that the relevant states (and the associated folia) are everywhere primary (the von Neumann algebras resulting from the GNS representation of such states have only trivial centre for a sufficiently small neighbourhood of a point). Each primary folium of local physical states provides us with a realization of the principle of local definiteness in the sense of the work [13], where a fixed gravitational background was assumed. The characteristic change here is that, unlike the

† To have some idea from the folium of a given representation we would like to mention that in the conventional theory there is a conjecture that the class of local Hadamard states defines the folium of a unique (up to unitary equivalence) representation.

‡ One should note, however, that the whole information about the physical laws contained in the algebra of observables can be expressed by direct specification of the two-sided ideals in the algebra of procedures as well. This alternative is widely used in the traditional treatments of quantum field theory. But for the treatment of gravity in quantum field theory it appears to be inevitable to convert the physical laws into appropriate mathematical constraints on states rather than observables.

situation in that work, for each sufficiently small neighbourhood of a point there will now be different primary folia of local states. This fact can be understood on the basis of our interpretation of the local algebras as the algebras of observation procedures.

Our main objective is, first, the question of how to specify the primary folia of local physical states.

We can formulate now one general criterion selecting the primary folia of physical interest. Let us comment first on the physical background. The axioms of quantum field theory in Minkowski space exclude the existence of observables at a single point. In that theory, due to the exact Lorentz invariance, the observables in spacelike complement of a single point generate the total algebra. It is not hard to see that this statement ignores the existence of the Planck length, $l_p = (\hbar\kappa/c)^{1/2} \approx 10^{-33}$ cm (κ is the gravitational constant), as the smallest possible length scale that can be measured even in principle by experiments. In reality the above statement need not hold in the gravitational case. The best we can do is to require the validity of that statement in the Minkowskian limit $\kappa \to 0$ where the Planck length tends to zero. Therefore in the limit $\kappa \to 0$ the algebra $\mathscr{A}_{\rm obs}(\mathcal{O})$ has to move into the commutant of the total algebra as $\mathcal{O}$ contracts to a single point. Thinking in terms of states this requires that, if we ignore the Planck regime, two states in the same primary folium should become indistinguishable in a sufficiently small neighbourhood of a point. Clearly, this statement converts the ignorance of the Planck regime into the requirement of a common leading short distance singularity (ultraviolet tail) of different states in the same primary folium. The full significance of the primary folia exhibiting this property will become evident in the light of our considerations in this work.

The required features of the algebra of observation procedures are incorporated in a simple model, the so-called tensor algebra over the space of scalar test functions on the spacetime manifold. The monomials of the local algebra $\mathscr{A}(\mathcal{O})$ in this model are smooth functions $f^{(n)}: M \times \ldots \times M \to C$ with support in $\mathcal{O}$. The algebraic product is the tensor product of functions

$$\begin{aligned} &f^{(n)} \cdot g^{(m)} = h^{(n+m)} \\ &h^{(n+m)}(p_1, \ldots, p_{n+m}) = f^{(n)}(p_1, \ldots, p_n) g^{(m)}(p_{n+1}, \ldots, p_{n+m}). \end{aligned} \tag{5}$$

The involution is the complex conjugation together with the inversion of the sequence of arguments. A diffeomorphism sending the point p to χp acts as the automorphism α_χ on $\mathscr{A}$ according to

$$(\alpha_\chi f^{(n)})(p_1, \ldots, p_n) = f^{(n)}(\chi^{-1} p_1, \ldots, \chi^{-1} p_n). \tag{6}$$

A state ω on $\mathscr{A}(\mathcal{O})$ is given by a hierarchy of distributions (the n-point functions) $\omega^{(n)} \in \mathscr{D}'(\mathcal{O} \times \ldots \times \mathcal{O})$. $\omega^{(n)}(f^{(n)})$ is the expectation value of the monomials $f^{(n)}$ in the state ω. In the present work we shall take this model as the kinematical model for the local algebras of a scalar field. It must be emphasized that this intepretation departs from the similar Borchers interpretation of the ordinary Wightman field theory [14] in an essential feature. We do not admit, namely, any *a priori* relations between observables. In order to work with the more familiar notion of a covariant 'quantum field ϕ' we shall write for the degree 1 elements of the algebra $\phi(f^{(1)})$ instead of $f^{(1)}$. Heuristically we may pass from $\phi(f^{(1)})$ in each chart $x = \varphi(p)$ to $\phi(x)$ according to

$$\phi(f^{(1)}) = \int d^4x\, \phi(x) f^{(1)}(x). \tag{7}$$

Correspondingly we may pass from $\omega^{(n)}(f^{(n)})$ to $W^{(n)}(x_1,\ldots,x_n)$, where $W^{(n)}(x_1,\ldots,x_n)$ is referred to as the n-point function of the state.

Depending on the specific theory in mind we also need concern ourselves in the following with the hierarchy of truncated n-point functions, $W_{\mathrm{T}}^{(n)}$, in terms of which the hierarchy $W^{(n)}$ is obtained by standard formulae.

The formalism described so far does not initially include any notion of spacetime geometry. Therefore the central problem is how one can transform it in a semiclassical theory. We address ourselves now to this problem.

2. The local structure of physical states

The concept of spacetime metric is naturally tied to the subjective ignorance of the Planck regime. On the other hand, as was already indicated, that ignorance requires a common leading short distance singularity of different states in the same primary folium. This raises the question of whether we can in some sense combine these two aspects.

In this section we want to exhibit the precise correspondence between the spacetime metric and the local structure of states in one primary folium. So we shall, first, ignore the Planck regime and consider its effect later.

On general grounds we expect that the two-point function plays the dominant role in the theory. Specifically, the spacetime metric should be encoded basically in the local structure of that function. Therefore, in this work our attention will be focused on the specification of the local structure of the two-point function, leaving the specification of higher functions to future work.

Let us now consider a 'sufficiently small' contractible neighbourhood $\mathcal{O}_p$ of a point $p\in M$ and a primary folium, denoted by $\mathcal{F}_{\mathcal{O}_p}$, of local states on $A(\mathcal{O}_p)$. We set in some chart $x=\{x^\mu\}=\varphi(p)$

$$\mathrm{d}_{\mathcal{O}_p}=\sup_{x'\in\mathcal{O}_p}|x^\mu-x'^\mu|.$$

For a given state $\omega\in\mathcal{F}_{\mathcal{O}_p}$ we shall assume that there exists at least one smooth scalar function $F^{(2)}:\mathcal{O}_p\times\mathcal{O}_p\to R$, so that $\tau_x^{(2)}(x')\equiv F^{(2)}(x,x')\,W_{\mathrm{T}}^{(2)}(x,x')$ is bounded as a function of x' in $\mathcal{O}_p$ and the limit

$$\|\tau_x^{(2)}\|=\lim_{d_{\mathcal{O}_p}\to 0}\sup_{x'\in\mathcal{O}_p}|F^{(2)}(x,x')\,W_{\mathrm{T}}^{(2)}(x,x')| \tag{8}$$

exists and is non-vanishing. Here $W_{\mathrm{T}}^{(2)}$ is the truncated two-point function of the state ω. For practical reasons the quantity arising from the above limit is assumed to be dimensionless†.

One might think of the function $F^{(2)}$ as describing the structure of the leading short distance singularity of the two-point function of the state involved. Since the structure of this singularity should be common for all states in the same primary folium‡ in what follows the function $F^{(2)}$ is taken to be universal, i.e. independent on the individual

† We shall adopt in our discussion the natural units in which $c=\hbar=1$. Accordingly the field ϕ will have the dimension of an inverse length.

‡ In the notation of the work [11] this statement corresponds to the well established fact that the scaling limit coincides for all states in one primary folium.

states. Concerning the specification of that function we shall assume that the limit

$$\lim_{d_{\mathcal{O}_p}\to 0} d_{\mathcal{O}_p}^{-2} \sup_{x'\in\mathcal{O}_p} |F^{(2)}(x, x')|$$

exists and is non-vanishing. Expanding now the function $F^{(2)}(x, x')$ in the coordinate differences $\xi^\mu = x'^\mu - x^\mu$, the above condition asserts that the leading term in this expansion must be of second order, namely

$$F^{(2)}(x, x') = \tilde{g}_{\mu\nu}(x)\xi^\mu\xi^\nu + \dots . \tag{9}$$

The dimensionless quantity $\tilde{g}_{\mu\nu}(x)$ that arises from this expansion transforms like a tensor and is determined by the above assumption up to a conformal factor (note that ξ^μ does not transform in general like a vector). In view of this fact one may conclude that the macroscopic metric $g_{\mu\nu}(x)$ can be obtained from $\tilde{g}_{\mu\nu}(x)$ by a conformal transformation, namely

$$g_{\mu\nu}(x) = \Omega^{-1}(x)\tilde{g}_{\mu\nu}(x). \tag{10}$$

This observation may be regarded as the quantum version of the classical result that the knowledge of the null cone at each point of the spacetime enables one to measure the metric at this point up to a conformal factor, see [15]. We can use this analogy further to give the function $F^{(2)}(x, x')$ an intrinsic geometrical meaning by requiring that the equation $F^{(2)}(x, x') = 0$ define the null cone at point x. Therefore by this requirement $F^{(2)}(x, x')$ can be identified up to the conformal factor $\Omega^{-1}(x)$ with the square of the geodesic distance $\sigma(x, x')$ between the points x and x', namely

$$\sigma(x, x') = \Omega^{-1}(x)F^{(2)}(x, x'). \tag{11}$$

We may determine the conformal factor in the last equation by normalizing $\|\tau_x^{(2)}\|$ in (8) to one which results in $F^{(2)}(x, x')$ coinciding with $\sigma(x, x')$.

Having introduced the notion of local macroscopic metric, we now take on the problem of writing down an expansion determining the local structure of the two-point function of the state considered. At this point there are several ways to proceed. The most convenient way consists in applying the techniques of covariant Taylor expansion, developed in [16] and [3]. We shall base our analysis on an expansion for the symmetric part of the truncated two-point function $W^{(2)}_{\mathrm{T,S}}$ of the form

$$W^{(2)}_{\mathrm{T,S}}(x, x') = \sigma^{-1}(1 + a_\mu\sigma^{;\mu} + a_{\mu\nu}\sigma^{;\mu}\sigma^{;\nu} + \dots). \tag{12}$$

Here a_μ, $a_{\mu\nu}$, ... are (smooth) tensors at point x and the semicolon denotes covariant derivatives with respect to the symmetric affine connection defined by the metric. It should be noted that this is not to say that such an expansion could not include additional singular terms which respect the norm condition (8). For example we could allow $W^{(2)}_{\mathrm{T,S}}(x, x')$ to involve an additional logarithmic singularity, such as in the case of Hadamard expansion. But, in that expansion the logarithmic singularity occurs because the equations governing the dynamics of the quantum field are supposed to be linear. As already mentioned in the introduction we are not satisfied with this idea. Generally there is no real justification for regarding such additional singularities as fundamental. We therefore adopt the view that additional singularities are not present. It is quite likely that at some future time we may have the occasion for improving the expansion (12), e.g. by a return to an additional singularity. But at this stage we must adhere to the principle of simplicity. In this sense the expansion is the simplest thing that one can write.

Another point is that in general there would be states in one primary folium whose behaviour do differ from that given by (12). We assert to have in (12) only a condition singling out the subclass of 'smooth states'. These are such that the amount of the energy-momentum density produced by them is finite. This point will be illustrated in section 4.

Now, for a reason which is apparent from mathematics we shall refer to the expansion (12), when terminated at some order, as the jet class associated with this order. For example the jet class of order two is determined by the tensors a_μ and $a_{\mu\nu}$. This terminology will help us to avoid confusion.

One important point should be noted about the expansion (12). In reality we must always confine ourselves in (12) to a separation of the points x and x' of scales greater than Planck length, as we are dealing with semiclassical quantum gravity. Further, we must always avoid the possibility that the separation of the points x and x' becomes too large, as we have in (12) a local expansion. In actual situations there would be always a domain of many orders of magnitude on which the expansion (12) can be valid.

Thus, if we want to develop the theory with the expansion (12) that part of the (symmetric) truncated two-point function which corresponds to a separation of the points x and x' of scales comparable with the Planck length remains unspecified. Basically, one is dealing here with a lack of determinacy. There are, however, important indications that the theory should become finite at scales below the Planck length. Once this assumption is made the Planck length would act as natural cut-off in the semiclassical theory and hence wherever we use the expansion (12) to make some calculations the end results must be replaced by their average value over the Planck regime as $x' \to x$. In this way one gets a theory in which no singularity occurs.

We shall adopt this point of view in our discussion. It will be used in the form that the average of σ^{-1} over the Planck regime gets replaced by the inverse value of the gravitational constant, κ^{-1}. We then have in the theory a sort of general principle which asserts that the effect of gravity should always be included in the local structure of states. We shall refer to this principle as the Planck structure hypothesis. This hypothesis reduces the occurrence of singularities to a peculiarity of the Minkowskian limit $\kappa \to 0$.

The discussion so far has led to a semiclassical interpretation of the theory, i.e. disregarding the Planck regime, the local macroscopic geometry arises as a common intrinsic property of a primary folium of local physical states. The next central question concerns the physical significance of jet classes and the problem of their specification. At this stage we need the notion of dynamical laws in order to proceed.

3. The local laws

The problem of specifying the jet classes in the present context is closely related to what one calls in the conventional approach the problem of renormalization of the energy-momentum tensor operator. First we note that if we wish to have a theory based on differential equations the actual construction of the jet classes must be subjected to a certain 'maximal set' of differential equations relating them to the macroscopic geometry defined by the primary folium employed. There is an objective criterion telling us what kind of equations one should incorporate in a semiclassical theory. Indeed, following the intuitive idea that the admissible physical states should carry a finite inertial and gravitational mass the equations employed have to provide us with a realization of the principle of equivalence (equality of inertial and gravitational mass). Thus the problem becomes one of how to convert this idea into appropriate mathematical constraint equations on states.

As a first preliminary step towards this goal let us assume that among all local observables of a bounded region $\mathcal{O}$ there is a specified observable, called Q, whose expectation value vanishes in each 'smooth' state belonging to a primary folium $\mathscr{F}_{\mathcal{O}}$ of local physical states, namely

$$\langle Q\rangle_\omega = 0 \qquad \forall\omega \in \mathscr{F}'_{\mathcal{O}} \tag{13}$$

where $\mathscr{F}'_{\mathcal{O}}$ denotes the class of smooth states as a subset of $\mathscr{F}_{\mathcal{O}}$. One may think of Q as being for each state sensitive to a deviation of the inertial mass from the gravitational mass. Viewed in this way the condition (13) is an essential constraint to which the relevant states must be subjected. Therefore we shall try to present the theory directly in terms of some postulates about Q.

In the present work we are primarily concerned with one feature of Q, its scaling behaviour at a point $p \in M$. On the heuristic level we shall assume here that as $\mathcal{O}$ contracts to a single point p the scaling behaviour of Q is controlled in each chart $x = \varphi(p)$ by a symmetric tensor operator $Q_{\mu\nu}(x)$. Heuristically we may then replace equation (13) by the following equations at a single point

$$\langle Q_{\mu\nu}\rangle_\omega = 0 \qquad \forall\omega \in \mathscr{F}'_{\mathcal{O}}. \tag{14}$$

Now, as we are dealing with the principle of equivalence we would expect that the operator $Q_{\mu\nu}$ involves the field operator ϕ in a non-linear manner.

It is important to realize that the constraints imposed by (14) are macroscopic in character, i.e. they need not be visible on small scales appropriate for the microscopic dynamics of the states†. This is an immediate consequence of the principle of equivalence which requires the relevant scale for the constraints (14) to be the macroscopic scale of metric inhomogeneity. The vanishing of the right-hand side of (14), therefore, turns out to be a general feature valid on large scales. We would like to emphasize that there are not any compelling reasons for the constraints (14) to hold at scales smaller than the macroscopic scale of metric inhomogeneity. Rather, we would expect that on such small scales the right-hand side of (14) will differ from zero, leading to a state-dependent residual quantity. The origin of such quantities may be found in the irreversible (dissipative) structure of the theory on small scales‡, an issue which, although very important, we will not discuss in the present article. We would expect, therefore, that the theory to be developed by means of equations (14) cannot take account of the 'relaxation processes' visible on the small scales appropriate for the microscopic dynamic of the states. In this sense we shall interpret (14) as a condition characterizing the local equilibrium§.

† Here we try to use intuitive physical arguments to understand the whole physical content of the equations (14). But, we would like to note that the assumption of the existence of small scales appropriate for the microscopic dynamics can be motivated by the physics involved in the Hawking radiation, see [18]. A further remark is needed. The order of magnitude of such scales is always imagined to be measured in an appropriate local coordinate system, e.g. the free-falling frame defined by the folium of local states employed.

‡ The motivation for this comes from our experience with the dynamical behaviour of systems with many degrees of freedom.

§ To explain the extent to which equations (14) are the defining characteristic of local equilibrium, and to establish their structural connection with some local stability group remains to be explored.

It should be clearly understood that the behaviour of the ϕ field established by (14) does not happen in the ordinary Minkowski theories. It is an entirely new feature emerging in theories including the gravitational interaction. Therefore we are led to formulate a correspondence principle. According to this principle the physical effect of the equations (14) should disappear in the non-gravitational limit $\kappa \to 0$ where the spacetime metric should become globally the Minkowski metric. We may establish this fact by requiring that the expectation value $\langle Q_{\mu\nu}\rangle_\omega$ in every state of one primary folium should satisfy the asymptotic condition

$$\langle Q_{\mu\nu}\rangle_\omega \xrightarrow{\kappa\to 0} \kappa^{-1} G_{\mu\nu} \tag{15}$$

where $G_{\mu\nu}$ is the Einstein tensor corresponding to the macroscopic metric defined by the folium of local states considered. This ensures, indeed, that in the limit $\kappa \to 0$ the requirement (14) is no longer a constraint on the states but is reduced to the identity $G_{\mu\nu} = 0$, as already satisfied in the Minkowski theories.

We may also expect here a close relationship between equations (14) and the semiclassical Einstein equations. In the next section we shall establish this relationship in more specific terms. Notice now that by (15) the whole of $Q_{\mu\nu}$ must have the dimension of a length to the power -4.

To construct $Q_{\mu\nu}$ in terms of the field operator ϕ we may start from the statement that equations (14), as local equilibrium condition, need not hold for an arbitrary field configuration but only for fields which satisfy the dynamical laws. Thus the question arises of how to supplement them by a field equation. Here we are, of course, greatly hampered by the absence of a natural approach. But, tentatively, we may write the field equation in the form

$$\Box\phi + \kappa\phi Q_\alpha{}^\alpha = 0 \tag{16}$$

where $\Box$ is the invariant d'Alembertian depending on the local primary folium employed. Notice now that as a consequence of (15) and (16) in the non-gravitational limit $\kappa \to 0$ the theory becomes one of a scalar massless field propagating in Minkowski spacetime.

In view of (16) we would expect now that the operator $Q_{\mu\nu}$ involves the derivatives of the field ϕ up to first order (otherwise we would obtain certain pathologies). Further, because of complete homogeneity of spacetime under equilibrium condition we would expect that $Q_{\mu\nu}$ cannot explicitly contain the field operator ϕ and hence must be expressible only in terms of derivatives of ϕ.

The simplest candidate for $Q_{\mu\nu}$ incorporating all the expected features will be

$$Q_{\mu\nu} = \phi_{;\mu}\phi_{;\nu}. \tag{17}$$

The hypothesis that we want to advance is that the necessary dynamical information for the semiclassical quantum gravity situation is always contained in equations (14), (16) and (17).

4. The Einstein equations

In this section we study more closely the kind of restrictions which the constraint equations (14) impose upon the structure of jet classes†. Before entering into the

† The considerations made in this section are heuristically in character. They should be understood as a first step towards a more rigorous treatment.

discussion we want to collect some technical facts. First, in the standard notation of the point separation method, see [3], equation (14) may be expressed as†

$$\langle Q_{\mu\nu}\rangle_\omega = \lim_{x'\to x} g_\nu{}^{\nu'} W^{(2)}_{\mathrm{S};\mu;\nu'}(x,x') = 0. \tag{18}$$

Here $W^{(2)}_{\mathrm{S}}$ is the symmetric part of the two-point function of ω, and $g_\nu^{\nu'}$ is the bivector of the parallel transport (here and in what follows the unprimed indices refer to tensors in tangent space at x while the primed indices refer to the tensor in tangent space at x').

An important feature of this equation is that it restricts only the structure of the jet class of order four. We refer for the discussion of the analogous situation in the frame of the conventional approach to the publications [4, 5], where attention was directed to the problem of renormalizing the energy–momentum tensor operator and singularities arising from the Hadamard expansion of the two-point function. Now, let us write down explicitly the expansion that would determine the local structure of the symmetric part of the truncated two-point function, $W'^{(2)}_{\mathrm{T,S}}$, corresponding to the jet class of order four

$$W^{(2)}_{\mathrm{T,S}}(x,x') = -12\sigma^{-1}(1 + a_\mu\sigma^{;\mu} + a_{\mu\nu}\sigma^{;\mu}\sigma^{;\nu} + a_{\mu\nu\delta}\sigma^{;\mu}\sigma^{;\nu}\sigma^{\delta} + a_{\mu\nu\delta\gamma}\sigma^{;\mu}\sigma^{;\nu}\sigma^{;\delta}\sigma^{;\gamma}) \tag{19}$$

which is similar to (12)‡. The requirement of symmetry determines the tensors a_μ and $a_{\mu\nu\delta}$ to

$$a_\mu = 0 \tag{20}$$

$$a_{\mu\nu\delta} = -\tfrac{1}{2}a_{\mu\nu;\delta}. \tag{21}$$

The simple proof may be found by looking at the symmetric covariant Taylor series, see [11].

We are now prepared to give the calculational results concerning the local behaviour of the expectation value $\langle Q_{\mu\nu}\rangle_\omega$. Using the expansion (19) and the formula (18) we find after collecting terms in like powers of $\sigma^{;\mu}$

$$\langle Q_{\mu\nu}\rangle_\omega = \langle Q_{\mu\nu}\rangle_\omega^{\mathrm{quartic}} + \langle Q_{\mu\nu}\rangle_\omega^{\mathrm{quadratic}} + \langle Q_{\mu\nu}\rangle_\omega^{0} \tag{22}$$

where

$$\langle Q_{\mu\nu}\rangle_\omega^{\mathrm{quartic}} = -12 \lim_{x'\to x} \sigma^{-2}(-2\sigma^{-1}\sigma_{;\mu}\sigma_{;\nu} + g_{\mu\nu}) \tag{23}$$

$$\begin{aligned}\langle Q_{\mu\nu}\rangle_\omega^{\mathrm{quadratic}} = -12 \lim_{x'\to x} \{&\sigma^{-3}(-2a_{\alpha\beta}\sigma^{;\alpha}\sigma^{;\beta}\sigma_{;\mu}\sigma_{;\nu}) + \sigma^{-2}[(\tfrac{1}{6}R_{\mu\alpha\nu\beta} + g_{\mu\nu}a_{\alpha\beta})\sigma^{;\alpha;}\sigma^{;\beta}\\ &+ 2a_{\mu\alpha}\sigma^{;\alpha}\sigma_{;\nu} + 2a_{\nu\alpha}\sigma^{;\alpha}\sigma_{;\mu}] - 2\sigma^{-1}a_{\mu\nu}\}\end{aligned} \tag{24}$$

$$\begin{aligned}\langle Q_{\mu\nu}\rangle_\omega^{0} = W^{(1)}_{;\mu}W^{(1)}_{;\nu} - 12 \lim_{x'\to x} \{&\sigma^{-3}(-2a_{\alpha\beta\delta\gamma}\sigma^{;\alpha}\sigma^{;\beta}\sigma^{;\delta}\sigma^{;\gamma}\sigma_{;\mu}\sigma_{;\nu})\\ &+ \sigma^{-2}[(\tfrac{1}{40}R_{\mu\alpha\nu\beta;\delta\gamma} + \tfrac{7}{360}R^{\tau}_{\alpha\mu\beta}R_{\tau\delta\nu\gamma} + g_{\mu\nu}a_{\alpha\beta\delta\gamma} + \tfrac{1}{6}R_{\mu\alpha\nu\beta}a_{\delta\gamma})\sigma^{;\alpha}\sigma^{;\beta}\sigma^{;\delta}\sigma^{;\gamma}\\ &+ (-\tfrac{2}{3}a_{\alpha\lambda}R^{\lambda}{}_{\beta\mu\delta} + a_{\alpha\beta\delta;\mu} + 4a_{\mu\alpha\beta\delta})\sigma^{;\alpha}\sigma^{;\beta}\sigma^{;\delta}\sigma_{;\nu}\\ &+ (\tfrac{1}{3}a_{\lambda\alpha}R^{\lambda}{}_{\beta\nu\delta} + 4a_{\nu\alpha\beta\delta})\sigma^{;\alpha}\sigma^{;\beta}\sigma^{;\delta}\sigma_{;\mu}]\\ &+ \sigma^{-1}(-24a_{\mu\nu\alpha\beta} - 3a_{\mu\nu\alpha;\beta} - 3a_{\mu\nu\beta;\alpha})\sigma^{;\alpha}\sigma^{;\beta}\}.\end{aligned} \tag{25}$$

† In the following expression a symmetrization with respect to the indices μ and ν must be done so that $Q_{\mu\nu}$ becomes symmetric. For simplicity we shall make the symmetrization only at the end.

‡ For technical reasons we have separated off the factor -12.

Here $W^{(1)}$ is the one-point function. In writing the above expressions we have suppressed direction-dependent terms involving odd powers of $\sigma^{;\alpha}$, since such terms may be eliminated by averaging over a separation of the point x in the $\sigma^{;\alpha}$ direction and one in the $-\sigma^{;\alpha}$ direction. There remains still a difficulty concerning direction-dependent terms involving even powers of $\sigma^{;\mu}$. To get rid of direction-dependence of such terms, that is, in order for $\langle Q_{\mu\nu}\rangle_\omega$ to be a true tensor at point x, one has to average over all directions using a suitable measure. Following the work of Adler, Lieberman and Ng [4] in what follows we use the elementary averaging procedure which consists in making the replacements

$$
\begin{aligned}
&\sigma^{;\alpha}\sigma^{;\beta} \to \tfrac{1}{2}\sigma g^{\alpha\beta}\\
&\sigma^{;\alpha}\sigma^{;\beta}\sigma^{;\delta}\sigma^{;\gamma} \to \tfrac{1}{6}\sigma^2(g^{\alpha\beta}g^{\delta\gamma}+g^{\alpha\delta}g^{\beta\gamma}+g^{\alpha\gamma}g^{\beta\delta})\\
&\sigma^{;\mu}\sigma^{;\nu}\sigma^{;\alpha}\sigma^{;\beta}\sigma^{;\delta}\sigma^{;\gamma}\\
&\qquad \to \tfrac{1}{24}\sigma^3\{g^{\mu\nu}(g^{\alpha\beta}g^{\delta\gamma}+g^{\alpha\delta}g^{\beta\gamma}+g^{\alpha\gamma}g^{\beta\delta})\\
&\qquad\quad +g^{\mu\alpha}(g^{\nu\beta}g^{\delta\gamma}+g^{\nu\delta}g^{\beta\gamma}+g^{\nu\gamma}g^{\beta\delta})\\
&\qquad\quad +g^{\mu\beta}(g^{\nu\alpha}g^{\delta\gamma}+g^{\nu\delta}g^{\alpha\gamma}+g^{\nu\gamma}g^{\alpha\delta})+g^{\mu\delta}(g^{\nu\alpha}g^{\beta\gamma}+g^{\nu\beta}g^{\alpha\gamma}+g^{\nu\gamma}g^{\beta\alpha})\\
&\qquad\quad +g^{\mu\gamma}(g^{\nu\alpha}g^{\beta\delta}+g^{\nu\beta}g^{\alpha\delta}+g^{\nu\delta}g^{\alpha\beta})\}.
\end{aligned}
\tag{26}
$$

In consequence of this averaging, the term $\langle Q_{\mu\nu}\rangle_\omega^{\text{quartic}}$ vanishes identically. For the second term in (22) we find

$$
\langle Q_{\mu\nu}\rangle_\omega^{\text{quadratic}} = -\lim_{x'\to x}\sigma^{-1}(R_{\mu\nu}-8a_{\mu\nu}+2g_{\mu\nu}a_\alpha{}^\alpha). \tag{27}
$$

Now, according to our Planck structure hypothesis, stated in section 3, we have to replace expression (27) by

$$
\langle Q_{\mu\nu}\rangle_\omega^{\text{quadratic}} = -\kappa^{-1}(R_{\mu\nu}-8a_{\mu\nu}+2g_{\mu\nu}a_\alpha{}^\alpha). \tag{28}
$$

Turning now to the evaluation of the last term in (22) we find after averaging

$$
\langle Q_{\mu\nu}\rangle_\omega^0 = W^{(1)}_{;\mu}W^{(1)}_{;\nu}+\tau_{\mu\nu}+H_{\mu\nu} \tag{29}
$$

where

$$
\tau_{\mu\nu} = 156a_{\mu\nu\lambda}{}^\lambda - 36g_{\mu\nu}a_\alpha{}^\alpha{}_\lambda{}^\lambda \tag{30}
$$

and

$$
\begin{aligned}
H_{\mu\nu} = {}& -\tfrac{1}{20}(\Box R_{\mu\nu}+R_\mu{}^\lambda{}_\nu{}^\xi{}_{;\lambda;\xi}+R_\mu{}^\lambda{}_\nu{}^\xi{}_{;\xi;\lambda})-\tfrac{7}{180}(R^\tau{}_{\lambda\mu}{}^\lambda R_{\tau\xi\nu}{}^\xi+R^\tau{}_{\lambda\mu\xi}R_\tau{}^\lambda{}_\nu{}^\xi+R^\tau{}_{\lambda\mu\xi}R_\tau{}^\xi{}_\nu{}^\lambda)\\
& -\tfrac{1}{3}(a_\alpha{}^\alpha R_{\mu\nu}+2R_{\mu\lambda\nu\alpha}a^{\lambda\alpha})+\tfrac{4}{3}(a_{\alpha\lambda}R^{\lambda\alpha}{}_{\mu\nu}+a_{\alpha\lambda}R^\lambda{}_{\nu\mu}{}^\alpha+a_{\nu\lambda}R^\lambda{}_\mu)\\
& -\tfrac{2}{3}(a_{\lambda\alpha}R^{\lambda\alpha}{}_{\nu\mu}+a_{\lambda\alpha}R^\lambda{}_{\mu\nu}{}^\alpha+a_{\lambda\mu}R^\lambda{}_\nu)+2a_{\mu\lambda}R^\lambda{}_{\alpha\nu}{}^\alpha-4(a_{\nu\lambda}R^\lambda{}_{\alpha\mu}{}^\alpha\\
& +a_{\alpha\lambda}R^\lambda{}_{\nu\mu}{}^\alpha+a_{\alpha\lambda}R^{\lambda\alpha}{}_{\mu\nu})-2(a^\lambda{}_{\lambda\nu;\mu}+2a_{\mu\lambda}{}^\lambda{}_{;\nu})+18a_{\nu\lambda}{}^\lambda{}_{;\mu}.
\end{aligned}
\tag{31}
$$

Now putting all these results together and looking back at (18) we find

$$
\langle Q_{\mu\nu}\rangle_\omega = -\kappa^{-1}(R_{\mu\nu}-8a_{\mu\nu}+2g_{\mu\nu}a_\alpha{}^\alpha)+W^{(1)}_{;\mu}W^{(1)}_{;\nu}+H_{\mu\nu}+\tau_{\mu\nu}=0. \tag{32}
$$

We may use at this point the correspondence principle (15) to obtain

$$
a_{\mu\nu} = \tfrac{1}{4}R_{\mu\nu}. \tag{33}
$$

This determines the jet class of order two. Consequently, equation (32) take the form†

$$G_{\mu\nu} = -\kappa S_{(\mu\nu)}\{\omega\} \tag{34}$$

where

$$S_{\mu\nu}\{\omega\} = W^{(1)}_{;\mu} W^{(1)}_{;\nu} + H_{\mu\nu} + \tau_{\mu\nu}. \tag{35}$$

In (34) we have a set of 10 equations to which the actual construction of the allowed jet class of order four has to be subjected. These equations relate the one- and two-point function and correspond to the standard form of the semiclassical Einstein equations, the quantum source of gravity being $S_{\mu\nu}\{\omega\}$.

Let us now look at the tensor $\tau_{\mu\nu}$. We immediately see a connection between that tensor and the amount of energy momentum contained in the local part of the two-point function. Actually, the tensor $\tau_{\mu\nu}$ is the basic dynamical variable occurring in the theory and one should always imagine different states in one primary folium to differ in the behaviour of $\tau_{\mu\nu}$. Only in this way can we get a theory which is basically in accord with the standard ideas of general relativity.

From the standpoint of the Cauchy problem, equation (34) alone does not provide a determinate mathematical problem. We need, namely, an equation by which the quantum source of gravity can be computed independently. This gap is now filled by taking into account the field equation (16). Indeed, using the point separation method, we may derive from (16) the following equations for the one-point function and the two-point function‡

$$\Box W^{(1)}(x) = -\kappa \lim_{x'\to x} \lim_{x''\to x'} g^{\alpha''}_{\alpha'} W^{(3);\alpha'}_{;\alpha''}(x, x', x'') \tag{36}$$

$$\Box W^{(2)}(x, x') = -\kappa \lim_{x''\to x} \lim_{x'''\to x''} g^{\alpha''}_{\alpha'''} W^{(4);\alpha'''}_{;\alpha''}(x, x'', x''', x') \tag{37}$$

where $W^{(3)}$ and $W^{(4)}$ are the three-point function (symmetrized in x' and x'') and the four-point function (symmetrized in x'', x''') respectively.

There is a technical problem if we try to treat the Cauchy problem, because of the term $H_{\mu\nu}$ in (35). That term involves, namely, the fourth-order derivatives of the metric and terms which are quadratic in curvature.

Now, appealing to the idea of the local equilibrium, one might argue that the effect of $H_{\mu\nu}$ will appear small in comparison with other terms in (34) and hence one could put the theory in a more sensible form by neglecting that tensor. But, one cannot get a reasonable interpretation of equations by adopting this picture. It is, namely, quite likely that the tensor $H_{\mu\nu}$, even if it is small, would lead to inappropriate stability properties of solutions§. At first sight this seems to prevent us from obtaining a reasonable set of equations. But, we would like the reader to notice that the scheme developed might open up a new possibility to overcome this difficulty. It may be that $H_{\mu\nu}$ could be entirely compensated for by a corresponding counter-term in the remainder of $S_{\mu\nu}\{\omega\}$. Such a counter-term could be produced by the tendency of the state towards local equilibrium. But a consistent incorporation of this idea requires a detailed

† In the following the parentheses around the indices denotes the symmetric part of a tensor.
‡ From the field equation (16) one obtains an infinite sequence of equations, each relating $W^{(n)}$ to $W^{(n+2)}$. Of particular importance is the question of how to truncate this sequence at some suitable order by means of the equations (14) and some reasonable approximations to obtain the whole dynamics of local equilibrium.
§ The requirement that the right-hand side of the Einstein equations should not contain terms like $H_{\mu\nu}$ was originally suggested in [6].

knowledge about the structure of local equilibrium, i.e. about the whole dynamics involved in the local equilibrium. Further investigations are needed here to resolve the issue.

At this point a remark concerning the position of our approach with respect to the conventional theory appears to be in place. We have already mentioned that the conventional theory is based on the idealized assumption of a linear dynamical equation for the quantum field which dismisses a possible correlation between the dynamics of the two-point function and that of the entire hierarchy of other functions. Remarkably the significance and the validity of this assumption have never been seriously discussed in the literature†.

How difficult in the framework of the conventional theory the problem of defining the right-hand side of the Einstein equations (1) is, may be illustrated by the following example. In many problems one studies a massless quantum field obeying the equation

$$\Box\phi = 0 \tag{38}$$

which differs from (16) by the absence of the self-interacting term $\kappa\phi Q_\alpha{}^\alpha$. Starting from the standard expression of $T_{\mu\nu}$ for this field, calculations show, [4], that the divergent part of $\langle T_{\mu\nu}\rangle$ is universal for all states whose local structure is restricted to the Hadamard form. The leading divergency of $\langle T_{\mu\nu}\rangle$ has the form $\alpha G_{\mu\nu}+\beta g_{\mu\nu}$, α and β being quadratically divergent. The term $\alpha G_{\mu\nu}$ gives rise to the renormalization of the gravitational constant. The term $\beta g_{\mu\nu}$ can be cancelled by introducing into the Einstein equations a diverging cosmological counter-term, leading to the renormalization of the cosmological constant‡. $\langle T_{\mu\nu}\rangle$ contains further a divergent term which diverges logarithmically and a finite-state-dependent term. There is no conclusive treatment of these terms powerful enough to decide whether the theory is capable of unambiguous definition of the renormalized energy-momentum tensor. One might argue that the logarithmic divergent term could be dropped by introducing into the left-hand side of the Einstein equations a corresponding diverging counter-term, but this procedure is somewhat arbitrary, because the diverging counter-term is only unique up to an arbitrary finite part and this ambiguity evidently affects the finite term of $\langle T_{\mu\nu}\rangle$§. It is therefore unclear what meaning can be attributed to the right-hand side of the Einstein equations by means of the renormalization prescriptions. An axiomatic approach might be more serious, see [6, 7], but we would like the reader to notice that, at a more fundamental level, the conventional theory suffers from the inability to explain why, in many interesting situations, the dynamics contained in the Einstein equations is clearly separated from the scales on which the quantum fields can be probed, see [18].

Our own interest in the formulation of a non-linear theory results from the realization that a suitable non-linear generalization of the conventional theory is needed to overcome the difficulties in defining the right-hand side of the Einstein equations and in interpreting them as a condition of local equilibrium. The principal virtue of our

† A justification of the conventional scheme on the basis of many attractive results that have been obtained in that scheme seems to be less obvious. For example a serious difficulty of the conventional scheme concerns the role played by sub-Planckian scales in the usual derivation of the Hawking radiation. For the discussion of this point we refer the reader to the works [17, 18].

‡ We would like to mention that the term $\beta g_{\mu\nu}$ can also be cancelled by introducing into the field equation (38) a diverging mass term.

§ We expect that in cancelling the leading divergency of $\langle T_{\mu\nu}\rangle$ in the massless case this sort of ambiguity, on dimensional grounds, will not affect the finite term of $\langle T_{\mu\nu}\rangle$.

approach as compared to the conventional theory is that the right-hand side of the Einstein equations comes out in a natural way by means of the condition of local equilibrium and the assumption of a Planck scale cut-off. Our basic strategy was that the whole (state-dependent) dynamics of the local equilibrium carries the local characteristics of the non-linear field equation (16) on large scales.

5. Concluding remarks

We hope to have demonstrated a new possibility of thinking about semiclassical quantum gravity. Let us summarize once again the basic steps.

Starting from the principle of equivalence we have attributed the corresponding non-linear constraint equations (14) to quantum gravitation. The basic input here was the assumption that the relevant local states belong to one primary folium exhibiting a specific universal short distance structure. The latter property was essential in introducing the notion of macroscopic spacetime metric. This acts as a superselection quantity separating different folia of local states. To answer the question which folium of local states is actually realized we have to solve the non-linear field equation (16) together with the constraint equations (14), (17) subject to appropriate boundary conditions. In this sense different folia of local states are connected by dynamical laws.

The nature of the dynamics in this scenario is, however, at this stage of development obscure, e.g. it is still not clear whether the Cauchy development respects the local structure of the truncated two-point function assumed in (19), on which the results of this work are based. But, to this problem some understanding of the local behaviour of the higher functions seems to be an essential prequisite. We feel confidence that a rigorous justification of this scenario can be given. Concerning 'the thermodynamic aspects' of the theory there is the problem of a deeper understanding of constraint equations which we have called the condition of local equilibrium. There must also be some changes to be introduced into these equations in order to include the effect of local entropy production.

The other important question that remains to be answered concerns the relation of our approach to a 'Lagrangian' and its corresponding energy–momentum tensor. From the conceptual point of view it is, of course, entirely open whether investigations in quantum gravity should follow the orthodox picture of Lagrangian formalism. Here we merely note that it is perhaps possible that the basic nature of the macroscopic metric to be essentially a state-dependent quantity limits the effectiveness of such a picture.

In conclusion, let us point out that we have concentrated in this article on the broad line of the development of a 'possible theory', rather than on any attempts at a rigorous justification of our assumptions. It is our belief that a rigorous formulation of a theory along the lines suggested will have a beneficial effect upon our understanding of quantum gravity.

Acknowledgments

I am indebted to Professor R Haag for many stimulating discussions and valuable suggestions and I thank Professor K Fredenhagen for a helpful discussion. Thanks are also due to an anonymous referee for useful suggestions and comments. Finally, I would like to thank Professor H D Doebner for kind hospitality at the Arnold Sommerfeld Institute where this work was revised.

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