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Dispersion relations for vacuum-polarization functions in electroweak physics *

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We propose a class of once-subtracted dispersion relations for the vacuum-polarization functions of massive fermions, in which the subtraction constants are determined explicitly from Ward identities. We show that in perturbation theory to $O(\alpha)$ and $O(\alpha\alpha_s)$ this approach, the alternative dispersion relations proposed by Chang, Gaemers and van Neerven, and dimensional regularization all give the same contributions to electroweak observables such as $\Delta\rho$ and Δr. The threshold behaviours of the subtraction integrands are, however, very different and the two dispersion methods are expected to lead to significantly variant estimates of contributions arising from non-perturbative $t\bar{t}$ threshold effects.

The study of radiative corrections of $O(\alpha\alpha_s)$ has become a matter of considerable interest in electroweak physics. In fact, it has been shown that QCD corrections to the vacuum-polarization functions associated with the $W^{\pm}$ and Z^0 bosons are significant for large m_t values [1–5]. This in turn affects the detailed study of such basic corrections as $\Delta\rho$ and Δr and, as a consequence, the predicted value of m_W and the m_t upper bound.

One approach in the study of the QCD corrections has been the perturbative evaluation, using dimensional regularization, of the two-loop diagrams

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involving the top and bottom quarks and a virtual gluon [1]. An alternative method, based on dispersion relations, goes back to the pioneering work of Chang et al. [6]. Writing the vacuum-polarization tensors for vector and axial vector currents as

$$\Pi_{\mu\nu}^{\mathrm{V,A}}(q,m_1,m_2) = \Pi^{\mathrm{V,A}}(s,m_1,m_2)\, g_{\mu\nu} + \lambda^{\mathrm{V,A}}(s,m_1,m_2)\, q_\mu q_\nu\,, \tag{1}$$

where $s = q^2$ and m_1 and m_2 denote the masses of the two virtual quarks, these authors proposed the dispersion relations

$$\begin{aligned} &\Pi^{\mathrm{V,A}}(s,m_1,m_2) \\ &= \frac{1}{\pi}\left[\int_{(m_1+m_2)^2}^{\Lambda^2} \mathrm{d}s'\,\frac{\mathrm{Im}\,\Pi^{\mathrm{V,A}}(s',m_1,m_2)}{s'-s-i\varepsilon}\right. \\ &\left. -\frac{1}{2}\left(\int_{4m_1^2}^{\Lambda^2}\mathrm{d}s'\,\frac{\mathrm{Im}\,\Pi^{\mathrm{V}}(s',m_1,m_1)}{s'} + \int_{4m_2^2}^{\Lambda^2}\mathrm{d}s'\,\frac{\mathrm{Im}\,\Pi^{\mathrm{V}}(s',m_2,m_2)}{s'}\right)\right]. \end{aligned} \tag{2}$$

The subtraction terms remove the quadratically divergent part of the first term; there remains a logarithmically divergent part, which is regulated by the cutoff Λ^2. The peculiar form of the subtraction terms was predicated on the grounds that, at the one-loop level, the resulting expressions amount to finite renormalizations of the corresponding results obtained through dimensional regularization and the observation that such terms cancel in physical quantities such as the corrections to the vector-boson masses.

In the present paper we propose a new class of dispersion relations in which the subtraction constants are derived explicitly from Ward identities. Writing

$$\Pi_{\mu\nu}(q) = -i\int \mathrm{d}^4x\, \mathrm{e}^{iq\cdot x}\langle 0|T^*\left[J_\mu(x)J_\nu^\dagger(0)\right]|0\rangle\,, \tag{3}$$

where T^* denotes the covariant time-ordered product, J_μ represents vector or axial vector currents, and we have suppressed for brevity the dependence on m_1 and m_2, we obtain the Ward identity

$$q^\mu\Pi_{\mu\nu}(q) = \int \mathrm{d}^4x\, \mathrm{e}^{iq\cdot x}\langle 0|T\left[\partial^\mu J_\mu(x)J_\nu^\dagger(0)\right]|0\rangle \equiv \Delta(s)\, q_\nu\,, \tag{4}$$

Combining eqs. (1) and (4), we have

$$\Pi(s) = -\lambda(s)\,s + \Delta(s)\,. \tag{5}$$

We now observe that both $\lambda(s)$ and $\Delta(s)$ are only logarithmically divergent. In the case of $\lambda(s)$ this is due to the extraction of two powers of the external momentum in eq. (1). In the case of $\Delta(s)$ this arises from the fact that J_μ

is broken softly by mass terms (so that $\partial^\mu J_\mu$ involves operators of canonical dimension 3) and the further point that one power of the external momentum is extracted in eq. (4). Our proposal is to evaluate the logarithmically divergent quantities $\lambda(s)$ and $\Delta(s)$ by means of unsubtracted (but regularized) dispersion relations. The quadratically divergent quantity $\Pi(s)$ is then uniquely determined by the Ward identity of eq. (5), namely

$$\Pi^{\mathrm{V,A}}(s,m_1,m_2) = -\frac{s}{\pi}\int_{(m_1+m_2)^2}^{\Lambda^2} \mathrm{d}s' \, \frac{\mathrm{Im}\,\lambda^{\mathrm{V,A}}(s',m_1,m_2)}{s'-s-i\varepsilon} + \frac{1}{\pi}\int_{(m_1+m_2)^2}^{\Lambda^2} \mathrm{d}s' \, \frac{\mathrm{Im}\,\Delta^{\mathrm{V,A}}(s',m_1,m_2)}{s'-s-i\varepsilon}. \quad (6)$$

Eq. (6) can be written in the equivalent form

$$\Pi^{\mathrm{V,A}}(s,m_1,m_2) = \frac{1}{\pi}\int_{(m_1+m_2)^2}^{\Lambda^2} \mathrm{d}s' \left(\frac{\mathrm{Im}\,\Pi^{\mathrm{V,A}}(s',m_1,m_2)}{s'-s-i\varepsilon} + \mathrm{Im}\,\lambda^{\mathrm{V,A}}(s',m_1,m_2) \right). \quad (7)$$

For vector currents with $m_1 = m_2$, $\partial^\mu J_\mu(x) = 0$; therefore $\Delta(s) = 0$ holds, $\mathrm{Im}\,\lambda$ is related to $\mathrm{Im}\,\Pi$ by eq. (5), and one verifies that eqs. (2) and (7) are identical. For vector currents with unequal masses and for axial vector currents, the subtraction terms in eqs. (2) and (7) are, at first hand, very different. We will see later on that this is expected to lead to significantly variant estimates of non-perturbative threshold effects. However, as far as purely perturbative calculations are concerned, we will now show that (i) the prescriptions of eqs. (2) and (7) coincide for arbitrary values of m_1 and m_2 at the one-loop level; (ii) for axial vector currents with $m_1 = m_2$ or vector and axial vector currents with $m_2 = 0$, eqs. (2) and (7) differ at the two-loop level, i.e. to leading order in QCD, by finite renormalizations that cancel in $\Delta\rho$ and Δr. (iii) Both calculations give the same contributions to $\Delta\rho$ and Δr as dimensional regularization.

At the one-loop level, a simple procedure is to evaluate directly $\Pi_{\mu\nu}$ from eq. (3) using dimensional regularization and then to extract the imaginary parts of $\Pi^{\mathrm{V,A}}(s,m_1,m_2)$ and $\lambda^{\mathrm{V,A}}(s,m_1,m_2)$, as these are independent of the regularization procedure. (An equivalent method is to evaluate directly the imaginary parts by means of Cutkosky's rule.) As a check, we have calculated also $\Delta(s)\,q_\nu$ via eq. (4) and verified that, as expected, the Ward identity of eq. (5) is satisfied in dimensional regularization. In these calculations we normalize the currents so that $J_\mu = \bar\psi_2\gamma_\mu\psi_1$ for vector currents and $J_\mu = \bar\psi_2\gamma_\mu\gamma_5\psi_1$ for axial vector currents, where ψ_1 and ψ_2 are the field operators describing quarks of mass m_1 and m_2, respectively. For the imaginary parts at lowest order we find, when $s > (m_1 + m_2)^2$:

$$\operatorname{Im}\lambda_0^{\mathrm{V}}(s,m_1,m_2) = -\frac{\sqrt{\omega}}{4\pi s^3}\left[s^2 + s\left(m_1^2 + m_2^2\right) - 2\left(m_1^2 - m_2^2\right)^2\right], \quad (8)$$

$$\begin{aligned}\operatorname{Im}\Pi_0^{\mathrm{V}}(s,m_1,m_2) &= -s\operatorname{Im}\lambda_0^{\mathrm{V}}(s,m_1,m_2)\\ &\quad - \frac{3\sqrt{\omega}}{8\pi s^2}(m_1 - m_2)^2\left[s - (m_1 + m_2)^2\right], \end{aligned} \quad (9)$$

where the subscript 0 means "lowest order", we have included the colour factor 3, and $\omega \equiv \left[s - (m_1 + m_2)^2\right]\left[s - (m_1 - m_2)^2\right]$. (Note that $|\boldsymbol{p}| \equiv \frac{1}{2}\sqrt{\omega/s}$ is the momentum of either particle in the centre-of-mass system (c.m.s.) defined by $q = (q^0, \mathbf{0})$.) Comparison with eq. (5) shows that the second term in eq. (9) equals $\operatorname{Im}\Delta_0^{\mathrm{V}}(s,m_1,m_2)$.

Due to γ_5 reflection symmetry, we have $\Pi^{\mathrm{A}}(s,m_1,m_2) = \Pi^{\mathrm{V}}(s,m_1,-m_2)$ and $\lambda^{\mathrm{A}}(s,m_1,m_2) = \lambda^{\mathrm{V}}(s,m_1,-m_2)$ up to $\mathrm{O}(\alpha_s)$. Thus we see from eqs. (8) and (9) that, although $\operatorname{Im}\Pi_0^{\mathrm{V}} \neq \operatorname{Im}\Pi_0^{\mathrm{A}}$, $\operatorname{Im}\lambda_0^{\mathrm{V}} = \operatorname{Im}\lambda_0^{\mathrm{A}}$. The reason is easy to understand mathematically: when propagators are rationalized, the only odd terms in m_2 are proportional to $m_1 m_2$ and, at the one-loop level, their cofactors involve only two γ matrices. Therefore, after the traces are evaluated they become proportional to $g_{\mu\nu}$ and thus contribute to $\operatorname{Im}\Pi_0^{\mathrm{V,A}}$ but not to $\operatorname{Im}\lambda_0^{\mathrm{V,A}}$. Since such odd terms are the only ones that distinguish the vector and axial vector functions, our previous conclusions follow. However, the equality $\operatorname{Im}\lambda^{\mathrm{V}} = \operatorname{Im}\lambda^{\mathrm{A}}$ is not expected to survive in higher orders.

Using eq. (8), we find by explicit calculation, for arbitrary m_1 and m_2 and $\Lambda^2 \gg m_1^2, m_2^2$, that the constant in eq. (7) is given by

$$\frac{1}{\pi}\int_{(m_1+m_2)^2}^{\Lambda^2} \mathrm{d}s' \operatorname{Im}\lambda_0^{\mathrm{V,A}}(s',m_1,m_2) = -\frac{1}{4\pi^2}\left[\Lambda^2 - 3\left(m_1^2 + m_2^2\right)\right]. \quad (10)$$

As $\operatorname{Im}\Pi_0^{\mathrm{V}}(s,m,m) = -s\operatorname{Im}\lambda_0^{\mathrm{V,A}}(s,m,m)$, it immediately follows that the subtraction constants in eqs. (2) and (7) are identical at lowest order.

In order to extend the analysis to the level of the leading QCD corrections, it is convenient to write

$$\Pi^{\mathrm{V,A}}(s,m_1,m_2) = \Pi_0^{\mathrm{V,A}}(s,m_1,m_2) + \frac{\alpha_s}{\pi}\Pi_1^{\mathrm{V,A}}(s,m_1,m_2) + \ldots, \quad (11a)$$

$$\lambda^{\mathrm{V,A}}(s,m_1,m_2) = \lambda_0^{\mathrm{V,A}}(s,m_1,m_2) + \frac{\alpha_s}{\pi}\lambda_1^{\mathrm{V,A}}(s,m_1,m_2) + \ldots. \quad (11b)$$

The real and imaginary parts of $\Pi_1^{\mathrm{V,A}}(s,m_1,m_2)$ have been extensively studied in the literature. In order to evaluate $\operatorname{Im}\lambda_1^{\mathrm{V,A}}$, we have restricted ourselves to the two cases of greatest current interest, namely $m_1 = m_2$ and $m_2 = 0$, which to very good approximation can be applied to the t-b isodoublet. We find

$$\mathrm{Im}\,\lambda_1^{\mathrm{V}}(s,m,m) = -\frac{1}{s}\mathrm{Im}\,\Pi_1^{\mathrm{V}}(s,m,m)\,, \tag{12}$$

$$\mathrm{Im}\,\lambda_1^{\mathrm{A}}(s,m,m) = \mathrm{Im}\,\lambda_1^{\mathrm{V}}(s,m,m) - \frac{1}{4\pi r^2}\left(\frac{\phi}{r} + \sqrt{1-\frac{1}{r}}\right), \tag{13}$$

$$\begin{aligned}\pi\mathrm{Im}\,\lambda_1^{\mathrm{V,A}}(s,m,0) = {} & \frac{1}{3}\left(1+\frac{2}{x}\right)\left(1-\frac{1}{x}\right)^2\left[2\mathrm{Li}_2\left(\frac{1}{1-x}\right) + \beta(-\alpha+\beta)\right] \\ & -\frac{\alpha}{3}\left(1+\frac{2}{x}-\frac{2}{x^2}\right) + \frac{\beta}{3}\left(1+\frac{5}{x}-\frac{3}{2x^2}\right)\left(1-\frac{1}{x}\right)^2 \\ & -\frac{1}{4}\left(1-\frac{1}{x}\right)\left(1+\frac{3}{x}-\frac{16}{3x^2}\right),\end{aligned} \tag{14}$$

where $r \equiv s/(4m^2)$, $x \equiv s/m^2$, $\phi \equiv \ln(\sqrt{r} + \sqrt{r-1}\,) = \mathrm{arcosh}\sqrt{r}$, $\alpha \equiv \ln x$, $\beta \equiv \ln(x-1)$, and $\mathrm{Li}_2(z) \equiv -\int_0^1 \mathrm{d}t\,\ln(1-zt)/t$ is the dilogarithmic function. Eq. (12) follows from the Ward identity of eq. (5). Eqs. (13) and (14) have been obtained by applying Cutkosky's rule to the relevant two-loop diagrams. We note that $\mathrm{Im}\,\lambda_1^{\mathrm{A}}(s,m,m) \neq \mathrm{Im}\,\lambda_1^{\mathrm{V}}(s,m,m)$. The second term in eq. (13) gives a finite contribution to the subtraction constant in eq. (7) and, indeed, we find

$$\frac{1}{\pi}\int_{4m^2}^{\Lambda^2} \mathrm{d}s'\,\mathrm{Im}\,\lambda_1^{\mathrm{A}}(s',m,m) = \frac{1}{\pi}\int_{4m^2}^{\Lambda^2} \mathrm{d}s'\,\mathrm{Im}\,\lambda_1^{\mathrm{V}}(s',m,m) - \frac{m^2}{\pi^2}. \tag{15a}$$

Furthermore, eq. (14) leads to

$$\frac{1}{\pi}\int_{m^2}^{\Lambda^2} \mathrm{d}s'\,\mathrm{Im}\,\lambda_1^{\mathrm{V,A}}(s',m,0) = -\frac{m^2}{\pi^2}\left(\frac{\Lambda^2}{4m^2} + \frac{3}{2}\ln\frac{\Lambda^2}{m^2} + \frac{1}{4}\right). \tag{15b}$$

Comparison with the subtraction constants in eq. (2) shows that the difference between the approaches of eqs. (2) and (7) is given by the terms $-m^2/\pi^2$ in eq. (15a) and $-m^2/(4\pi^2)$ in eq. (15b). We now explain why, in spite of these differences, the two methods as well as dimensional regularization give the same perturbative results for the contributions of the t-b isodoublet to convergent quantities such as $\Delta\rho$ and Δr. Detailed examination shows that in the three approaches the vacuum-polarization functions can be written as

$$\frac{\pi^2}{m^2}\Pi_i^{\mathrm{V}}(s,m,m) = rX_i + V_i(r)\,, \tag{16a}$$

$$\frac{\pi^2}{m^2}\Pi_i^{\mathrm{A}}(s,m,m) = rX_i + Y_i + A_i(r)\,, \tag{16b}$$

$$\frac{\pi^2}{m^2}\Pi_i^{\mathrm{V,A}}(s,m,0) = \frac{1}{4}(xX_i + Y_i) + F_i(x)\,, \tag{16c}$$

where $i = 0, 1$ labels the one- and two-loop results, $V_i(r)$, $A_i(r)$, and $F_i(x)$ are finite functions listed in ref. [4], and X_i and Y_i are divergent constants. The exact expressions for these constants depend on the regularization scheme. For example, in the dimensional-regularization scheme X_0, Y_0, and X_1 contain terms proportional to ε^{-1} and ε^0 while Y_1 involves also contributions of $O(\varepsilon^{-2})$.

Calling $L \equiv \ln(\Lambda^2/m^2)$, one finds $X_0 = X_1 = L$ and $Y_0 = -3L/2$ in both dispersive approaches. However, $Y_1 = 3L^2/2 - 9L/2$ in the case of eq. (2) and $Y_1 = 3L^2/2 - 9L/2 - 1$ in the case of eq. (7). The important thing to note is that the X_0, Y_0, X_1, and Y_1 cancel identically among themselves in the evaluation of convergent quantities such as $\Delta\rho$ and Δr, independently of their specific representations. We illustrate this point by considering the contribution of the t-b isodoublet (with $m_b \to 0$) to Δr. One finds

$$\begin{aligned}\frac{s_w^2}{\pi\alpha}\Delta r^{(tb)} &= \frac{4}{9}s_w^2\Big(4\Pi^{V'}(0,m_t,m_t) + \Pi^{V'}(0,0,0)\Big)\\ &+ \frac{1}{m_W^2}\Big(\Pi^V(0,m_t,0) - \mathrm{Re}\,\Pi^V(m_W^2,m_t,0)\Big)\\ &+ \frac{c_w^2}{s_w^2 m_W^2}\mathrm{Re}\Big[\Pi^V(m_W^2,m_t,0) - \frac{1}{4}\Big(v_t^2\Pi^V(m_Z^2,m_t,m_t)\\ &+\Pi^A(m_Z^2,m_t,m_t) + v_b^2\Pi^V(m_Z^2,0,0) + \Pi^A(m_Z^2,0,0)\Big)\Big], \qquad (17)\end{aligned}$$

where the derivatives in the first term are with respect to s and are evaluated at $s = 0$, and $v_t = 1 - 8s_w^2/3$ and $v_b = -1 + 4s_w^2/3$ are the neutral-current vector couplings of t and b, respectively. As usual, $c_w^2 = 1 - s_w^2 = m_W^2/m_Z^2$. Strictly speaking, $\Pi^{V'}(0, m_b, m_b)$ exhibits a mass singularity for $m_b \to 0$. This difficulty is usually circumvented by writing

$$\Pi^{V'}(0,0,0) = \frac{\mathrm{Re}\,\Pi^V(m_Z^2,0,0)}{m_Z^2} + \left(\Pi^{V'}(0,0,0) - \frac{\mathrm{Re}\,\Pi^V(m_Z^2,0,0)}{m_Z^2}\right), \qquad (18)$$

where the first term is perturbatively well-defined and the expression contained within the parentheses is ultraviolet-finite and can be related to experimental data of σ ($e^+e^- \to b\bar{b}$) via a conventional once-subtracted dispersion relation. Inserting eqs. (16a–c) into eq. (17), one verifies that the X_i and Y_i cancel identically, proving the equivalence of the three methods for perturbative calculations of $O(\alpha)$ and $O(\alpha\alpha_s)$.

Although the above mentioned agreement bears witness to the ingenuity of the authors of ref. [6], some important features of the subtractions in eq. (2) are, in our view, surprising and perhaps unphysical. One notices, for example, that the subtractions in eq. (2) involve in many cases quantities with different channels, thresholds, and currents than the direct dispersion integral! To put it more directly: if we are interested in evaluating, for example, the W-boson vacuum-polarization functions, which involve the $t\bar{b}$ channel, why should we subtract quantities that seem to describe completely different physics, namely the $t\bar{t}$ and $b\bar{b}$ channels? In contrast, the subtraction constants in eq. (7), derived explicitly from Ward identities, involve the same currents and channels as the direct integral. Because of these facts, the integrands of the subtraction constants in eqs. (2) and (7) have, in general, very different threshold behaviours. To illustrate this point, we give the leading threshold behaviours of the relevant vacuum-polarization functions:

$$\frac{\pi}{m^2}\mathrm{Im}\,\Pi_0^{\mathrm{V}}(s,m,m) = -4\pi r\mathrm{Im}\,\lambda_0^{\mathrm{V,A}}(s,m,m) = \frac{3}{2}v + \mathrm{O}(v^3), \tag{19a}$$

$$\frac{\pi}{m^2}\mathrm{Im}\,\Pi_0^{\mathrm{A}}(s,m,m) = v^3 + \mathrm{O}(v^5), \tag{19b}$$

$$\frac{\pi}{m^2}\mathrm{Im}\,\Pi_1^{\mathrm{V}}(s,m,m) = -4\pi r\mathrm{Im}\,\lambda_1^{\mathrm{V}}(s,m,m) = \pi^2 + \mathrm{O}(v), \tag{19c}$$

$$\frac{\pi}{m^2}\mathrm{Im}\,\Pi_1^{\mathrm{A}}(s,m,m) = \frac{2}{3}\pi^2 v^2 + \mathrm{O}(v^3), \tag{19d}$$

$$-4\pi r\mathrm{Im}\,\lambda_1^{\mathrm{A}}(s,m,m) = \pi^2 + \mathrm{O}(v), \tag{19e}$$

where $v = \sqrt{1-1/r}$ is the velocity of either quark in the c.m.s. The constant behaviour in eqs. (19c) and (19e) is due to the well-known Coulomb effect which is proportional to $(1/v)$ times the lowest-order contribution. In the $m_2 = 0$ case we have

$$\frac{\pi}{m^2}\mathrm{Im}\,\Pi_0^{\mathrm{V,A}}(s,m,0) = \frac{3}{2}v^2 + \mathrm{O}(v^3), \tag{20a}$$

$$\pi\mathrm{Im}\,\lambda_0^{\mathrm{V,A}}(s,m,0) = -3v^2 + \mathrm{O}(v^3), \tag{20b}$$

$$\frac{\pi}{m^2}\mathrm{Im}\,\Pi_1^{\mathrm{V,A}}(s,m,0) = v^2\left(-3\ln(2v) + \frac{2}{3}\pi^2 + \frac{9}{2}\right) + \mathrm{O}(v^3\ln v), \tag{20c}$$

$$\pi\mathrm{Im}\,\lambda_1^{\mathrm{V,A}}(s,m,0) = v^2\left(6\ln(2v) - \frac{4}{3}\pi^2 - 11\right) + \mathrm{O}(v^3\ln v), \tag{20d}$$

where $v = (x-1)/(x+1)$ is the velocity of the massive quark in the c.m.s.

In the equal-mass case we see that the integrands of the subtraction terms in eqs. (2) and (7) have the same leading threshold behaviour. However, in the unequal-mass case (with $m_2 = 0$) the situation is completely different: while the contribution of $\mathrm{Im}\,\lambda_i^{\mathrm{V,A}}(s,m,0)$ is greatly suppressed at threshold, that of $\mathrm{Im}\,\Pi_i^{\mathrm{V}}(s,m,m)/s$ is not.

Over the last several years, a number of authors have pointed out that there are interesting and possibly significant effects associated with the $t\bar{t}$ threshold. These may arise from resonances in the $t\bar{t}$ system [2] or from the resummation of Coulombic effects near threshold [7]. As these effects are usually expressed in terms of contributions to the absorptive parts of vacuum-polarization functions, a dispersive approach seems to be most suitable in order to determine the corresponding real parts. It is apparent, however, that eqs. (2) and (7) give qualitatively different answers for such "non-perturbative effects." While eq. (2) tells us that the vector and axial vector parts of both $\mathrm{Re}\,\Pi^{ZZ}(s,m,m)$ and $\mathrm{Re}\,\Pi^{WW}(s,m,0)$ obtain potentially significant $t\bar{t}$ threshold contributions from the subtraction constants, eq. (7) informs us that this is only the case for $\mathrm{Re}\,\Pi^{ZZ}(s,m,m)$! As eq. (7) is constructed explicitly from Ward identities and involves subtraction constants associated with the same channels as the direct integrals, we believe it provides a consistent framework to study such non-perturbative threshold effects. It should be emphasized, however, that the application of eq. (7) requires the evaluation of the non-perturbative threshold effects on $\mathrm{Im}\,\lambda^{V,A}(s,m,m)$. While $\mathrm{Im}\,\lambda^{V}(s,m,m)$ is related to $\mathrm{Im}\,\Pi^{V}(s,m,m)/s$ by the Ward identity of eq. (5), $\mathrm{Im}\,\lambda^{A}(s,m,m)$ may require a special analysis. Specific numerical studies of perturbative and threshold effects based on eq. (7) will be given in a separate communication.

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