BRIEF REPORTS

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Measuring the weak isospin of b quarks in e^+e^- annihilation

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From measurements of the Z decay width to b quarks $\Gamma(Z\to b\bar{b})$ and the forward-backward asymmetry $A_{FB}(b)$ of b quarks at e^+e^- colliders, the weak-isospin components of the left- and right-handed b quarks can unambiguously be determined: $I_3^L(b)=-\frac{1}{2}$ and $I_3^R(b)=0$. From the nonzero value of $I_3^L(b)$ the top quark can be inferred as the isospin partner of the b quark.

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For quite some time there has been considerable interest in measurements of the weak-isospin quantum numbers of b quarks. If the isomultiplet structure of the first two families is reiterated in the third family, the left-handed b quark is the lower component of an isodoublet, demanding the existence of top quarks as the upper isopartner to the b quarks. The right-handed particles are both isosinglets.

Several arguments have been advanced in the past which experimentally rule out the assignment $I_3^L(b)=0$ to the left-handed b quarks. They are based on the SU(2)×U(1) gauge structure of the electroweak interactions. If both right- and left-handed components of the b quarks are assumed to be isosinglets, several consequences can be derived which are in conflict with experimental observations. (i) Assigning zero-isospin quantum numbers to left-handed b quarks breaks the Glashow-Iliopoulos-Maiani (GIM) mechanism which demands the left-handed quarks to belong to the same isomultiplet in all three families. If the GIM mechanism is broken the mixing between the quarks induces flavor-changing neutral currents which give rise to large branching ratios of B decays to charged-lepton pairs [1]: $B(B\to l^+l^-X)/B(B\to l\nu_l X)>0.12$. This ratio is three orders of magnitude bigger than the bound recently set by UA1 [2]: $B(B\to\mu^+\mu^-X)/B(B\to\mu\nu_\mu X)<5.0\times10^{-5}/(0.110\pm0.009)$. (ii) The partial Z decay width to b quarks were reduced by more than an order of magnitude if b quarks were isosinglets [3]. (iii) The forward-backward asymmetry of b quarks in e^+e^- annilation is determined by the Z axial charge of b quarks which is given by the difference between the left- and right-handed isospin quantum numbers. Were the isospin of b_L zero and equal to the isospin of b_R, the forward-backward asymmetry would vanish across the entire e^+e^- energy range. This however is in clear conflict with nonzero values of the forward-backward asymmetry as observed at DESY PETRA, SLAC PEP, KEK TRISTAN, and CERN LEP (for a summary see Ref. [4]).

Additional arguments for top quarks follow from the observation of rapid B-$\bar{B}$ oscillations [5] and the theoretical requirement of the electroweak gauge theory to be anomaly free, demanding the sum over all electric charges in the third family to be zero.

In this Brief Report we exploit data on the width $\Gamma(Z\to b\bar{b})$ and the forward-backward asymmetry $A_{FB}(b)$ of b quarks from e^+e^- colliders, to determine accurately the isospin components $I_3^L(b)$ and $I_3^R(b)$ of the left- and right-handed b quarks. Earlier analyses [6] were based on various subsets of these data. This method corroborates the results of the flavor-changing neutral-current (FCNC) analysis; it is nevertheless worth exploiting as it provides an easy and transparent access to the quantum numbers of the b quark.

The decay width and the forward-backward asymmetry are given by the Z vectorial and axial charges in the (improved) Born approximation (see, e.g., [7]) as

$$\Gamma(Z\to b\bar{b})=\frac{G_F M_Z^3}{2\pi\sqrt{2}}(v_b^2+a_b^2)\ , \tag{1}$$

$$A_{FB}(b)=\frac{3}{4}\frac{2v_e a_e}{v_e^2+a_e^2}\frac{2v_b a_b}{v_b^2+a_b^2}\ . \tag{2}$$

The Born approximation, as defined in this form, is not only sufficient for our analysis aiming at half-integer quantum numbers, but we are forced to this approximation if we refrain from major extensions of the Higgs sector to keep the model renormalizable for generalized isospin assignments [8]. Expressing the Z charges by the isospin component of the b quark (briefly denoted I_3^L and I_3^R from now on),

$$v_b = (I_3^L + I_3^R) - 2e_b \sin^2\bar{\theta}_W \ , \quad (3)$$

$$a_b = I_3^L - I_3^R \quad (4)$$

(and for the leptons $v_e = -\frac{1}{2} + 2\sin^2\bar{\theta}_W$, $a_e \doteq -\frac{1}{2}$), the width defines a circle in the (I_3^L, I_3^R) plane while the forward-backward asymmetry defines a pair of straight lines:

$$(I_3^L + \tfrac{1}{3}\sin^2\bar{\theta}_W)^2 + (I_3^R + \tfrac{1}{3}\sin^2\bar{\theta}_W)^2 = R^2 \ , \quad (5)$$

$$|I_3^R + \tfrac{1}{3}\sin^2\bar{\theta}_W| = \gamma |I_3^L + \tfrac{1}{3}\sin^2\bar{\theta}_W| \ . \quad (6)$$

The radius (squared) is given by

$$R^2 = \frac{\pi\sqrt{2}\Gamma(Z\to b\bar{b})}{G_F M_Z^3} \quad (7)$$

and the slope parameter (squared)

$$\gamma^2 = \left[1 - \frac{4A_{\rm FB}(b)}{3}\frac{v_e^2 + a_e^2}{2v_e a_e}\right] \Bigg/ \left[1 + \frac{4A_{\rm FB}(b)}{3}\frac{v_e^2 + a_e^2}{2v_e a_e}\right] . \quad (8)$$

The center of the circle as well as the crossing point of the lines are displaced from the origin of the (I_3^L, I_3^R) plane down to the third quadrant by a small shift $-\frac{1}{3}\sin^2\bar{\theta}_W$ in both variables. The physical solution for the isospin components are those values where the circle and the straight lines both cross one and the same point with half-integer coordinates. The width alone allows for two solutions with L and R interchanged; this ambiguity is resolved by the forward-backward asymmetry which is asymmetric in L and R and selects finally one single point.

It is interesting to note the high sensitivity of the width and of the forward-backward asymmetry to the pairs of possible half-integer isospin values. This is demonstrated in Table I for the lowest values of the quantum numbers. The analysis appears quite robust.

Corroborating evidence can be derived from the forward-backward asymmetries of b quarks measured at PETRA, PEP, and TRISTAN. While the FB asymmetry on the Z is due to the coherent superposition of vectorial and axial Z charges, the asymmetry at the low PETRA/PEP energies is a result of the interference between vectorial γ exchange and axial Z exchange. At TRISTAN all amplitudes contribute with similar strength and the asymmetry is of the general form

$$A_{\rm FB}(b) = \frac{3}{4}\frac{-2e_e e_b a_e a_b \,{\rm Re}\chi + 4v_e v_b a_e a_b |\chi|^2}{e_e^2 e_b^2 - 2e_e e_b v_e v_b \,{\rm Re}\chi + (v_e^2 + a_e^2)(v_b^2 + a_b^2)|\chi|^2} \quad (9)$$

with the electric charges $e_e = -1$, $e_b = -\frac{1}{3}$ and

$$\chi = -\frac{1}{4\sin^2\bar{\theta}_W \cos^2\bar{\theta}_W}\frac{s}{s - M_Z^2 + iM_Z\Gamma_Z} .$$

For a given experimental value of $A_{\rm FB}(b)$, Eq. (9) corresponds to a conic section in the (I_3^L, I_3^R) plane, i.e., ellipse, hyperbola, or a pair of straight lines. At low PETRA/PEP energies, the expression is reduced to a simple straight line

$$I_3^R \approx I_3^L + \frac{A_{\rm FB}(b)}{9\,{\rm Re}\chi} . \quad (10)$$

We now confront this discussion with data from e^+e^- storage rings, summarized in Tables II and III. The width $\Gamma_{b\bar{b}}$ and the forward-backward asymmetry $A_{\rm FB}^{b\bar{b}}$ on

TABLE I. Values of the partial width $\Gamma(Z\to b\bar{b})$ and the forward-backward asymmetry $A_{\rm FB}(b)$ on the Z for the lowest possible isospin quantum numbers of the left- and right-handed b quarks. The electroweak mixing angle is chosen as $\sin^2\bar{\theta}_W = 0.2327$.

	$\Gamma_{b\bar{b}}$ (MeV)	I_3^L: -1	$-\frac{1}{2}$	0	$+\frac{1}{2}$	$+1$
	$+1$	3386	2048	1705	2357	4003
	$+\frac{1}{2}$	2048	710	367	1019	2665
I_3^R	0	1705	**367**	24	676	2322
	$-\frac{1}{2}$	2357	1019	676	1327	2974
	-1	4003	2665	2322	2974	4621

	$A_{\rm FB}^b$	I_3^L: -1	$-\frac{1}{2}$	0	$+\frac{1}{2}$	$+1$
	$+1$	0.000	−0.068	−0.102	−0.045	0.016
	$+\frac{1}{2}$	0.068	0.000	−0.097	0.031	0.076
I_3^R	0	0.102	**0.097**	0.000	0.100	0.103
	$-\frac{1}{2}$	0.045	−0.031	−0.100	0.000	0.057
	-1	−0.016	−0.076	−0.103	−0.057	0.000

TABLE II. Averages of Z-decay data used in the analysis. The value for $A_{\rm FB}(b)$ is corrected for $B\bar{B}$ mixing.

Observable	Ref.	Result
$A_{\rm FB}(b)$	[25]	0.126 ± 0.022
$\Gamma(Z\to b\bar{b})$ (MeV)	[26]	361 ± 19
χ_B	[27]	0.144 ± 0.020
M_Z (GeV)	[25]	91.175 ± 0.021
a_e^2	[25]	0.2492 ± 0.0012
v_e^2	[25]	0.0012 ± 0.0003

TABLE III. PETRA/PEP/TRISTAN results for $A_{\rm FB}(b)$. The values quoted include the correction for $B\bar{B}$ mixing based on the LEP average $\chi_B = 0.144\pm0.020$.

$\sqrt{s}$ (GeV)	Ref.	$A_{\rm FB}(b)$
29	[14–17]	-0.055 ± 0.086
35	[18–21]	-0.228 ± 0.053
44	[18],[19]	-0.489 ± 0.156
55	[22–24]	-0.844 ± 0.185

TABLE IV. Fit results for the weak isospin of the b quark. In all fits the LEP results for $a^2_{\rm lept}$ and $v^2_{\rm lept}$ have been included as constraint.

	Z^0 resonance only solution I	Z^0 resonance only solution II	PETRA+PEP +TRISTAN	Z^0 resonance +PETRA+PEP +TRISTAN
I_3^L	$-0.493^{+0.011}_{-0.013}$	$0.339^{+0.013}_{-0.011}$	-0.29 ± 0.16	$-0.490^{+0.015}_{-0.012}$
I_3^R	-0.077 ± 0.088	-0.077 ± 0.088	$0.15^{+0.17}_{-0.14}$	-0.028 ± 0.056
$a^2_{\rm lept}$	0.2492 ± 0.0012	0.2492 ± 0.0012	0.2492 ± 0.0012	0.2492 ± 0.0012
$v^2_{\rm lept}$	0.0014 ± 0.0003	0.0014 ± 0.0003	0.0012 ± 0.0003	0.0014 ± 0.0003
$\chi^2/N_{\rm DOF}$	$1.4/(4-4)$	$1.4/(4-4)$	$2.6/(6-4)$	$6.0/(8-4)$

the Z have been measured at LEP [9–12] and the SLAC Linear Collider (SLC) [13]. Our determination of $I_3^l(b)$ and $I_3^R(b)$ is based on recent averages shown in Table II. The values are given for the quark final states, including the correction for B-$\bar{B}$ oscillations χ_B in the transition from hadron to quark final states (see Ref. [7]). To compare these values directly to the formulas (1) and (2), we have to apply QED and QCD corrections to the measured values:

$$\Gamma(Z\to b\bar{b})=\Gamma(Z\to b\bar{b})^{\rm expt}/r_{\rm QCD}=345\pm18\ {\rm MeV}\ ,$$

$$A_{\rm FB}(b)=\frac{A_{\rm FB}(b)^{\rm expt}}{1-\alpha_s/\pi}+\Delta_{\rm QED}=0.135\pm0.024$$

with $r_{\rm QCD}=1.045$, $\alpha_s(M_Z^2)=0.12$ and $\Delta_{\rm QED}=0.004$ to account for a shift in $A_{\rm FB}(b)$ due to photonic corrections. Table IV column 2 displays the result of a fit to the data. In this fit the LEP averages for $a^2_{\rm lept}$ and $v^2_{\rm lept}$ have been included as constraints. The value for $\sin^2\bar{\theta}_W(b)$ has been approximated by $\sin^2\bar{\theta}_w({\rm lept})=0.2327^{+0.0027}_{-0.0018}$, derived from the ratio of $v^2_{\rm lept}/a^2_{\rm lept}$ assuming the standard-model isospin assignments for leptons. The nonvanishing χ^2 for this fit is a consequence of the fact that for fixed a_e and v_e the condition $2a_bv_b/(a_b^2+v_b^2)<1$ imposes an upper bound on $A_{\rm FB}(b)$, which is exceeded slightly by the average value of the data. The result of this fit can also be expressed in terms of the radius R and the slope parameter γ of Eqs. (7) and (8):

$$R=0.416^{+0.015}_{-0.018},\quad \gamma=0.00\pm0.20\ .$$

The corresponding 1-σ bounds in the (I_3^L,I_3^R) plane are shown in Fig. 1. There are two crossover regions for the bounds from $\Gamma(Z\to b\bar{b})$ and $A_{\rm FB}(b)$, but only one contains a grid point of half integer isospin values I_3^L and I_3^R, whereas the other one, corresponding to solution II in Table IV, is far off any such grid point.

The ambiguity can however also be solved purely experimentally by including the measurements of $A_{\rm FB}(b)$ from PEP [14–17], PETRA [18–21], and TRISTAN [22–24], which are summarized in Table III. In Fig. 1 we show as an example the 1-σ bounds from data of $A_{\rm FB}(b)$ at 35 GeV. As explained above, the contribution of the various terms to the forward-backward asymmetry in Eq. (10) is energy dependent. Therefore the data for $A_{\rm FB}(b)$ from PETRA, PEP, and TRISTAN alone are sufficient in principle to derive I_3^L and I_3^R simultaneously. The result of a fit to $A_{\rm FB}(b)$ from these storage rings is given in Table IV column 3. The 90% confidence region extends along the band for $A_{\rm FB}(b)$ at 35 GeV and includes only two pairs of half-integer isospin values.

Combining the measurements at the Z resonance with those at lower energies, only one solution for the weak isospins of the left- and right-handed b quarks remains finally. The result of a fit to the combined data set is

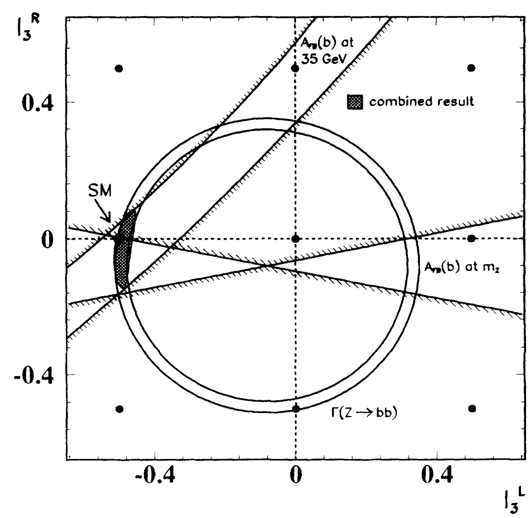

FIG. 1. Experimental constraints for the isospin of the b quark in the (I_3^L,I_3^R) plane. The circle and the pair of straight lines, centered in $e_b\sin^2\bar{\theta}_W=-0.08$, represent 1-$\sigma$ bounds derived from averages for $\Gamma(Z\to b\bar{b})$ and $A_{\rm FB}(b)$ at the Z resonance. The effective electroweak mixing angle is chosen as $\sin^2\bar{\theta}_W(b)\approx\sin^2\bar{\theta}_w({\rm lept})=0.2327$ as obtained from the ratio $v^2_{\rm lept}/a^2_{\rm lept}$ measured at LEP. The grid points mark the half-integer values of I_3^L and I_3^R allowed by the isospin algebra. Only one of the two crossover regions contains a grid point, which coincides with the standard-model isospin assignment for the b quarks, $I_3^L(b)=-\frac{1}{2}$ and $I_3^R(b)=0$. Experimentally the $b\bar{b}$ asymmetry at PETRA, PEP, and TRISTAN energies can discriminate between the two solutions mathematically allowed by Z data. As an example we show the 1-σ bounds derived from the measured FB asymmetry at a c.m. energy of 35 GeV. Combining all e^+e^- collider data results in the 90% confidence region in the (I_3^L,I_3^R) plane indicated by the hatched area.

given in Table IV column 4, the 90% confidence region in the (I_3^L, I_3^R) plane is indicated by the small hatched area in Fig. 1. The only solution compatible with data is the isospin assignment of the standard model:

$$[I_3^L(b)]_{\rm expt} = -0.490^{+0.015}_{-0.012} \Longrightarrow I_3^L(b) = -\tfrac{1}{2} \ ,$$

$$[I_3^R(b)]_{\rm expt} = -0.028 \pm 0.056 \Longrightarrow I_3^R(b) = 0 \ .$$

This analysis of Z-decay data in conjunction with $A_{\rm FB}(b)$ measurements from PETRA, PEP, and TRISTAN corroborates in a most transparent way the isomultiplet structure of the standard model for the b quark in the third family, underlying the existence of the top quark as isopartner of the b quark.

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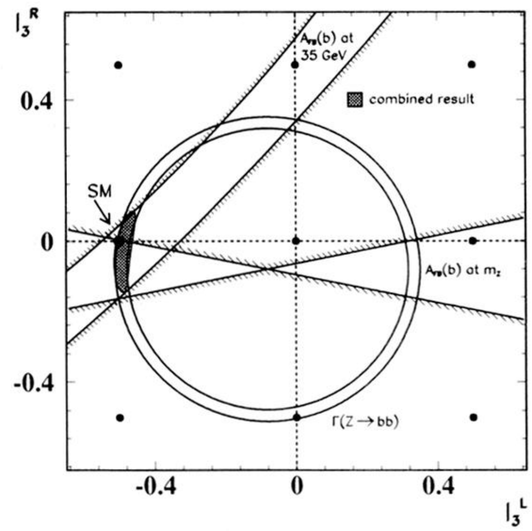

FIG. 1. Experimental constraints for the isospin of the b quark in the (I_3^L, I_3^R) plane. The circle and the pair of straight lines, centered in $e_b \sin^2\bar{\theta}_W = -0.08$, represent 1-$\sigma$ bounds derived from averages for $\Gamma(Z \to b\bar{b})$ and $A_{\rm FB}(b)$ at the Z resonance. The effective electroweak mixing angle is chosen as $\sin^2\bar{\theta}_W(b) \approx \sin^2\bar{\theta}_w(\text{lept}) = 0.2327$ as obtained from the ratio $v_{\text{lept}}^2/a_{\text{lept}}^2$ measured at LEP. The grid points mark the half-integer values of I_3^L and I_3^R allowed by the isospin algebra. Only one of the two crossover regions contains a grid point, which coincides with the standard-model isospin assignment for the b quarks, $I_3^L(b) = -\frac{1}{2}$ and $I_3^R(b) = 0$. Experimentally the $b\bar{b}$ asymmetry at PETRA, PEP, and TRISTAN energies can discriminate between the two solutions mathematically allowed by Z data. As an example we show the 1-σ bounds derived from the measured FB asymmetry at a c.m. energy of 35 GeV. Combining all e^+e^- collider data results in the 90% confidence region in the (I_3^L, I_3^R) plane indicated by the hatched area.