# Constraints on anomalous WW $\gamma$ and WW $\gamma \gamma$ couplings at a 500 GeV linear $\mathrm{e}^{+} \mathrm{e}^{-}$collider 

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#### Abstract

We examine the potential of a future 500 GeV linear $\mathrm{e}^{+} \mathrm{e}^{-}$collider (NLC) to probe the "minimal" set ( $\kappa_{\gamma}, \lambda_{\gamma}$ ) of anomalous $W W \gamma$ and $W W \gamma \gamma$ couplings via the (sub) processes $\gamma \gamma \rightarrow W^{+} W^{-}, \mathrm{e}^{-} \gamma \rightarrow W^{-} v$ and $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathbf{W}^{+} \mathbf{W}^{-}$, for comparison. Photon beams both from classical Bremsstrahlung and, notably, from backscattering laser light off $e^{ \pm}$beams are considered. The differential cross sections $\mathrm{d} \sigma / \mathrm{d} \cos \theta$ of the three reactions are calculated analytically and used as observables in a $\chi^{2}$ analysis under identical assumptions on machine parameters. The constraints emerging from the use of laser photon beams appear very encouraging, are independent of $\kappa_{\mathrm{Z}}$ and $\lambda_{\mathrm{Z}}$ and impressively underline the importance of realizing laser photon beams at the NLC.


Research and development on linear $\mathrm{e}^{+} \mathrm{e}^{-}$colliders at SLAC, KEK, DESY, CERN and Novosibirsk has been progressing rapidly and the physics potential of such future machines is presently under intensive study. A very important task at a future 500 GeV linear $\mathrm{e}^{+} \mathrm{e}^{-}$collider (NLC) is to verify the non abelian gauge structure of the electroweak standard model by probing the three and four gauge boson couplings as directly and precisely as possible.

Of particular interest is the "classical" set of anomalous $W W \gamma$ and $W W Z$ couplings $\kappa_{\gamma, Z}$ and $\lambda_{\gamma, Z}$, with $\kappa_{\gamma}, \lambda_{\gamma}$ related to the W magnetic dipole and electric quadrupole moments $\mu_{\mathrm{W}}$ and $Q_{\mathrm{W}}$, respectively, as
$\mu_{\mathrm{w}}=\frac{e}{2 m_{\mathrm{w}}}\left(1+\kappa_{\mathrm{y}}+\lambda_{\gamma}\right), \quad Q_{\mathrm{w}}=-\frac{e}{m_{\mathrm{w}}^{2}}\left(\kappa_{\mathrm{y}}-\lambda_{\gamma}\right)$,
and
$k_{\mathrm{v}}=1, \quad \lambda_{\mathrm{v}}=0, \quad \mathrm{~V}=\gamma, \mathrm{Z}$,
at tree level in the standard model. The possibilities of probing these crucial couplings both at future $\mathrm{e}^{+} \mathrm{e}^{-}$and hadron colliders have been extensively investigated in the literature [1-8].

At $\mathrm{e}^{+} \mathrm{e}^{-}$colliders starting with LEP II, an important process in this respect is $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}$. However, here, the $W W \gamma$ and $W W Z$ vertices both enter and the associated anomalous couplings conspire to weaken the bounds in absence of further theoretical input [3]. At NLC energies of 500 GeV , say, the situation improves considerably [8], yet separate tests of photon and Z anomalous couplings are desirable.

In this letter, we report on a comparative study of the sensitivity to the anomalous couplings $\kappa_{\gamma}$ and $\lambda_{\gamma}$, in the following (sub) processes

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\(\gamma \gamma \rightarrow W^{+} W^{-} \quad\left(e^{+} \mathrm{e}^{-} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-} \mathrm{W}^{+} \mathrm{W}^{-}\right)\),
\(e^{-\gamma} \boldsymbol{\gamma} W^{-} v \quad\left(e^{+} e^{-} \rightarrow e^{+} W^{-} v\right)\),
\(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}\),
\(e^{-\gamma} \boldsymbol{\gamma} W^{-} v \quad\left(e^{+} e^{-} \rightarrow e^{+} W^{-} v\right)\),
\(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}\),
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under identical assumptions on NLC parameters.

Our new results refer mainly to the reactions (4) and (5), while the "classical" reaction (6) has been recalculated mainly to serve as a reference under the same conditions. Particular emphasis will be put in this paper on the promising possibility [ $5,9,10$ ], to obtain colliding $\gamma \gamma$ and $\gamma e$ beams at the NLC with approximately the same energies and luminosities as in $\mathrm{e}^{+} \mathrm{e}^{-}$collisions by means of Compton (back) scattering of laser light. For comparison, we also discuss the results obtained with the photon spectra from classical Bremsstrahlung in Weiz-säcker-Williams approximation.
The reactions (4) and (5) enjoy a number of features making them potentially very interesting at linear $\mathrm{e}^{+} \mathrm{e}^{-}$ colliders of high energy:

- Due to the exchange of a (massive) vector boson in the $t$ channel the total cross sections for both reactions (4) and (5) approach a constant ( $\approx 86 \mathrm{pb}$ and 47 pb , respectively) in the standard model at high energies, while the $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}$cross section vanishes asymptotically.
- Both reactions (4) and (5) are sensitive only to the photon anomalous couplings $\kappa_{\gamma}$ and $\lambda^{\gamma}$.
- Given the anomalous $W W \gamma$ vertex, local $U(1)_{\text {e.m. }}$ invariance requires the existence of an anomalous $W W \gamma \gamma$ vertex with strength proportional to $\lambda_{\gamma}$ (see eqs. (7), (8) below). At future linear $\mathrm{e}^{+} \mathrm{e}^{-}$colliders, this new four gauge boson interaction can be probed for the first time in the process (4) and to our knowledge, has not been considered before (for a recent application to heavy ion colliders, see ref. [11]).
Let us start by considering the most general $C$ - and $P$-conserving WW $\gamma$ interaction term in the lagrangian [ 1], suplemented with (minimal) WW $\gamma \gamma$ and $W W \gamma \gamma \gamma$ terms to render it (manifestly) invariant under local electromagnetic $\mathrm{U}(1)$ gauge transformations
$\mathscr{L}_{\mathrm{W} \gamma}=\mathscr{L}_{\mathrm{W} \gamma}^{\mathrm{SM}}-\mathrm{i} e\left(\left(\kappa_{\gamma}-1\right) W_{\mu}^{\dagger} W_{\nu} F^{\mu \nu}+\frac{\lambda_{\gamma}}{m_{\mathrm{W}}^{2}} W_{\sigma \mu}^{\dagger} W_{\nu}^{\mu} F^{v \sigma}\right)$,
where
$W_{\mu \nu}=\left(\partial_{\mu}-\mathrm{i} e A_{\mu}\right) W_{\nu}-\left(\partial_{\nu}-\mathrm{i} e A_{\nu}\right) W_{\mu}, \quad F_{\mu \nu}=\partial_{\mu} A_{\nu}-\partial_{\nu} A_{\mu}$.
Moreover, for the process $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}$we add the standard $W W Z$ anomalous vertex [1] obtained from eq. (7) by the replacements $\gamma \rightarrow \mathrm{Z}$ and $\mathrm{e} \rightarrow \mathrm{e} \cot \theta_{\mathrm{w}}$, where $\theta_{\mathrm{w}}$ is the weak mixing angle.

The next step consists in calculating the tree level differential cross sections for the three reactions (4)-(6) by means of the Feynman rules from the specified (effective) lagrangian. The tree level Feynman graphs corresponding to reactions (4)-(6) are displayed in figs. la-Ic. All our calculations were performed analytically without any further approximations by means of the algebraic manipulation packages REDUCE and MATHEMATICA.
Our results take the following form in the respective center-of-mass systems with total (sub)energy $\hat{s}$ and CMS angle $\theta=\angle(\gamma, \mathrm{W})$.

- For $\gamma \gamma \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}$, we find
$m_{\mathrm{w}}^{2} \frac{\mathrm{~d} \sigma_{\gamma \gamma+\mathrm{w}+\mathrm{w}-}}{\mathrm{d} \cos \theta}=\frac{\pi \alpha^{2}}{32} \frac{\left(1-Y^{-1}\right)^{1 / 2}}{D^{2}} F(Y, \theta)$,
where

$$
\begin{align*}
& F(Y, \theta)=F_{1}+\chi_{\gamma} F_{2}-\lambda_{\gamma} F_{3}+\chi_{\gamma}^{2} F_{4}-\chi_{\gamma} \lambda_{\gamma} F_{5}+\lambda_{\gamma}^{2} F_{6}+\chi_{\gamma}^{3} F_{\gamma}-\chi_{\gamma}^{2} \lambda_{\gamma} F_{8}+\chi_{\gamma} \lambda_{\gamma}^{2} F_{9}-\lambda_{\gamma}^{3} F_{10} \\
& \quad+\chi_{\gamma}^{4} F_{11}-\chi_{\gamma}^{3} \lambda_{\gamma} F_{12}+\chi_{\gamma}^{2} \lambda_{\gamma}^{2} F_{13}-\chi_{\gamma} \lambda_{\gamma}^{3} F_{14}+\lambda_{\gamma}^{4} F_{15}, \tag{10}
\end{align*}
$$

with
$F_{1}=8\left[3 \frac{D^{2}}{Y}-2 D\left(8+\frac{3}{Y}\right)+32 Y+\frac{6}{Y}\right], \quad F_{2}=32(5 D-16 Y), \quad F_{3}=64 D$,
(a)

$=$

$+$

$+$

(b)

$=$

$+$

(c)

$=$

$+$

$+$


Fig. 1. Feynman diagrams for the processes (a) $\gamma \gamma \rightarrow W^{+} W^{-}$, (b) $e^{-\gamma} \boldsymbol{\gamma} W^{-} v$ and (c) $e^{+} e^{-} \rightarrow W^{+} W^{-}$at tree level. The black dots indicate the vertices where anomalous vector boson couplings enter.
$F_{4}=32\left[D^{2}(Y-2)+3 D Y-D+10 Y\right], \quad F_{5}=64\left(-2 D^{2}-3 D Y+2 Y\right)$,
$F_{6}=16\left[-D^{3}+2 D^{2}(6 Y-1)+2 D Y(4 Y-7)+4 Y\right], \quad F_{7}=16\left[-2 D^{2}(2 Y-1)+D(5 Y+1)-4 Y\right]$,
$F_{8}=16\left[2 D^{2}(2 Y+3)+D(8 Y-3)-8 Y\right], \quad F_{9}=16\left[-2 D^{2}(7 Y-3)-D Y(8 Y-17)-3 D-4 Y\right]$,
$F_{10}=16 D\left[4 D^{2} Y+2 D\left(4 Y^{2}-8 Y+1\right)+4 Y-1\right], \quad F_{11}=-D^{3}+2 D^{2}(7 Y-1)+2 D(6 Y+1)+4 Y$,
$F_{12}=4\left[-D^{3}-2 D^{2}(5 Y+1)+2 D+4 Y\right]$,
$F_{13}=2\left[D^{3}(10 Y-3)+2 D^{2}\left(8 Y^{2}+3 Y-3\right)+2 D\left(8 Y^{2}-18 Y+3\right)+12 Y\right]$,
$F_{14}=4\left[-D^{3}(6 Y+1)-2 D^{2}\left(8 Y^{2}-21 Y+1\right)-2 D(12 Y-1)+4 Y\right]$,
$F_{15}=12 D^{4} Y+D^{3}\left(32 Y^{2}-52 Y-1\right)+2 D^{2}\left(32 Y^{3}-80 Y^{2}+55 Y-1\right)+2 D\left(16 Y^{2}-18 Y+1\right)+4 Y$,
( 11 cont'd)
and the abbreviations introduced in ref. [7]
$\chi_{\gamma}=1-\kappa_{\gamma}$,
$Y=\hat{s} / 4 m_{\mathrm{w}}^{2}, \quad D=\cos ^{2} \theta+Y \sin ^{2} \theta$.
For the special case $\lambda_{\gamma}=0$, our result agrees with that of ref. [7] apart from a term $-16 D Y$ missing in eq. (4.6) of ref. [7].

- For $\mathrm{e}^{-} \gamma \rightarrow \mathrm{W}^{-} v$ with arbitrary circular photon polarization characterized by the Stokes parameter $-1 \leqslant \xi_{2} \leqslant 1$, the result is
$m_{\mathrm{w}}^{2} \frac{\mathrm{~d} \sigma_{\mathrm{ve}} \cdot \mathrm{w}_{\mathrm{v}}}{\mathrm{d} \cos \theta}=\frac{\pi \alpha^{2}}{512 \sin ^{2} \theta_{\mathrm{w}}} \frac{Y-\frac{1}{4}}{Y^{3}(Y-Z)^{2}} G(Y, \theta)$,
where
$G(Y, \theta)=G_{\mathrm{unpol}}+\xi_{2} G_{\mathrm{pol}}$,
with

$$
\begin{align*}
G_{\text {unpol }} & =Z\left(8 Y-4+\frac{8 Z^{2}+4 Z+1}{Y}\right)-8 \chi_{\gamma} Z(Y+Z)+\left(\chi_{\gamma}+\lambda_{\gamma}\right)^{2}\left[\left(Y^{2}+Z^{2}\right)(4 Y-4 Z-1)+4 Y Z\right] \\
& -32\left(\chi_{\gamma}+\lambda_{\gamma}\right) \lambda_{\gamma} Y Z(Y-Z)+64 \lambda_{\gamma}^{2} Y Z(Y-Z)^{2},  \tag{16}\\
G_{\text {pol }} & =-4 Z\left(2 Y-1-\frac{Z(2 Z+1)}{Y}\right)+4 \lambda_{\gamma} Z-4\left(\chi_{\gamma}+2 \lambda_{\gamma}\right) Z(2 Y-2 Z-1)+\left(\chi_{\gamma}+\lambda_{\gamma}\right)^{2}\left(Y^{2}-Z^{2}\right)(4 Y-4 Z-1), \tag{17}
\end{align*}
$$

and the abbreviation
$Z=\frac{1}{2}\left(Y-\frac{1}{4}\right)(1+\cos \theta)=-\hat{u} / 4 m_{\mathrm{W}}^{2}$.
In the special case $\lambda_{\gamma}=0, \xi_{2}=0$, our result agrees with the one given in refs. [12,13], and in the standard model limit for $\xi_{2} \neq 0$, with the cross section of refs. [14,15]. Moreover, our numerical results for $\xi_{2}=0$ but $\chi_{\gamma}, \lambda_{y} \neq 0$, (see below) are similar to the ones of Yehudai as reported in ref. [5]. The consequences of probing polarization effects corresponding to other nonvanishing components of the Stokes vector, $\xi_{1} \neq 0$ (for $C P$ violating anomalous couplings) and $\xi_{3} \neq 0$, will be reported elsewhere [16].

- For $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}$the cross section as a function of $\kappa_{r, Z}$ and $\lambda_{r, Z}$ is well known and may e.g. be found in ref. [1]. For simplicity, we shall, henceforth, assume $1-\kappa_{z}=\lambda_{z}=0$. Other plausible choices, compatible with lowenergy information [4] or $\operatorname{SU}(2)$ symmetry arguments, like $\kappa_{Z}=1, \lambda_{Z}=\lambda_{\gamma}$ or $\kappa_{Z}=\kappa_{\gamma}, \lambda_{z}=\lambda_{\gamma}$ actually lead to stronger bounds on ( $\kappa_{\gamma}, \lambda_{\gamma}$ ) if combined with the results from the reactions (4), (5). The subprocesses $\gamma \gamma \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}$and $\mathrm{e}^{-} \gamma \rightarrow \mathrm{W}^{-} \nu$ may be (approximately) related to $\mathrm{e}^{+} \mathrm{e}^{-}$collisions by folding the respective cross sections with appropriate differential $\gamma \gamma$ and $\gamma$ e luminosity functions, respectively ${ }^{\# 1}$
$\frac{\mathrm{d} \sigma_{\mathrm{c}+\mathrm{c}-\cdots \mathrm{c}+\mathrm{c}-\mathrm{w}+\mathrm{w}-}}{\mathrm{d} \cos \theta} \simeq \int_{4, n^{2} / s}^{1} \mathrm{~d} \tau \frac{\mathrm{~d} L_{\gamma \gamma}(\tau)}{\mathrm{d} \tau} \frac{\mathrm{d} \sigma_{\gamma \gamma}+\mathrm{w}+\mathrm{w}-}{\mathrm{d} \cos \theta}$,
$\frac{\mathrm{d} \sigma_{\mathrm{c}+\mathrm{e}-\mathrm{c}+\mathrm{w}-\mathrm{v}}}{\mathrm{d} \cos \theta} \simeq \int_{m_{\mathrm{w}}^{2} / s}^{1} \mathrm{~d} \tau \frac{\mathrm{~d} L_{\mathrm{yc}}(\tau)}{\mathrm{d} \tau} \frac{\mathrm{d} \sigma_{\mathrm{ye}}-\cdot \mathrm{w}-v}{\mathrm{~d} \cos \theta}$,
where the differential luminosity $\mathrm{d} L_{i j} / \mathrm{d} \tau$ is defined as usual in terms of the momentum distributions $f_{i j}$ of particles $i$ and $j$
$\frac{\mathrm{d} L_{i j}}{\mathrm{~d} \tau}=\int_{\tau / / \mathrm{m} \mathrm{m}}^{\mathrm{l}_{\mathrm{m}}} \frac{\mathrm{d} x}{x} f_{i}(x) f_{j}(\tau / x), \quad \tau=\frac{\hat{s}}{s}$,
with $s$ being the total $\mathrm{e}^{+} \mathrm{e}^{-} \mathrm{CM}$ energy squared and $x_{\mathrm{m}} \leqslant 1$ corresponding to the maximum momentum allowed kinematically.

We consider next two very different sources for the photon spectrum in eqs. (19) and (20).
(i) Classical photon Bremsstrahlung. In this case, corresponding to the familiar Weizsäcker-Williams approximation, the photon luminosity spectrum is very soft and with the help of eq. (21) and $x_{\mathrm{m}}=1$ takes the wellknown form

$$
\begin{align*}
\frac{\mathrm{d} L_{\gamma \mathrm{c}}}{\mathrm{~d} \tau}(\tau) & =\left(\frac{\alpha}{2 \pi} \ln \frac{s}{4 m_{\mathrm{e}}^{2}}\right) \frac{1+(1-\tau)^{2}}{\tau} \equiv f_{\gamma}(\tau),  \tag{22}\\
\frac{\mathrm{d} L_{\gamma \gamma}}{\mathrm{d} \tau}(\tau) & =\left(\frac{\alpha}{2} \ln \frac{s}{4 m_{\mathrm{e}}^{2}}\right)^{2}\left(\frac{1}{\tau}(2+\tau)^{2} \ln \frac{1}{\tau}-\frac{2}{\tau}(1-\tau)(3+\tau)\right) . \tag{23}
\end{align*}
$$

[^0](ii) Laser photons. The idea of Compton scattering laser light off $\mathrm{e}^{-}$and $\mathrm{e}^{+}$beams of a single pass $\mathrm{e}^{+} \mathrm{e}^{-}$ collider was first discussed a long time ago by Akerlof [17] and Ginzburg et al. [9,10]. This possibility appears quite realistic [5] and is most interesting in our context, since the spectrum of the scattered laser photons is hard and the total luminosities for $\gamma e$ and $\gamma \gamma$ collisions turn out to be of the same order as the one for $\mathrm{e}^{+} \mathrm{e}^{-}$ collisions [9,10]. In this case, the corresponding differential luminosity functions are obtained from the momentum distribution of the Compton scattered laser photons ${ }^{* 2}$ [9,10]
$f_{\gamma}^{\text {tascr }}\left(x_{0}, x\right) \equiv \frac{1}{\sigma_{\mathrm{C}}} \frac{\mathrm{d} \sigma_{\mathrm{C}}}{\mathrm{d} x}=\frac{1-x+1 /(1-x)-4 x / x_{0}(1-x)+4 x^{2} / x_{0}^{2}(1-x)^{2}}{\left(1-4 / x_{0}-8 / x_{0}^{2}\right) \ln \left(1+x_{0}\right)+\frac{1}{2}+8 / x_{0}-1 / 2\left(1+x_{0}\right)^{2}}$,
where the dimensionless parameter
$x_{0}=\frac{2 \sqrt{s} \omega_{\text {1aser }}}{m_{\mathrm{e}}^{2}}$
characterizes the dependences on the total $\mathrm{e}^{+} \mathrm{e}^{-}$energy $\sqrt{s}$, the laser energy $\omega_{\text {lascr }}$ and the electron mass $m_{e}$, and will be taken [5] to be $x_{0}=4.82$, corresponding to $\omega_{\text {laser }} \simeq 1.26 \mathrm{eV}$. The differential luminosity functions are then obtained as
$\frac{\mathrm{d} L_{\mathrm{rc}}^{\text {laser }}}{\mathrm{d} \tau}=f_{\gamma}^{\text {laser }}\left(x_{0}, \tau\right)$,
and $\mathrm{d} L_{\underset{\gamma}{\text { aser }} / \mathrm{d} \tau}$ by integrating eq. (21) analytically with eq. (24) and $x_{\mathrm{m}}=x_{0} /\left(1+x_{0}\right)$. The resulting expression is, however, too long to be quoted here. In figs. 2,3 the luminosity functions weighting the differential cross sections in the integrals (19), (20) are plotted, with the dashed curves corresponding to the classical Bremsstrahlung (Weizsäcker-Williams) and the solid lines to the laser photon spectrum. The differences are dramatic in both cases. Due to the hardness of the laser photon spectra, subenergies of the order $\sqrt{\hat{s}} \simeq 0.8 \sqrt{s}$ are involved
*2 For the e- $\gamma$ conversion factor $k=O(1)$ we tacitly assume [5] $k=1$.


Fig. 2. Equivalent photon spectra that may be obtained from an electron beam with energy 250 GeV . The dotted line shows the Weiz-säcker-Williams spectrum due to classical Bremsstrahlung. The spectrum from a backscattered laser beam is shown as the solid line for the same electron energy and laser energy $\omega_{\text {aser }} \simeq 1.26 \mathrm{eV}\left(x_{0}=4.82\right)$.


Fig. 3. Differential $\gamma \gamma$ luminosity for virtual $\gamma \gamma$ collisions due to classical Bremsstrahlung (dotted line) and for backscattered laser photons (solid line) from $\mathrm{e}^{+} \mathrm{e}^{-}$beams with the CM energy $\sqrt{s}=500 \mathrm{GeV}$ and $x_{0}=4.82$.
here, which leads us to expect a much enhanced sensitivity to the anomalous couplings $\kappa_{r}$ and $\lambda_{r}$ as compared to the Weizsäcker-Williams case.
Given the differential cross sections for our three reactions (4)-(6), a $\chi^{2}$ analysis ${ }^{\# 3}$ was performed by comparing the standard model predictions with those corresponding to nonvanishing anomalous couplings $\kappa_{\gamma}$ and $\lambda_{\gamma}$, including a free normalisation constant $f_{\text {norm }}$
$\chi^{2}=\sum_{i}\left(\frac{X_{i} f_{\text {norm }}-Y_{i}}{\Delta_{\text {stat }}^{\prime} f_{\text {norm }}}\right)^{2}+\left(\frac{f_{\text {norm }}-1}{\Delta_{\text {sys }}}\right)^{2}$,
with
$X_{i} \equiv \frac{\mathrm{~d} \sigma^{\mathrm{SM}}}{\mathrm{d} \cos \theta_{i}}, \quad Y_{i} \equiv \frac{\mathrm{~d} \sigma\left(\kappa_{\gamma}, \lambda_{\gamma}\right)}{\mathrm{d} \cos \theta_{i}}$.
The statistical errors $\Delta_{\text {stat }}^{i}$ were computed from the following (conservative) set of NLC parameters: $-\sqrt{s}=500 \mathrm{GeV}, \int \mathscr{L}_{\mathrm{c}+\mathrm{e}-} \mathrm{d} t=10 \mathrm{fb}^{-1},|\cos \theta| \leqslant 0.7$,

- WW reconstruction efficiency (including branching ratios) $=0.15$,
- W reconstruction efficiency (including branching ratios) $=0.1$,
and the standard model cross sections $X_{i}$.
\#3 We think that the treatment of the systematic error by eq. (27) is more appropriate than another form frequently employed (notably by theorists)
$\chi^{2}=\sum_{i}\left(\frac{X_{i}-Y_{i}}{A_{i}}\right)^{2}, \quad \Delta_{i}^{2}=\Delta_{\text {sau }}^{i 2}+\Delta_{\text {sys }}^{2} X_{i}^{2}$,
which actually implies uncorrelated systematic errors, bin by bin. In general, for given $A_{\text {sys }}$ this form leads to more optimistic bounds than eq. (27) where the bin by bin systematics is totally correlated. We thank D. Haidt and H.-U. Martyn for helpful discussions on this point.

The systematic uncertainty was taken to be ${ }^{\# 4}$
$\Delta_{\mathrm{sys}}= \pm 2 \%$.
Bounds on the anomalous couplings $\kappa_{y}, \lambda_{\gamma}$ at $90 \%$ confidence level for two degrees of freedom are derived by displaying the contours in the ( $1-\kappa_{\gamma}, \lambda_{\gamma}$ ) plane corresponding to
$\chi^{2}=4.61$,
with $\chi^{2}$ being minimized with respect to $f_{\text {norm }}$. This fixes $f_{\text {norm }}$ to first order in $\Delta_{\text {sys }}^{2}$
$f_{\text {norm }} \simeq \frac{\sum_{i}\left(Y_{i} / \Delta_{\text {stat }}^{i}\right)^{2}+1 / \Delta_{\text {sys }}^{2}}{\sum_{i} X_{i} Y_{i} / \Delta_{\text {stat }}^{i 2}+1 / \Delta_{\text {sys }}^{2}}$.
Note that $\sum_{i} X_{i} Y_{i} / \Delta_{\text {stat }}^{i 2}=N_{\text {tot }}(|\cos \theta|<0.7)$, the total number of events for given $\kappa_{r}, \lambda_{r}$ and our $\cos \theta$ cut.
Next, we turn to a discussion of our results.
(i) Classical Bremsstrahlung. In this case, we apply the Weizsäcker-Williams approximation, eqs. (22), (23), for the photon luminosity function. The corresponding exclusion contours at $90 \%$ confidence level are displayed in fig. 4 for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-} \mathrm{W}^{+} \mathrm{W}^{-}$(cf. eq. (19)), for $\mathrm{e}^{+} \mathrm{e} \rightarrow \mathrm{e}^{+} \mathrm{W}^{-} v$ (cf. eq. (20)) and for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}$(with $1-\kappa_{z}=\lambda_{z}=0$ ), for comparison. Due to the softness of the photon luminosity spectra (cf. figs. 2,3), in this case, only comparatively low subenergies $\sqrt{\hat{s}}=\sqrt{\tau s}$ are available for probing the anomalous couplings and the total number of (reconstructed) events is correspondingly low,

$$
\begin{align*}
N_{\mathrm{iol}}^{\mathrm{SM}}(|\cos \theta|<0.7) & \simeq 340 \ll \Delta_{\mathrm{sys}}^{-2} \text { for } \mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-} \mathrm{W}^{+} \mathrm{W}^{-}, \\
& \simeq 1000<\Delta_{\mathrm{syy}}^{-2} \text { for } \mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{e}^{+} \mathrm{W}^{-} \mathrm{v} . \tag{32}
\end{align*}
$$

[^1]

Fig. 4. Exclusion domains for the anomalous couplings $\kappa_{\gamma}$ and $\lambda_{\gamma}$ from $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-} \mathrm{W}^{+} \mathrm{W}^{-}, \mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{e}^{+} \mathrm{W}^{-} v$ in the equivalent photon approximation, and from $\mathrm{e}^{+} \mathrm{e}^{-\rightarrow} \mathrm{W}^{+} \mathbf{W}^{-}$. The areas surrounding the cross symbol (standard model at tree level) are allowed at the $90 \%$ confidence level.

For comparison
$N_{\text {lot }}^{\mathrm{SM}}(|\cos \theta|<0.7) \simeq 1800<\Delta_{\text {sys }}^{-2}$ for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}$.
The exclusion contours are, therefore, largely determined by statistics in these three cases.
From fig. 4 we conclude that in case of classical Bremsstrahlung, we obtain (at $90 \% \mathrm{CL}$ ) the following: - individually,
from $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{e}^{+} \mathrm{W}^{-} \mathrm{v}:-0.06 \leqslant 1-\kappa_{\gamma} \leqslant 0.09, \quad\left|\lambda_{\gamma}\right| \leqslant 0.11$,
from $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-} \mathrm{W}^{+} \mathrm{W}^{-}$: Comparatively little valuable new information on $1-\kappa_{\gamma}$ and $\lambda_{\gamma}$.
Only if $\lambda_{r} \approx 0$ then $\left|1-\kappa_{y}\right| \leqslant 0.06$,

- jointly from all three reactions (cf. shaded area in fig. 4),
$-0.05 \leqslant 1-\kappa_{\gamma} \leqslant 0.01, \quad-0.05 \leqslant \lambda_{\gamma} \leqslant 0.06 \quad\left(1-\kappa_{Z}=\lambda_{Z}=0\right)$.
Altogether, for classical Bremsstrahlung, the constraints on the anomalous couplings $1-\kappa_{\gamma}$ and $\lambda_{\gamma}$ from $e^{+} e^{-} \rightarrow e^{+} e^{-} W^{+} W^{-}$and $e^{+} e^{-} \rightarrow e^{+} W^{-} v$ are much weaker than those from $e^{+} e^{-} \rightarrow W^{+} W^{-}$. However, it should be remembered that they are (essentially) independent of $\kappa_{\mathrm{z}}$ and $\lambda_{\mathrm{z}}$ !
(ii) Laser photons. In this case the situation is much more favorable due to the hardness of the laser photon spectra, (cf figs. 2, 3). The corresponding exclusion contours at $90 \%$ confidence level are displayed in fig. 5 for $\gamma \gamma \rightarrow W^{+} W^{-}$, for $\mathrm{e}^{-} \gamma \rightarrow \mathrm{W}^{-} v$ and for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}$(with $1-\kappa_{z}=\lambda_{z}=0$ ), again for comparison. Due to the high subenergies available, the cross sections are much larger for reactions (4), (5) and we find

$$
\begin{array}{rlrl}
N_{\text {tot }}^{\mathrm{SM}}(|\cos \theta|<0.7) & \simeq 25200 \gg \Delta_{\text {sys }}^{-2} & & \text { for } \gamma \gamma \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}, \\
& \simeq 4600>\Delta_{\text {sys }}^{-2} & \text { for } \mathrm{e}^{-} \gamma \rightarrow \mathrm{W}^{-} v . \tag{34}
\end{array}
$$

The exclusion contours are, therefore, largely determined by systematics, eq. (29), in this case. Moreover, as explained in the preceding footnotes, they also significantly depend on the way the systematic errors are accounted for in the $\chi^{2}$ expression.


Fig. 5. Exclusion domains for the anomalous couplings $\kappa_{\gamma}$ and $\lambda_{\gamma}$ from $\gamma \gamma \rightarrow W^{+} W^{-}, \mathrm{e}^{-} \gamma \rightarrow W^{-} v$, and $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathbf{W}^{+} \mathbf{W}^{-}$from using backscattered laser photon beams. The areas surrounding the cross symbol (standard model at tree level) are allowed at the $90 \%$ confidence level.

From fig. 5 we conclude that in case of laser photons, we obtain very significant new constraints both individually from reactions (4), (5) and - even more so - jointly, from combining all three of them (cf. shaded area in fig. 5). We find (at $90 \% \mathrm{CL}$ ):

- individually
from $\gamma \gamma \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}:-0.02 \leqslant 1-\kappa_{\gamma} \leqslant 0.04, \quad-0.06 \leqslant \lambda_{\gamma} \leqslant 0.1$,
from $\mathrm{e}^{-} \gamma \rightarrow \mathrm{W}^{-} \mathrm{v}:-0.04 \leqslant 1-\kappa_{\gamma} \leqslant 0.06, \quad\left|\lambda_{\gamma}\right| \leqslant 0.06$.
- jointly from all three reactions (4)-(6) (cf. shaded area in fig. 5),
$\left|1-\kappa_{\gamma}\right| \leqslant 0.02, \quad-0.04 \leqslant \lambda_{\gamma} \leqslant 0.05 \quad\left(1-\kappa_{Z}=\lambda_{z}=0\right)$.
The bounds from $\gamma \gamma \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}$and $\mathrm{e}^{-} \gamma \rightarrow \mathrm{W}^{-} v$ appear very encouraging, are independent of $\kappa_{\mathrm{Z}}$ and $\lambda_{\mathrm{Z}}$ and impressively underline the importance of realizing laser photon beams at a future 500 GeV linear $\mathrm{e}^{+} \mathrm{e}^{-}$collider.

Let us conclude.
We have examined the potential of a future 500 GeV linear $\mathrm{e}^{+} \mathrm{e}^{-}$collider (NLC), to probe the "minimal" set ( $\kappa_{\gamma}, \lambda_{\gamma}$ ) of anomalous WW $\gamma$ and $W W \gamma \gamma$ couplings, via the (sub) processes $\gamma \gamma \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}, \mathrm{e}^{-} \gamma \rightarrow \mathrm{W}^{-} \nu$ and $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}$, for comparison. We considered photon beams, both from classical Bremsstrahlung and, notably, from backscattering laser light off $\mathrm{e}^{ \pm}$beams. The differential cross sections $\mathrm{d} \sigma / \mathrm{d} \cos \theta$ of the three reactions were calculated analytically and used as observables in a $\chi^{2}$ analysis under identical assumptions on machine parameters. The use of laser photon beams leads to very encouraging constraints on ( $\kappa_{\gamma}, \lambda_{\gamma}$ ), representing a dramatic improvement over a photon spectrum from classical Bremsstrahlung. Unlike $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}$, the constraints from the (sub) processes $\gamma \gamma \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}$and $\mathrm{e}^{-} \gamma \rightarrow \mathrm{W}^{-} v$ are independent of ( $\kappa_{\mathrm{z}}, \lambda_{z}$ ) in the light of which also the relatively weak constraints via classical Bremsstrahlung appear useful. The role of polarization in the processes considered here is presently under study.

Note added. After completion of this work, we learned from a recent preprint by Yehudai [18] where similar constraints on ( $\kappa_{\gamma}, \lambda_{\gamma}$ ) are derived from the process $\gamma \gamma \rightarrow \mathrm{W}^{+} \mathrm{W}^{-}$and laser photon beams.

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[^0]:    \#1 A small background from subprocesses with photons replaced by $Z$ 's is, as usual, neglected.

[^1]:    \#4 When comparing our results with others, please note that in processes with dominating systematic uncertainties (like $\gamma \gamma \rightarrow W^{+} W^{-}$and $\mathrm{e}^{-} \gamma \rightarrow W^{-} \nu$ ), a $\pm 2 \%$ systematic error in eq. (27) gives quite similar bounds on the anomalous couplings as a $\pm 5 \%$ systematic error in the $\chi^{2}$ formula quoted in the preceding foot note!

