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Effects of heavy Majorana neutrinos and neutral vector bosons on electroweak observables

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Extensions of the standard model with extra neutral vector bosons predict additional fermions. In the minimal case, where the extended gauge group is contained in the unified group SO(10), one has one right-handed neutrino for each quark–lepton generation. For this theory we systematically study the effects of both, the mixing of neutral vector bosons and the neutrino mixing, on a variety of observables used for precision tests of the electroweak theory. In most quantities the effects of neutrino and vector boson mixing tend to compensate each other. In particular the shift in the invisible width of the Z boson may be positive or negative. The most stringent bounds follow from tests for universality of the charged current. A comparable sensitivity could be attained from a measurement of the left–right asymmetry and forward–backward asymmetries at the Z pole. We emphasize the importance of measuring τ decays with improved accuracy.

1. Introduction

In the standard model of strong and electroweak interactions $B-L$, the difference of baryon and lepton number, is conserved up to a gravitational anomaly and, as a consequence, neutrinos are exactly massless. However, in unified theories based on the gauge group SO(10) or E_6, which contain the standard model as effective low-energy theory, $B-L$ plays the role of a spontaneously broken local symmetry and neutrinos acquire masses related to the scale of $B-L$ breaking. This is a theoretically very appealing extension of the standard model which implies lepton number violating processes at low and high energies.

At present very little is known about the scale Λ_{B-L} of $B-L$ breaking. The experimental upper bounds on neutrino masses and also cosmological constraints are compatible with a mass scale Λ_{B-L} as low as 1 TeV *. This is in fact expected in superstring theories which predict a low-energy gauge group which contains one or two U(1) factors [2] in addition to the standard model gauge group. In the minimal case, where the extended gauge group is contained in SO(10), one has one

* For a review, see ref. [1].

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extra Z′ vector boson with mass of order Λ_{B-L}. Furthermore, anomaly freedom requires the existence of right-handed neutrinos whose Majorana masses satisfy an upper bound [3] which is proportional to the Z′ vector boson mass.

Heavy Majorana neutrinos with masses below 1 TeV can be produced in ep and e^+e^- collisions if their mixing with light neutrinos is sufficiently large. Recently a detailed analysis has been performed of the range in neutrino masses and mixings which can be explored at present and future ep and e^+e^- colliders [4]. The results show that these colliders have the potential to discover lepton-number violating processes if the scale of $B-L$ breaking is of order 1 TeV.

Precision tests of the electroweak theory * provide stringent constraints on physics beyond the standard model [6–8]. However, up to now effects of neutrino mixings [7,9,10] and of extra neutral vector bosons [6,7] on electroweak processes have only been analysed separately. Since heavy Majorana neutrinos and an extra Z′ vector boson are likely to come together, and since their effects on electroweak observables may very well be of the same order of magnitude it is important to analyse neutrino and vector boson mixings together. As we shall see, both effects tend to compensate each other in almost all processes. Hence, only a complete analysis of as many observables as possible will allow for a definite interpretation of the physical origin of conceivable deviations from standard model predictions.

The paper is organized as follows: In sect. 2 we describe the extended gauge model with Majorana neutrinos and list various formulae needed to compute the different electroweak observables. Sect. 3 is then devoted to charged current processes, sect. 4 deals with e^+e^- physics on the Z pole, and in sect. 5 we discuss low-energy neutrino processes. Finally, in sect. 6 the various observables are compared and the present limits on mixings between light and heavy Majorana neutrinos are determined.

2. Extended gauge model with Majorana neutrinos

We consider an extended gauge theory with the local symmetry $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_{Y'}$ contained in the unified group SO(10). The smallest anomaly-free fermion representation is the 16-plet of SO(10) which contains a "right-handed" neutrino ν_R in addition to the 15 Weyl fermions of one quark–lepton generation. Compared to the standard model the lagrangian contains the following new terms (cf. ref. [3]):

$$\begin{aligned}\delta\mathscr{L} = & -\tfrac{1}{4}C_{\mu\nu}C^{\mu\nu} + i\overline{\nu_R}\not{D}\nu_R \\ & - \bar{l}\varphi g_\nu \nu_R - \overline{\nu_R} g_\nu^\dagger \varphi^\dagger l \\ & - \tfrac{1}{2}\chi\overline{\nu_R^c} h \nu_R - \tfrac{1}{2}\chi^\dagger \overline{\nu_R} h^\dagger \nu_R^c + \delta L_{\text{Higgs}}.\end{aligned} \tag{2.1}$$

* For a detailed review of theoretical foundations and experimental facts, see ref. [5].

Here $C_{\mu\nu}$ is the field strength of the new vector field C_μ, $\varphi = (\varphi^0, \varphi^-)$ is the doublet of Higgs fields, $l = (\nu_L, e_L^-)$ is the doublet of left-handed leptons and χ is a SU(2) singlet Higgs field whose vacuum expectation value generates the Majorana masses for the right-handed neutrinos. g_ν and h are complex 3×3 matrices and δL_{Higgs} contains kinetic and interaction terms of the new Higgs field χ and possibly further scalar fields. All gauge covariant derivatives differ from the standard model expressions by

$$\delta D_\mu = -i\sqrt{\tfrac{2}{3}}\, g' Y' C_\mu, \tag{2.2}$$

where

$$Y' = Y - \tfrac{5}{4}(B - L) \tag{2.3}$$

is the properly normalized new U(1) charge and g' the $\mathrm{U}(1)_Y$ gauge coupling of the standard model.

The vacuum expectation values of the Higgs fields $v' = \langle \chi \rangle_0$ and $v = \langle \varphi^0 \rangle_0$ break the gauge group to $\mathrm{SU}(3)_C \times \mathrm{U}(1)_{\text{em}}$, and one obtains two massive neutral vector bosons Z and Z′,

$$Z_\mu = \cos\zeta\, Z_\mu^0 - \sin\zeta\, C_\mu, \qquad Z'_\mu = \sin\zeta\, Z_\mu^0 + \cos\zeta\, C_\mu, \tag{2.4}$$

where $Z_\mu^0 = -\sin\Theta\, B_\mu + \cos\Theta\, W_\mu^3$ is the neutral vector boson of the standard model. The mixing of the vector bosons modifies weak angle and ρ parameter which, at tree level, satisfy the exact relations (cf. refs. [3,11]) ($\mu^2 = \pi\alpha/\sqrt{2}\,G$)

$$\sin^2\Theta = \frac{1}{2} - \left(\frac{1}{4} - \frac{\mu^2}{\rho m_Z^2}\right)^{1/2}, \tag{2.5}$$

$$\sin 2\zeta = -2\sqrt{2/3}\,\sin\Theta \frac{\rho m_Z^2}{m_{Z'}^2 - m_Z^2}, \tag{2.6}$$

$$\Delta\rho \equiv \rho - 1 = \sin^2\zeta \frac{m_{Z'}^2 - m_Z^2}{m_Z^2}, \tag{2.7}$$

where α is the electromagnetic fine structure constant and G is the Fermi constant of the standard model which, at tree level, is related to the vacuum expectation value of the Higgs field φ^0 by $G = (2\sqrt{2}\,v^2)^{-1}$.

In the extended gauge model under consideration all observables can be expressed in terms of α, G, m_Z^2, $m_{Z'}^2$ and fermion mass matrices, because the Higgs sector is specified. These quantities represent a convenient choice of parameters, since α, m_Z^2 and the muon lifetime τ_μ are precisely measured.

An important difference between the standard model and the extended gauge model is the mixing in the neutrino sector due to the presence of the right-handed neutrinos. As we shall see in sect. 3, this mixing affects the expression for the μ decay width and, consequently, the relation between the Fermi constant and the vacuum expectation value of the Higgs field. In our analysis of electroweak observables this effect will be taken into account.

From eqs. (2.1)–(2.4) one can derive the two neutral currents of the model. A straightforward calculation yields the interaction lagrangian (cf. ref. [3]):

$$\mathscr{L} = J^{\mu}_{\mathrm{NC}} Z_{\mu} + J'^{\mu}_{\mathrm{NC}} Z'_{\mu}, \tag{2.8}$$

where

$$J^{\mu}_{\mathrm{NC}} = \frac{g \cos\zeta}{2\cos\Theta} \sum_i \overline{\psi}_i \gamma^{\mu}\left(V^i - A^i\gamma_5\right)\psi_i, \tag{2.9}$$

$$\begin{aligned} V^i = {} & T_{3\mathrm{L}}(i) - 2\sin^2\Theta\, Q_i \\ & + \sqrt{\tfrac{2}{3}} \tan\zeta \sin\Theta\left(T_{3\mathrm{L}}(i) - 2Q_i + \tfrac{5}{2}(B-L)_i\right), \end{aligned} \tag{2.10}$$

$$A^i = \left(1 + \sqrt{\tfrac{2}{3}} \tan\zeta \sin\Theta\right) T_{3\mathrm{L}}(i), \tag{2.11}$$

$$J'^{\mu}_{\mathrm{NC}} = \frac{g \sin\zeta}{2\cos\Theta} \sum_i \overline{\psi}_i \gamma^{\mu}\left(V'^i - A'^i\gamma_5\right)\psi_i, \tag{2.12}$$

$$\begin{aligned} V'^i = {} & T_{3\mathrm{L}}(i) - 2\sin^2\Theta\, Q_i \\ & - \sqrt{\tfrac{2}{3}} \operatorname{ctg}\zeta \sin\Theta\left(T_{3\mathrm{L}}(i) - 2Q_i + \tfrac{5}{2}(B-L)_i\right), \end{aligned} \tag{2.13}$$

$$A'^i = \left(1 - \sqrt{\tfrac{2}{3}} \operatorname{ctg}\zeta \sin\Theta\right) T_{3\mathrm{L}}(i). \tag{2.14}$$

Here the sum extends over all quarks and leptons, and $T_{3\mathrm{L}}(i)$, Q_i and $(B-L)_i$ denote weak isospin, electric charge and $B-L$ of the ith fermion.

The neutral currents (2.9) and (2.12) are expressed in terms of weak eigenstates. For quarks and charged leptons the transiton to mass eigenstates is identical to the standard model. In the neutrino sector one has the Dirac mass matrix $m_{\mathrm{D}} = g_{\nu} v$ and the Majorana mass matrix $m = hv'$. The chiral fields ν_{L} and ν_{R} are then related to mass eigenstates ν and N via the transformation (cf. ref. [4]):

$$\nu = L + L^{\mathrm{c}}, \qquad N = R + R^{\mathrm{c}}, \tag{2.15}$$

where

$$\nu_{\mathrm{L}} = L + \xi R^{\mathrm{c}} - \tfrac{1}{2}\xi\xi^{\dagger}L + \mathrm{O}(\xi^3), \tag{2.16}$$

$$\nu_{\mathrm{R}} = R - \xi^{\mathrm{T}}L^{\mathrm{c}} - \tfrac{1}{2}\xi^{\mathrm{T}}\xi^{*}R + \mathrm{O}(\xi^3), \tag{2.17}$$

$$\xi = m_{\mathrm{D}}\frac{1}{m}. \tag{2.18}$$

To leading order in $1/m$ the masses of the light and heavy Majorana neutrinos ν_i and N_i are given by the eigenvalues of the mass matrices

$$m_{\nu} = -m_{\mathrm{D}}\frac{1}{m}m_{\mathrm{D}}^{\mathrm{T}}, \qquad m_{\mathrm{N}} = m. \tag{2.19}$$

Eqs. (2.16)–(2.18) lead to the following expressions for the neutrino terms in neutral and charged currents, neglecting contributions of order ξ^3 and $\xi\zeta$:

$$\begin{aligned} J_{\mathrm{NC}}^{\mu} = \frac{g\cos\zeta}{2\cos\Theta}\Bigg[&\left(1-\sqrt{\tfrac{3}{2}}\tan\zeta\sin\Theta\right)\bar{\nu}(1-\xi\xi^{\dagger})\gamma^{\mu}\frac{1-\gamma_5}{2}\nu \\ &+\left(1-4\sqrt{\tfrac{2}{3}}\tan\zeta\sin\Theta\right)\left(\bar{\nu}\xi\gamma^{\mu}\frac{1-\gamma_5}{2}N+\overline{N}\xi^{*}\gamma^{\mu}\frac{1-\gamma_5}{2}\nu\right) \\ &+\overline{N}\left(\tfrac{5}{2}\sqrt{\tfrac{2}{3}}\tan\zeta\sin\Theta+\xi^{\dagger}\xi\right)\gamma^{\mu}\frac{1-\gamma_5}{2}N\Bigg], \end{aligned} \tag{2.20}$$

$$\begin{aligned} J_{\mathrm{NC}}^{\prime\mu} = \frac{g\sin\zeta}{2\cos\Theta}\Bigg[&\bar{\nu}\left(1+\tfrac{3}{2}\sqrt{\tfrac{2}{3}}\operatorname{ctg}\zeta\sin\Theta-4\sqrt{\tfrac{2}{3}}\operatorname{ctg}\zeta\sin\Theta\;\xi\xi^{\dagger}\right)\gamma^{\mu}\frac{1-\gamma_5}{2}\nu \\ &+4\sqrt{\tfrac{2}{3}}\operatorname{ctg}\zeta\sin\Theta\left(\bar{\nu}\xi\gamma^{\mu}\frac{1-\gamma_5}{2}N+\overline{N}\xi^{\dagger}\gamma^{\mu}\frac{1-\gamma_5}{2}\nu\right) \\ &-\overline{N}\left(\tfrac{5}{2}\sqrt{\tfrac{2}{3}}\operatorname{ctg}\zeta\sin\Theta-4\sqrt{\tfrac{2}{3}}\operatorname{ctg}\zeta\sin\Theta\;\xi^{\dagger}\xi\right)\gamma^{\mu}\frac{1-\gamma_5}{2}N\Bigg], \end{aligned} \tag{2.21}$$

$$J_{\mathrm{CC}}^{\mu} = \frac{g}{\sqrt{2}}\left[\bar{e}V\left(1-\tfrac{1}{2}\xi\xi^{\dagger}\right)\gamma^{\mu}\frac{1-\gamma_5}{2}\nu+\bar{e}V\xi\gamma^{\mu}\frac{1-\gamma_5}{2}N\right]. \tag{2.22}$$

In the leptonic part of the charged current a unitary mixing matrix V arises in the transition from weak to mass eigenstates of the charged leptons, which is the analog of the Kobayashi–Maskawa matrix in the quark sector. In general, the matrix V contains a CP violating phase. Hence, CP violating effects now also occur in leptonic processes. The mixing matrix $V(1-\frac{1}{2}\xi\xi^{\dagger})$ and the neutrino masses $m_{\nu i}$ are restricted by the experimental upper bound on the rate of neutrinoless double β decay (cf. ref. [3]).

Based on the currents (2.20)–(2.22) and the relations (2.5)–(2.7) among the electroweak parameters we can now compute deviations from standard model predictions and determine the sensitivity of various electroweak observables to neutrino and vector boson mixing.

In order to obtain bounds on the mixing parameters ζ and ξ we shall proceed as follows. Let O_{SM} be the standard model prediction of some observable O and ΔO_{mix} the deviation caused by neutrino and vector boson mixings. O_{SM} is the sum of the tree level contribution O_{Born} and the radiative correction ΔO_{rad}, i.e. $O_{\mathrm{SM}} = O_{\mathrm{Born}} + \Delta O_{\mathrm{rad}}$. The relative change is given by

$$\frac{\Delta O_{\mathrm{mix}}}{O_{\mathrm{SM}}} = \frac{\Delta O_{\mathrm{mix}}}{O_{\mathrm{Born}}} + \mathrm{O}\left(\frac{\Delta O_{\mathrm{mix}}\,\Delta O_{\mathrm{rad}}}{O_{\mathrm{Born}}^2}\right). \tag{2.23}$$

Let the measured value O_{exp} of the observable be known with an error $\pm\Delta O_{\mathrm{exp}}$. An estimate of the allowed range of mixing parameters is then obtained by requiring

$$\left|\frac{\Delta O_{\mathrm{mix}}}{O_{\mathrm{Born}}}\right| \leqslant \frac{\Delta O_{\mathrm{exp}}}{O_{\mathrm{exp}}}. \tag{2.24}$$

Our bounds on mixing parameters will be based on this inequality. We will thereby ignore small differences of O_{SM} and O_{exp} within the experimental error, a possible asymmetry of the experimental error and radiative corrections to ΔO_{mix}.

A further complication is the uncertainty of the standard model prediction O_{SM} for large values of the top quark mass, which may be comparable to or even larger than the experimental error. The leading radiative corrections proportional to m_{t}^2 can easily be incorporated by modifying the weak angle, the ρ parameter and the neutral current couplings of the b quark [12]. A complete analysis would have to include m_{t} as unknown parameter. In the following we shall ignore this complication, i.e. we restrict ourselves to the cases where the top mass is known or where it is sufficiently small.

3. Charged current processes

Predictions of the standard model for cross sections and decay rates can be expressed in terms of the electromagnetic fine structure constant α, the Fermi constant G and the Z boson mass m_{Z}. In the extended gauge model the mass $m_{\mathrm{Z}'}$ of the extra neutral vector boson is an additional parameter.

The Fermi constant is determined from the muon lifetime using the standard model formula. However, the mixing in the neutrino sector modifies the expression for the decay rate and, consequently, the Fermi constant depends on the mixing parameters. From the charged current (2.22) one obtains for the μ decay rate at tree level, summing over all neutrino species in the final state and neglecting electron and neutrino masses

$$\tau_\mu^{-1} = \frac{G^2 m_\mu^5}{192\pi^3}(1 - \lambda_e - \lambda_\mu), \tag{3.1}$$

where

$$\lambda_i = (\xi\xi^\dagger)_{ii} = \sum_j |\xi_{ij}|^2, \quad i, j = e, \mu, \tau. \tag{3.2}$$

Here we have neglected terms of order $\xi\xi^\dagger\delta V$, where $\delta V = V - 1$, since matrix elements of ξ and δV both have to be small. In eq. (3.1) we have also ignored radiative corrections to τ_μ, because we have to evaluate the change ΔO_{mix} of an observable only relative to the tree level standard model prediction O_{Born} (cf. eq. (2.23)). G is related to the vacuum expectation value of the Higgs field $\varphi^0 : G = (2\sqrt{2}v^2)^{-1}$. From eq. (3.1) we read off the "Fermi constant":

$$G_\mu = G\left(1 - \tfrac{1}{2}\lambda_e - \tfrac{1}{2}\lambda_\mu\right), \tag{3.3}$$

i.e. the quantity G which enters the weak angle (2.5) is larger than the "Fermi constant". In the case of mixing between three light and one heavy neutrino species the analogous result has been obtained by Bilenky et al. [13]. A similar modification of the Fermi constant is caused by new interactions which are described by dimension-6 operators at low energies [14].

The electroweak parameters Θ, ζ and $\Delta\rho$ can be expanded around the standard model limit

$$\sin^2\Theta = \sin^2\hat{\Theta}\left(1 - \frac{1}{6}\frac{\sin^2 2\hat{\Theta}}{\cos 2\hat{\Theta}}\epsilon - \frac{1}{2}\frac{\cos^2\hat{\Theta}}{\cos 2\hat{\Theta}}(\lambda_e + \lambda_\mu)\right), \tag{3.4}$$

$$\zeta = -\sqrt{\tfrac{2}{3}}\sin\hat{\Theta}\epsilon, \tag{3.5}$$

$$\Delta\rho = \tfrac{2}{3}\sin^2\hat{\Theta}\epsilon, \tag{3.6}$$

where

$$\epsilon = \frac{m_Z^2}{m_{Z'}^2}, \tag{3.7}$$

$$\sin^2\hat{\Theta} = \frac{1}{2} - \left(\frac{1}{4} - \frac{\hat{\mu}^2}{m_Z^2}\right)^{1/2}, \qquad \hat{\mu}^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu}. \tag{3.8}$$

Here $\sin\hat{\Theta}$ is the standard model expression for the weak angle, where G_μ is the "Fermi constant" defined by the standard model formula for the μ decay rate. Clearly, the induced dependence of the weak angle $\sin\Theta$ on the mixing parameters ϵ and λ_i, which enters the neutral currents, affects almost all electroweak observables. With $\alpha^{-1} \equiv \alpha^{-1}(m_Z) = 128$, $G_\mu = 1.166 \times 10^{-5}$ GeV^{-2} and $m_Z = 91.117$ GeV [15] one obtains numerically $\sin^2\hat{\Theta} = 0.234$.

Let us first consider the W boson mass. From eqs. (3.3), (3.4) and $g^2 = e^2/\sin^2\Theta$ one obtaines

$$m_W^2 = \cos^2\hat{\Theta} m_Z^2 \left(1 + \frac{1}{6} \frac{\sin^2 2\hat{\Theta}}{\cos 2\hat{\Theta}} \epsilon + \frac{1}{2} \frac{\sin^2\hat{\Theta}}{\cos 2\hat{\Theta}} (\lambda_e + \lambda_\mu) \right), \tag{3.9}$$

which yields

$$\frac{\delta m_W}{m_W} = 0.11\epsilon + 0.11(\lambda_e + \lambda_\mu). \tag{3.10}$$

Note that the neutrino mixing and the neutral vector boson mixing both increase the W boson mass.

The partial τ decay widths $\Gamma_{\tau\mu} \equiv \Gamma(\tau \to \mu\nu\nu)$ and $\Gamma_{\tau e} \equiv \Gamma(\tau \to e\nu\nu)$ test for the universality of the charged current. Summing over the neutrinos in the final state one finds (cf. (3.2))

$$\frac{\delta\Gamma_{\tau\mu}}{\Gamma_{\tau\mu}} = \lambda_e - \lambda_\tau, \tag{3.11}$$

$$\frac{\delta\Gamma_{\tau e}}{\Gamma_{\tau e}} = \lambda_\mu - \lambda_\tau. \tag{3.12}$$

The opposite sign of the two contributions to both partial widths follows from the effect of the neutrino mixing on the quantity G. As expected the partial τ widths remain unchanged for generation-independent mixing, i.e. $\lambda_e = \lambda_\mu = \lambda_\tau$.

Another measure of charged current universality is the ratio of π decay widths $R_\pi \equiv \Sigma_i \Gamma(\pi \to e\nu_i)/\Sigma_j \Gamma(\pi \to \mu\nu_j)$. One easily finds

$$\frac{\delta R_\pi}{R_\pi} = \lambda_\mu - \lambda_e. \tag{3.13}$$

A stringent test of quark–lepton universality of the charged current is finally provided by a comparison of the Fermi constants determined in μ decay and the semileptonic β and K_{e3} decays. Assuming unitarity of the Kobayashi–Maskawa

TABLE 1

Bounds on mixing parameters from charged current processes. The three theoretical models are: $\lambda_e = \lambda_\mu = \lambda_\tau = 10^{-2}$, $\epsilon = 2\times 10^{-2}$ (A); $\lambda_e, \lambda_\mu \ll \lambda_\tau = 10^{-2}$, $\epsilon = 2\times 10^{-2}$ (B); $\lambda_e = \lambda_\mu = \lambda_\tau = 10^{-2}$, $\epsilon = 0$ (C). The experimental errors are taken from ref. [16]

	Predicted deviation [%]			Exp. error [%]
	A	B	C	
$\delta m_W / m_W$	0.4	0.2	0.2	0.5
$\delta \Gamma_{\tau\mu} / \Gamma_{\tau\mu}$	0	−1	0	2
$\delta \Gamma_{\tau e} / \Gamma_{\tau e}$	0	−1	0	2
$\delta R_\pi / R_\pi$	0	0	0	1
$(G_\beta^2 - G_\mu^2)/G_\mu^2$	1	0	1	0.3

matrix one obtains in the usual manner (cf. (3.3))

$$\frac{G_\beta^2 - G_\mu^2}{G_\mu^2} = \lambda_\mu, \tag{3.14}$$

where G_β is the Fermi constant of the semileptonic decays. In analogy to eq. (3.3) one easily finds $G_\beta^2 = G^2\,(1 - \lambda_e)$. A bound on λ_μ is now obtained by identifying the ratio on the left-hand side with the measured quantity $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$, where $|V_{ub}|^2$ can be safely neglected. $(G_\beta^2 - G_\mu^2)/G_\mu^2$ is the only observable which is sensitive to a single mixing parameter.

In table 1 the deviations from standard model predictions for the different charged current observables are compared with current experimental precisions for the following three cases: (A) $\lambda_e = \lambda_\mu = \lambda_\tau = 10^{-2}$, $\epsilon = 2 \times 10^{-2}$; (B) λ_e, $\lambda_\mu \ll \lambda_\tau = 10^{-2}$, $\epsilon = 2 \times 10^{-2}$; (C) $\lambda_e = \lambda_\mu = \lambda_\tau = 10^{-2}$, $\epsilon = 0$. The value $\epsilon = m_Z^2/m_{Z'}^2 = 2 \times 10^{-2}$ corresponds to a Z–Z′ mixing ζ of about 10^{-2}. The strongest bound on neutrino mixings follows from quark–lepton universality: $\lambda_\mu < 3 \times 10^{-3}$. For the W boson mass a precision of about 400 MeV is sufficient in order to test the mixing parameters ϵ and λ_i at the level of 1%. A more accurate measurement of leptonic τ decays would provide a stringent constraint on τ neutrino mixings.

4. Z decay width and asymmetries

Observables at the Z pole in e^+e^- annihilation are affected by the modification of the neutral current couplings through Z–Z′ mixing, neutrino mixing and the dependence of the weak angle on the mixing parameters.

Particularly interesting with respect to neutrino mixing is the invisible width of the Z boson $\Gamma_{inv} = \Sigma_{i,j}\Gamma(Z \to \nu_i\nu_j)$. From the neutral current (2.20) one obtains at tree level

$$\Gamma_{inv} = \frac{G_\mu m_Z^3}{2\pi\sqrt{2}}\rho\left(1 + \frac{\lambda_e + \lambda_\mu}{2}\right)\left(\tfrac{1}{2} + \sin^2\hat{\Theta}\,\epsilon\right)\left(1 - \tfrac{2}{3}\mathrm{tr}(\xi\xi^\dagger)\right), \tag{4.1}$$

which, using eqs. (3.2) and (3.4)–(3.8), yields

$$\Gamma_{\text{inv}} = \frac{G_\mu m_Z^3}{4\pi\sqrt{2}}\left(1 + \tfrac{8}{3}\sin^2\hat{\Theta}\ \epsilon - \tfrac{1}{6}(\lambda_e + \lambda_\mu) - \tfrac{2}{3}\lambda_\tau\right) \tag{4.2}$$

and

$$\frac{\delta\Gamma_{\text{inv}}}{\Gamma_{\text{inv}}} = \frac{\delta\bar{n}}{\bar{n}} = 0.62\epsilon - 0.17(\lambda_e + \lambda_\mu) - 0.67\lambda_\tau, \tag{4.3}$$

where $\bar{n}$ denotes the effective number of neutrino species. Mixing with right-handed, SU(2) singlet neutrinos always reduces the invisible width [13,17]. Note, however, that the decrease is less than the naively expected last factor in eq. (4.1) because of the effect of the neutrino mixing on the Fermi constant. Furthermore, since the Z–Z′ mixing increases Γ_{inv}, the total change may be very small.

A straightforward calculation yields for the leptonic width

$$\Gamma_{ee} = \frac{G_\mu m_Z^3}{6\pi\sqrt{2}}\rho\left(1 + \frac{\lambda_e + \lambda_\mu}{2}\right)$$
$$\times\left[\tfrac{1}{2} - 2\sin^2\Theta + 4\sin^4\Theta - \sin^2\Theta\left(1 - \tfrac{8}{3}\sin^2\Theta\right)\epsilon\right], \tag{4.4}$$

where $\sin^2\Theta$ is still dependent on the mixing parameters ϵ and λ_i. Inserting eqs. (3.4) and (3.6) gives

$$\frac{\delta\Gamma_{ee}}{\Gamma_{ee}} = -0.17\epsilon + 0.59(\lambda_e + \lambda_\mu). \tag{4.5}$$

Contrary to Γ_{inv} there is no dependence on λ_τ. Vector boson and neutrino mixings again enter with opposite signs.

In order to obtain the total Z boson width

$$\Gamma_Z = \Gamma_{\text{inv}} + 3\Gamma_{ee} + 2\Gamma_{u\bar{u}} + 3\Gamma_{d\bar{d}} \tag{4.6}$$

we have to compute the hadronic partial widths $\Gamma_{u\bar{u}} \equiv \Gamma(Z \to u\bar{u})$ and $\Gamma_{d\bar{d}} \equiv \Gamma(Z \to d\bar{d})$. Using the neutral current couplings (2.20) one easily finds

$$\Gamma_{u\bar{u}} = \frac{3G_\mu m_Z^3}{6\pi\sqrt{2}}\rho\left(1 + \frac{\lambda_e + \lambda_\mu}{2}\right)$$
$$\times\left[\tfrac{1}{2} - \tfrac{4}{3}\sin^2\Theta + \tfrac{16}{9}\sin^4\Theta - \tfrac{1}{3}\sin^2\Theta\ \epsilon\right], \tag{4.7}$$

$$\Gamma_{\mathrm{d\bar{d}}} = \frac{3G_\mu m_Z^3}{6\pi\sqrt{2}}\rho\left(1+\frac{\lambda_e+\lambda_\mu}{2}\right)$$
$$\times\left[\tfrac{1}{2}-\tfrac{2}{3}\sin^2\Theta+\tfrac{4}{9}\sin^4\Theta+\left(\tfrac{1}{3}-\tfrac{8}{9}\sin^2\Theta\right)\sin^2\Theta\,\epsilon\right]. \tag{4.8}$$

From eqs. (4.2), (4.4), (4.6)–(4.8) and (3.4), (3.6) one then obtains

$$\frac{\delta\Gamma_Z}{\Gamma_Z} = 0.24\epsilon + 0.54(\lambda_e+\lambda_\mu) - 0.14\lambda_\tau. \tag{4.9}$$

For the three widths Γ_{inv}, Γ_{ee} and Γ_Z the coefficients of the mixing parameters ϵ and λ_i have the same order of magnitude, however different signs. Only λ_τ always enters with negative sign.

Finally, we consider the left–right asymmetry for longitudinally polarized electrons

$$A_{\mathrm{LR}} = \frac{\sigma\left(e_L^- e^+ \to \sum_f f\bar{f}\right) - \sigma\left(e_R^- e^+ \to \sum_f f\bar{f}\right)}{\sigma\left(e_L^- e^+ \to \sum_f f\bar{f}\right) + \sigma\left(e_R^- e^+ \to \sum_f f\bar{f}\right)}, \tag{4.10}$$

where f denotes any outgoing fermion, and the forward–backward asymmetry for a specific fermion pair with unpolarized incoming electrons

$$A_{\mathrm{FB}}(f) = \frac{\int_{\cos\vartheta>0} \mathrm{d}\Omega(\mathrm{d}\sigma/\mathrm{d}\Omega)(e^+e^-\to f\bar{f}) - \int_{\cos\vartheta<0} \mathrm{d}\Omega(\mathrm{d}\sigma/\mathrm{d}\Omega)(e^+e^-\to f\bar{f})}{\int_{\cos\vartheta>0} \mathrm{d}\Omega(\mathrm{d}\sigma/\mathrm{d}\Omega)(e^+e^-\to f\bar{f}) + \int_{\cos\vartheta<0} \mathrm{d}\Omega(\mathrm{d}\sigma/\mathrm{d}\Omega)(e^+e^-\to f\bar{f})}. \tag{4.11}$$

The quantity A_{LR} depends only on vector and axial vector couplings of the incoming electron, whereas for $A_{\mathrm{FB}}(f)$ the couplings of electron and outgoing fermion are relevant [18]

$$A_{\mathrm{LR}} = \frac{2V_e A_e}{V_e^2 + A_e^2}, \tag{4.12}$$

$$A_{\mathrm{FB}}(f) = \frac{3}{4}\frac{2V_e A_e}{V_e^2 \mp A_e^2}\frac{2V_f A_f}{V_f^2 + A_f^2}. \tag{4.13}$$

Due to the universal couplings of charged leptons to the Z boson, the forward–backward asymmetry of the experimentally clean μ pairs is determined by A_{LR}. Of particular interest is also the asymmetry of b quark jets [19].

Using the neutral current couplings (cf. (2.10), (2.11))

$$V_e = -\tfrac{1}{2} + 2\sin^2\Theta + \tfrac{2}{3}\sin^2\Theta\ \epsilon, \tag{4.14}$$

$$A_e = -\tfrac{1}{2} + \tfrac{1}{3}\sin^2\Theta\ \epsilon, \tag{4.15}$$

$$V_b = -\tfrac{1}{2} + \tfrac{2}{3}\sin^2\Theta - \tfrac{2}{3}\sin^2\Theta\ \epsilon, \tag{4.16}$$

$$A_b = -\tfrac{1}{2} + \tfrac{1}{3}\sin^2\Theta\ \epsilon, \tag{4.17}$$

and the expansion (3.4) for $\sin^2\Theta$, one obtains

$$\frac{\delta A_{LR}}{A_{LR}} = \frac{1}{2}\frac{\delta A_{FB}(\mu)}{A_{FB}(\mu)} = -1.4\epsilon + 10(\lambda_e + \lambda_\mu), \tag{4.18}$$

$$\frac{\delta A_{FB}(b)}{A_{FB}(b)} = -1.2\epsilon + 11(\lambda_e + \lambda_\mu). \tag{4.19}$$

The large coefficients of $\lambda_e + \lambda_\mu$ reflect the well-known sensitivity of the asymmetries with respect to the weak angle. This follows from the smallness of the standard model value of V_e. Since the coefficient of ϵ turns out to be an order of magnitude smaller, these asymmetries could provide strong constraints on neutrino mixings.

In table 2 deviations of Z boson widths and asymmetries from standard model predictions are compared with present or projected experimental errors. Due to cancellations between contributions from neutrino and vector boson mixings the deviations could well be below 1% for mixing angles of order 1%. A measurement of the left–right asymmetry and the forward–backward asymmetry of b quark jets with the expected accuracy of 2% would constrain the neutrino mixing $\lambda_e + \lambda_\mu$ to values below 2×10^{-3}. This corresponds to the present bound on λ_μ from quark–lepton universality of the charged current.

TABLE 2

Bounds on mixing parameters from observables at the Z pole. The three theoretical models A, B and C are the same as in table 1. The * denotes expected experimental errors

	Predicted deviation [%]			Exp. error [%]
	A	B	C	
$\delta\Gamma_{inv}/\Gamma_{inv}$	0.2	0.6	−1	3 [15]
$\delta\Gamma_{ee}/\Gamma_{ee}$	0.8	−0.3	1	0.8 [15]
$\delta\Gamma_Z/\Gamma_Z$	1	0.3	0.9	0.6 [15]
$\delta A_{LR}/A_{LR}$	17	−3	20	2 [20] *
$\delta A_{FB}(\mu)/A_{FB}(\mu)$	34	−6	40	20 [20] *
$\delta A_{FB}(b)/A_{FB}(b)$	19	−2	21	2 [19] *

5. Low-energy neutrino processes

Low-energy neutrino scattering processes are described by an effective lagrangian which accounts for the virtual exchange of Z, Z′ and W vector bosons. The neutral current part reads

$$\mathscr{L}_{\text{eff}}^{\text{NC}} = -\frac{1}{2m_{\text{Z}}^2} J_{\text{NC}}^{\mu} J_{\text{NC}\mu} - \frac{1}{2m_{\text{Z}'}^2} J_{\text{NC}}^{\prime\mu} J_{\text{NC}\mu}^{\prime}, \tag{5.1}$$

where J_{NC}^{μ} and $J_{\text{NC}}^{\prime\mu}$ are the two neutral currents given in eqs. (2.20) and (2.21). Inserting these currents in eq. (5.1) one obtains to leading order in λ_i, ξ and ϵ the following expression for neutrino interactions:

$$\begin{aligned}\mathscr{L}_{\text{eff}}^{\text{NC}\nu} = &-\sqrt{2}\, G_{\mu}\left(1 + \frac{\lambda_{\text{e}} + \lambda_{\mu}}{2}\right)\bar{\nu}(1 - \xi\xi^{\dagger})\gamma^{\mu}\frac{1 - \lambda_5}{2}\nu \\ &\times \sum_{i \neq \nu}\Big\{\bar{\psi}_i\big[2T_{3\text{L}}(i) - 2\sin^2\Theta\, Q_i \\ &+ \tfrac{10}{3}\sin^2\Theta\, \epsilon\big(\cos^2\Theta\, Q_i - \tfrac{5}{4}(B - L)_i\big)\big]\gamma_{\mu}\frac{1 - \gamma_5}{2}\psi_i \\ &+ \bar{\psi}_i\big[-2\sin^2\Theta\, Q_i + \tfrac{10}{3}\sin^2\Theta\, \epsilon\big(\cos^2\Theta\, Q_i - \tfrac{5}{4}(B - L)_i\big)\big] \\ &\times \gamma_{\mu}\frac{1 + \gamma_5}{2}\psi_i\Big\}.\end{aligned} \tag{5.2}$$

Similarly one obtains for the neutrino part of the charged current lagrangian

$$\begin{aligned}\mathscr{L}_{\text{eff}}^{\text{CC}\nu} = &-2\sqrt{2}\, G_{\mu}\left(1 + \frac{\lambda_{\text{e}} + \lambda_{\mu}}{2}\right) \\ &\times \bar{e}V(1 - \tfrac{1}{2}\xi\xi^{\dagger})\gamma^{\mu}\frac{1 - \gamma_5}{2}\nu\bar{\nu}(1 - \tfrac{1}{2}\xi\xi^{\dagger})V^{\dagger}\gamma_{\mu}\frac{1 - \gamma_5}{2}e,\end{aligned} \tag{5.3}$$

which, by means of a Fierz transformation, may be written in neutral-current ofrm as

$$\begin{aligned}\mathscr{L}_{\text{eff}}^{\text{CC}\nu} = &-2\sqrt{2}\, G_{\mu}\left(1 + \frac{\lambda_{\text{e}} + \lambda_{\mu}}{2}\right)\big[(1 - \tfrac{1}{2}\xi\xi^{\dagger})V^{\dagger}\big]_{ij}\big[V(1 - \tfrac{1}{2}\xi\xi^{\dagger})\big]_{kl} \\ &\times \bar{\nu}_i\gamma^{\mu}\frac{1 - \gamma_5}{2}\nu_l\bar{e}_k\gamma_{\mu}\frac{1 - \gamma_5}{2}e_j.\end{aligned} \tag{5.4}$$

This form is convenient for the calculation of νe elastic scattering where neutral and charged current contributions interfere.

It is now straightforward to compute cross sections for various neutrino–electron and neutrino–nucleon reactions. Since the neutrino species in the initial state is specified the result will depend on the matrix elements of V. This is different from the processes considered in the previous sections where neutrinos only appeared in the final state, and where the dependence on V disappeared in the sum over neutrino species due to the unitarity of V. The composition of the neutrino beams, which originate from decays of K and π mesons, also depends on the matrix elements of V. In the analysis of neutrino scattering experiments this effect has to be taken into account.

For neutrino–electron scattering one obtains the following cross sections:

$$\begin{aligned}\sigma_{\nu\mathrm{e}} &\equiv \sum_i \sigma\left(\nu_\mu \mathrm{e}^- \to \nu_i \mathrm{e}^-\right)\\ &= \left(G_\mu^2 s/4\pi\right)\left(1+\lambda_\mathrm{e}-\lambda_\mu\right)\\ &\quad\times\left[1-4z+\tfrac{16}{3}z^2+8z\,|V_{\mathrm{e}\mu}|^2-\tfrac{5}{3}z\left(1+\tfrac{4}{3}z-\tfrac{32}{3}z^2\right)\epsilon\right],\end{aligned} \tag{5.5}$$

$$\begin{aligned}\sigma_{\bar\nu\mathrm{e}} &\equiv \sum_i \sigma\left(\bar\nu_\mu \mathrm{e}^- \to \bar\nu_i \mathrm{e}^-\right)\\ &= \left(G_\mu^2 s/12\pi\right)\left(1+\lambda_\mathrm{e}-\lambda_\mu\right)\\ &\quad\times\left[1-4z+16z^2+8z\,|V_{\mathrm{e}\mu}|^2-\tfrac{5}{3}z\left(1-4z-32z^2\right)\epsilon\right],\end{aligned} \tag{5.6}$$

$$\begin{aligned}\sigma_{\nu_\mathrm{e}\mathrm{e}} &\equiv \sum_i \sigma\left(\nu_\mathrm{e} \mathrm{e}^- \to \nu_i \mathrm{e}^-\right)\\ &= \left(G_\mu^2 s/4\pi\right)\left(1-\lambda_\mathrm{e}+\lambda_\mu\right)\\ &\quad\times\left[1+4z+\tfrac{16}{3}z^2-8z\eta_\mathrm{e}+\tfrac{5}{3}z\left(1+\tfrac{20}{3}z+\tfrac{32}{3}z^2\right)\epsilon\right],\end{aligned} \tag{5.7}$$

$$\begin{aligned}\sigma_{\bar\nu_\mathrm{e}\mathrm{e}} &\equiv \sum_i \sigma\left(\bar\nu_\mathrm{e} \mathrm{e}^- \to \bar\nu_i \mathrm{e}^-\right)\\ &= \left(G_\mu^2 s/12\pi\right)\left(1-\lambda_\mathrm{e}+\lambda_\mu\right)\\ &\quad\times\left[1+4z+16z^2-8z\eta_\mathrm{e}+\tfrac{5}{3}z\left(1+12z+32z^2\right)\epsilon\right],\end{aligned} \tag{5.8}$$

$$\begin{aligned}\tilde\sigma_{\nu\mathrm{e}} &\equiv \sum_i \sigma\left(\nu_\mu \mathrm{e}^- \to \nu_i \mu^-\right)\\ &= \left(G_\mu^2 s/\pi\right)\left(1-\eta_\mu\right),\end{aligned} \tag{5.9}$$

$$\bar{\sigma}_{\bar{\nu}e} \equiv \sum_i \sigma\left(\bar{\nu}_\mu e^- \to \bar{\nu}_i \mu^-\right)$$
$$= \left(G_\mu^2 s/3\pi\right) |V_{e\mu}|^2. \qquad (5.10)$$

Here s is the center-of-mass energy squared, $z = \sin^2\Theta$ and

$$\eta_i = 1 - |V_{ii}|^2 \leqslant 1, \quad i = e, \mu, \tau, \qquad (5.11)$$

where we have used the unitarity of V. Recall that in our model all neutrinos are of Majorana type, i.e. $\nu = \nu^c$. ν and $\bar{\nu}$ denote left-handed and right-handed polarization states of these neutrinos and all mass effects are ignored.

At present the best measured quantity is the ratio

$$R_2 = \frac{\sigma_{\nu e}}{\sigma_{\bar{\nu}e}}. \qquad (5.12)$$

From eqs. (3.4), (5.5) and (5.6) one obtains for its dependence on mixing parameters:

$$\frac{\delta R_2}{R_2} = -1.1\epsilon + 1.3(\lambda_e + \lambda_\mu) + 3.3|V_{e\mu}|^2. \qquad (5.13)$$

Another ratio, which may be rather precisely measurable [7,21], reads

$$R_{12} = \frac{\sigma_{\nu e}}{\sigma_{\bar{\nu}e} + \sigma_{\nu_e e}}. \qquad (5.14)$$

Analogously, one might consider

$$\bar{R}_{12} = \frac{\sigma_{\bar{\nu}e}}{\sigma_{\nu e} + \sigma_{\bar{\nu}_e e}}. \qquad (5.15)$$

For these two ratios the dependence on mixing parameters reads

$$\frac{\delta R_{12}}{R_{12}} = -0.99\epsilon + 3.0\lambda_e - 0.53\lambda_\mu + 0.74\eta_e + 5.0|V_{e\mu}|^2, \qquad (5.16)$$

$$\frac{\delta \bar{R}_{12}}{\bar{R}_{12}} = 0.26\epsilon + 1.1\lambda_e - 1.8\lambda_\mu + 0.48\eta_e + 0.54|V_{e\mu}|^2. \qquad (5.17)$$

Contrary to R_2, both ratios depend on η_e.

Let us finally consider deep-inelastic neutrino scattering. A careful theoretical treatment has to include various corrections to the simple parton model predictions, such as quark mixing, the heavy flavour component of the sea, the charm quark threshold, QCD corrections etc. [22]. We will restrict ourselves to ratios of neutral and charged current cross sections for isoscalar targets, where most of these corrections and also experimental systematic uncertainties cancel. Our bounds on the mixing parameters will be based on the deviations of these ratios due to mixing relative to the simple parton model predictions.

For isoscalar targets one has

$$\sigma(\nu \mathrm{N} \to lX) = \tfrac{1}{2}\left(\sigma(\nu \mathrm{p} \to lX) + \sigma(\nu \mathrm{n} \to lX)\right), \tag{5.18}$$

where l denotes neutrino or charged lepton. The different parton model cross sections are given by

$$\begin{aligned} \sigma_{\nu \mathrm{N}} &\equiv \sum_i \sigma(\nu_\mu \mathrm{N} \to \nu_i X) \\ &= \left(G_\mu^2 s/4\pi\right)\langle n\rangle(1+\lambda_\mathrm{e}-\lambda_\mu) \\ &\quad\times\left\{1-2z+\tfrac{40}{27}z^2+\tfrac{10}{3}z\left(1-\tfrac{19}{9}z+\tfrac{40}{27}z^2\right)\epsilon\right. \\ &\quad\left.+\tfrac{1}{3}r\left[1-2z+\tfrac{40}{9}z^2+\tfrac{10}{3}z\left(1-\tfrac{13}{3}z+\tfrac{40}{9}z^2\right)\epsilon\right]\right\}, \end{aligned} \tag{5.19}$$

$$\begin{aligned} \sigma_{\bar{\nu} \mathrm{N}} &\equiv \sum_i \sigma(\bar{\nu}_\mu \mathrm{N} \to \bar{\nu}_i X) \\ &= \left(G_\mu^2 s/12\pi\right)\langle n\rangle(1+\lambda_\mathrm{e}-\lambda_\mu) \\ &\quad\times\left\{1-2z+\tfrac{40}{9}z^2+\tfrac{10}{3}z\left(1-\tfrac{13}{3}z+\tfrac{40}{9}z^2\right)\epsilon\right. \\ &\quad\left.+3r\left[1-2z+\tfrac{40}{27}z^2+\tfrac{10}{3}z\left(1-\tfrac{19}{9}z+\tfrac{40}{27}z^2\right)\epsilon\right]\right\}, \end{aligned} \tag{5.20}$$

$$\begin{aligned} \bar{\sigma}_{\nu \mathrm{N}} &\equiv \sigma(\nu_\mu \mathrm{N} \to \mu^- X) \\ &= \left(G_\mu^2 s/2\pi\right)|V_{\mathrm{ud}}|^2(1+\lambda_\mathrm{e})(1-\eta_\mu)\left(1+\tfrac{1}{3}r\right)\langle n\rangle, \end{aligned} \tag{5.21}$$

$$\begin{aligned} \bar{\sigma}_{\bar{\nu} \mathrm{N}} &\equiv \sigma(\bar{\nu}_\mu \mathrm{N} \to \mu^+ X) \\ &= \left(G_\mu^2 s/6\pi\right)|V_{\mathrm{ud}}|^2(1+\lambda_\mathrm{e})(1-\eta_\mu)(1+3r)\langle n\rangle. \end{aligned} \tag{5.22}$$

Here $z = \sin^2\Theta$, $r = \langle \bar{n}\rangle/\langle n\rangle \approx \frac{1}{8}$, where $\langle^{(}\bar{n}^{)}\rangle = \int_0^1 \mathrm{d}x\, x^{(}\bar{n}^{)}(x)$ with $n(x)$ ($\bar{n}(x)$) being the density of quarks (antiquarks) in the nucleon, and V_{ud} is the up–down matrix element of the Kobayashi–Maskawa matrix.

TABLE 3
Bounds on mixing parameters from low energy neutral current processes. The three models A, B and C are the same as in table. The * denotes an expected experimental error.

	Predicted deviation [%]			Exp. error [%]
	A	B	C	
$\delta R_2/R_2$	0.4	−2	3	10 [23]
$\delta R_{12}/R_{12}$	0.5	−2	2	2 [7,21] *
$\delta R_\nu/R_\nu$	1	2	−0.3	1 [6,22]
$\delta R_{\bar\nu}/R_{\bar\nu}$	−0.1	0.7	−0.9	5 [6,22]

From eqs. (5.19)–(5.22) one obtains for the cross section ratios

$$R_\nu = \frac{\sigma_{\nu \mathrm{N}}}{\bar\sigma_{\nu \mathrm{N}}}, \qquad R_{\bar\nu} = \frac{\sigma_{\bar\nu \mathrm{N}}}{\bar\sigma_{\bar\nu \mathrm{N}}}, \tag{5.23}$$

the relative deviations from the standard model predictions

$$\frac{\delta R_\nu}{R_\nu} = 0.83\epsilon + 0.34\lambda_e - 0.66\lambda_\mu + \eta_\mu, \tag{5.24}$$

$$\frac{\delta R_{\bar\nu}}{R_{\bar\nu}} = 0.37\epsilon + 0.07\lambda_e - 0.93\lambda_\mu + \eta_\mu. \tag{5.25}$$

Note that both ratios depend on η_μ. The various mixing parameters enter with positive and negative sign.

In table 3 predicted deviations and experimental errors are compared for leptonic and semi-leptonic neutral current observables. We have neglected the dependence on $|V_{e\mu}|^2$, η_e and η_μ, which from neutrino oscillation experiments are restricted to be smaller than about 2×10^{-3} [10]. In this approximation we can also neglect the change in the composition of the neutrino beams, which is of order $|V_{e\mu}|^2$. So far the data from neutrino electron scattering have not yet reached the precision of 1% which is required to test for mixing parameters at the 1% level. Neutrino–nucleon scattering data do have the necessary accuracy. In particular the ratio R_ν yields a useful bound for ϵ.

At least in principle, however, low-energy neutral current data are sensitive to matrix elements of V, the leptonic Kobayashi–Maskawa matrix which, to leading order, does not contribute to charged current processes and observables at the Z pole. Hence, low-energy neutrino neutral current data play a complementary role, and an increase in precision would be most welcome.

6. Discussion

In the preceding sections we have considered implications of precision tests of the electroweak theory on mixings of ν_e, ν_μ and ν_τ with heavy Majorana neutrinos. Such mixings necessarily occur if the ordinary neutrinos are massive and if the smallness of their masses is explained by the see-saw mechanism.

This mechanism of neutrino mass generation emerges most naturally in extensions of the standard model where $B-L$, the difference of baryon and lepton number, plays the role of a spontaneously broken local symmetry. At present little is known about the mass scale at which $B-L$ is broken. Some unified models, in particular most superstring theories, predict a low scale of $B-L$ breaking of order 1 TeV. As a consequence, these theories predict an extra Z′ vector boson and also heavy Majorana neutrinos with masses below 1 TeV, which may be kinematically accessible at present and projected colliders.

Production cross sections for heavy Majorana neutrinos in ep and e^+e^- collisions crucially depend on the parameter $\lambda_e = \Sigma_i |\xi_{ei}|^2$, where ξ_{ij} is the mixing matrix between light and heavy neutrinos. In general, charged and neutral current processes depend on the mixing parameters λ_i ($i = e, \mu, \tau$), $\eta_i = 1 - |V_{ii}|^2$ where V is the leptonic Kobayashi–Maskawa matrix, and the Z–Z′ mixing angle which is proportional to $m_Z^2/m_{Z'}^2$, where m_Z and $m_{Z'}$ are the masses of the Z and Z′ vector bosons. Hence, precision tests of the electroweak theory can be used to place upper bounds on mixing parameters, in particular on λ_e.

Previous analyses have treated the effects of neutrino mixing and neutral vector boson mixing on electroweak observables separately. However, in theories with a low scale of $B-L$ breaking both effects occur together and may be of comparable strength. Hence, both mixings must be analyzed simultaneously.

A qualitative result of our analysis is that the effects of neutrino mixing and Z–Z′ mixing generically tend to compensate each other. In fact, among the 15 observables which we have studied there are only two exceptions: the ratio of Fermi constants from μ and semileptonic decays, which depends only on a single mixing parameter, and the shift of the W boson mass where all mixing parameters enter with positive sign. Note that the shift of the invisible width of the Z boson, which is proportional to the effective number of light neutrinos, can be positive or negative.

Quantitatively, the most stringent bound follows from the comparison of the Fermi constants from μ and semileptonic decays, i.e. the test for quark–lepton universality of the charged current, which yields

$$\lambda_\mu < 3 \times 10^{-3}. \qquad (6.1)$$

This, together with the test of lepton universality in π decays, implies

$$\lambda_e < 1 \times 10^{-2}, \qquad (6.2)$$

which is the most stringent bound for the production of heavy Majorana neutrinos. From the shift of the W boson mass one finds

$$\lambda_e < 9.1 \frac{\delta m_W}{m_W}. \tag{6.3}$$

Hence, an improvement of the bound (6.2) requires a measurement of m_W to a precision of less than ± 100 MeV. A stringent bound could also be obtained by measuring forward–backward asymmetries and, with polarization, the left–right asymmetry in e^+e^- annihilation at the Z pole. The projected error of 2% for the forward–backward asymmetry of b quark jets and the left–right asymmetry would imply $\lambda_e < 2 \times 10^{-3}$.

The mixing of light and heavy Majorana neutrinos may be flavour dependent and largest for the third family. Since partial widths of the τ lepton are most sensitive to the mixing parameter λ_τ, a more precise measurement of τ decays is particularly interesting with respect to the problem of neutrino masses and mixings.

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