Physics Letters B 275 (1992) 495–505
North-Holland

PHYSICS LETTERS B

Polarization effects in exclusive semi-leptonic Λ_c and Λ_b charm and bottom baryon decays

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Received 29 October 1991

We discuss polarization effects in semi-leptonic decays of polarized and unpolarized heavy Λ-type baryons into heavy and into light Λ-type baryons. We use the non-leptonic decay of the daughter baryon and the leptonic decay of the $W_{\text{off-shell}}$ into a lepton pair as polarization analyzers to analyze the polarization of the daughter baryon and the $W_{\text{off-shell}}$. Technically this is done by writing down joint angular decay distributions. We calculate the values of the various asymmetry parameters that characterize the angular dependence of the angular decay distributions where we use the predictions of the heavy quark effective theory (HQET) supplemented by simple ansätze for the q^2-dependence of the form factors.

Recently the ARGUS Collaboration has reported on the observation of semi-leptonic (s.l.) charm Λ-baryon decays $\Lambda_c^+ \to \Lambda_s X \ell^+ \nu_\ell$ [1]. A major fraction of these decays is expected to consist of the exclusive s.l. decay mode $\Lambda_c^+ \to \Lambda_s \ell^+ \nu_\ell$ [1]. Also, the ALEPH Collaboration has recently seen an excess of correlated $\Lambda_s \ell^-$ pairs over $\Lambda_s \ell^+$ pairs (with high p_T leptons) from Z decays [2]. The $\Lambda_s \ell^+$ excess is readily interpreted as evidence for s.l. decays of bottom Λ-baryons via the chain $\Lambda_b \to \Lambda_c \to \Lambda_s$ [2]. Again, from the experience with s.l. bottom meson decays, one expects a significant fraction of the s.l. $\Lambda_b \to \Lambda_c^+ X$ transitions to consist of the exclusive mode $\Lambda_b \to \Lambda_c^+ \ell^- \bar{\nu}_\ell$.

Experimental results on exclusive s.l. decays of heavy baryons are eagerly awaited by the theoretical community as there has been significant progress in the description of current-induced transitions involving heavy hadrons in the last two years [3–8]. In the baryon sector there exist results for heavy baryon to heavy baryon transitions [9–15] including $1/m$ corrections [16,17], and on heavy baryon to light baryon transitions [12,13,18]. Λ-type heavy baryons are particularly simple in this regard as they consist of a heavy quark and a spin–isospin zero light diquark system.

In this letter we address the question of how to extract information on the structure of hadronic transition form factors from angular decay distributions using polarized and unpolarized exclusive s.l. $\Lambda_b \to \Lambda_c^+ \ell^- \bar{\nu}_\ell$ and $\Lambda_c^+ \to \Lambda_s \ell^+ \nu_\ell$ decays. In our analysis we use helicity amplitudes and polarization density matrix methods to analyze the joint decay distributions in these decays. This analysis complements and generalizes the analysis of ref. [19] where similar results were obtained in the language of spin and momenta correlations. We discuss the limiting behaviour of the joint angular decay distributions at the phase space boundaries $q^2_{\text{min}} = 0$ and $q^2_{\text{max}} = (M_1 - M_2)^2$. By further integrating the general multi-differential joint decay distributions we extract single angle decay distributions. We present theoretical predictions for the decay distributions using results of the heavy quark effective theory (HQET) supplemented by simple power behaved ansätze for the q^2-dependence of the transition form factors.

[1] Supported in part by the BMFT, FRG under contract 06MZ760.

Let us begin our discussion by defining a standard set of invariant form factors for the weak current-induced baryonic $\frac{1}{2}^+ \to \frac{1}{2}^+$ transitions. One has

$$\langle \Lambda_2(P_2)|J_\mu^{V+A}|\Lambda_1(P_1)\rangle = \bar{u}(P_2)[\gamma_\mu(F_1^V + F_1^A\gamma_5) + i\sigma_{\mu\nu}q^\nu(F_2^V + F_2^A\gamma_5)]u(P_1)\,, \qquad (1)$$

where J_μ^V and J_μ^A are vector and axial vector currents and $q_\mu = (P_1 - P_2)_\mu$ is the four-momentum transfer. The form factors $F_i^{V,A}$ are functions of q^2. In the following we shall always work in the zero-lepton-mass limit. Thus we have dropped invariants multiplying q_μ. [#1]

For our purpose it is convenient to regard the s.l. decay $\Lambda_1 \to \Lambda_2 + \ell + \nu_\ell$ as a quasi two-body decay $\Lambda_1 \to \Lambda_2 + W_{\text{off-shell}}$ followed by the leptonic decay $W_{\text{off-shell}} \to \ell + \nu_\ell$. To save on notation we shall drop the "off-shell" label in the following. In the zero-lepton-mass approximation only the $J^P = 1^-, 1^+$ component of the W participate in the decay (1^- vector current; 1^+ axial vector current). Accordingly we define helicity amplitudes $H^{V;A}_{\lambda_2\lambda_W}$ where λ_2 and λ_W are the helicities of the daughter baryon and the W-boson ($\lambda_W = 0$ longitudinal; $\lambda_W = \pm 1$ transverse). They are related to the invariant form factors through

$$\sqrt{q^2}H^V_{1/2\,0} = \sqrt{Q_-}\,[(M_1+M_2)F_1^V - q^2F_2^V]\,, \quad H^V_{1/2\,1} = \sqrt{2Q_-}\,[-F_1^V + (M_1+M_2)F_2^V]\,,$$
$$\sqrt{q^2}H^A_{1/2\,0} = \sqrt{Q_+}\,[(M_1-M_2)F_1^A + q^2F_2^A]\,, \quad H^A_{1/2\,1} = \sqrt{2Q_+}\,[-F_1^A - (M_1-M_2)F_2^A]\,, \qquad (2)$$

where we use the abbreviation $Q_\pm = (M_1 \pm M_2)^2 - q^2$. The remaining helicity amplitudes can be obtained with the help of the parity relations

$$H^{V(A)}_{-\lambda_2-\lambda_W} = +(-)H^{V(A)}_{\lambda_2\lambda_W}\,. \qquad (3)$$

One notes from eqs. (2) and (3) that the helicity amplitudes possess a simple structure at the kinematical boundaries $q^2_{\min} = 0$ and $q^2_{\max} = (M_1 - M_2)^2$. In the limit $q^2 \to 0$ the longitudinal helicity amplitudes are dominant. At the zero recoil limit $q^2_{\max} = (M_1 - M_2)^2$ the longitudinal and transverse axial vector helicity amplitudes become dominant. They become related by

$$H^A_{1/2\,1} = -\sqrt{2}\,H^A_{1/2\,0}\,, \quad q^2 \to q^2_{\max}\,. \qquad (4)$$

The behaviour of the helicity amplitudes at $q^2_{\max}$ can easily be understood from a partial wave analysis of the final state in the decay $\Lambda_1(\frac{1}{2}^+) \to \Lambda_2(\frac{1}{2}^+) + W(1^-, 1^+)$. As the phase space closes only the axial vector s-wave contribution survives.

Polarization effects in the decays arise from the fact that the population of helicities in the final state and thereby the corresponding density matrices are in general non-trivial. Let us first analyze the s.l. decay of an unpolarized parent Λ_1. The relevant (unnormalized) correlation density matrix is given by

$$\rho_{\lambda_2\lambda_W;\lambda_2'\lambda_W'} = H_{\lambda_2\lambda_W}H^*_{\lambda_2'\lambda_W'}\,. \qquad (5)$$

The correlation density matrix (5) may be probed by analyzing the angular decay distribution of the decay products of the W and the daughter baryon Λ_2. What is needed is a knowledge of the decay structure of the respective cascade decays. The decay $W \to \ell\nu_\ell$ is specified by the usual (V–A) charged current interaction and provides 100% analyzing power. In case of the $\Lambda_c \to \Lambda_s$ transitions the standard nonleptonic (n.l.) hyperon decays $\Lambda_s \to p\pi^-$, $n\pi^0$ are well suited for this purpose since their decay structure is experimentally well measured.

[#1] Muon mass effects have been investigated in mesonic s.l. $D \to K(K^*)$ decays and have been found to be $\leqslant 5\%$ in the total rate [20]. The biggest effect occurs for the partial rate into the longitudinal current component, where the effect of the muon mass amounts to O(10%), and is largest at small q^2. The muon mass effects in charm baryon decays are of similar small size [21]. This is different in s.l. $b \to c$ meson and baryon decays where lepton mass effects can be conveniently probed in the τ-channel [20–23].

In addition these decays provide good analyzing power in as far as the respective asymmetry parameters α_Λ are large ($\alpha_{\Lambda_s\to p\pi^-}=+0.64; \alpha_{\Lambda_s\to n\pi^0}=+0.66$ [24]). First measurements (and some theoretical prejudice [25–27]) exist on the corresponding asymmetry parameter in the n.l. decay $\Lambda_c^+\to\Lambda_s+\pi^+$ ($\alpha_{\Lambda_c}=-1.0^{+0.4}_{-0.0}$ [28]; $\alpha_{\Lambda_c}=-0.96\pm0.42$ [29]). In principle one could also use other decays of the Λ_c^+ as polarization analyzers in s.l. $\Lambda_b\to\Lambda_c$ transitions (as e.g. $\Lambda_c^+\to\Lambda_s+\rho^+$) once their decay structures become known. For the present purpose, however, we limit our discussion to the n.l. two-body decays $\Lambda_2(\frac{1}{2}^+)\to a(\frac{1}{2}^+)+b(0^-)$ as analyzers.

Using standard methods (see e.g. ref. [30]) one then obtains the normalized four-fold joint angular decay distribution for the two-sided cascade decay $\Lambda_1(\frac{1}{2}^+)\to\Lambda_2(\frac{1}{2}^+)[\to a(\frac{1}{2}^+)+b(0^-)]+W(\to\ell\nu_\ell)$:

$$\frac{d\Gamma}{dq^2\, d\cos\Theta\, d\chi\, d\cos\Theta_\Lambda}=B(\Lambda_2\to a+b)\tfrac{1}{2}\frac{G^2}{(2\pi)^4}|V_{Q_1Q_2}|^2\frac{q^2p}{24M_1^2}$$

$$\times\Big(\tfrac{3}{8}(1\pm\cos\Theta)^2|H_{1/2\,1}|^2(1+\alpha_\Lambda\cos\Theta_\Lambda)+\tfrac{3}{8}(1\mp\cos\Theta)^2|H_{-1/2\,-1}|^2(1-\alpha_\Lambda\cos\Theta_\Lambda)$$

$$+\tfrac{3}{4}\sin^2\Theta[|H_{1/2\,0}|^2(1+\alpha_\Lambda\cos\Theta_\Lambda)+|H_{-1/2\,0}|^2(1-\alpha_\Lambda\cos H_\Lambda)]$$

$$\mp\frac{3}{2\sqrt{2}}\alpha_\Lambda\cos\chi\sin\Theta\sin\Theta_\Lambda[(1\pm\cos\Theta)\mathrm{Re}(H_{-1/2\,0}H^*_{1/2\,1})+(1\mp\cos\Theta)\mathrm{Re}(H_{1/2\,0}H^*_{-1/2\,-1})]\Big). \qquad (6)$$

The polar angles Θ and Θ_Λ and the azimuthal angle χ are defined in fig. 1. In order to be more explicit fig. 1 was drawn for the specific cascade process $\Lambda_c^+\to\Lambda_s(\to p\pi^-)+W^+(\to\ell^+\nu_\ell)$. $B(\Lambda_2\to a+b)$ and α_Λ are the branching ratio and asymmetry parameter, respectively, of the daughter's Λ_2 decay into the channel a+b. G is the Fermi coupling constant and $V_{Q_1Q_2}$ is the Kobayashi–Maskawa matrix element for the weak transition $Q_1\to Q_2$. The upper and lower signs in eq. (6) hold for the $\ell^-\bar\nu_\ell$ and $\ell^+\nu_\ell$ leptonic final states, respectively. The CM momentum p is given by $p=\sqrt{Q_+Q_-}/2M_1$.

In writing down eq. (6) we have assumed that the invariant amplitudes (and thereby the helicity amplitudes) are relatively real. This is well justified in the decay region where one is below the particle production threshold $q^2=(M_1+M_2)^2$. Thus we have omitted so-called T-odd contributions proportional to $\sin\Theta\sin\chi\sin\Theta_\Lambda$ and $\sin(2\Theta)\sin\chi\sin\Theta_\Lambda$. We note in passing that the presence of such contributions could signal possible CP-violations in the s.l. decay process [31].

The structure of the decay distribution eq. (6) is quite similar to the corresponding four-fold decay distribution for the cascade decay $D\to K^*(\to K\pi)+\ell^++\nu_\ell$ [20,32–34] which has been proven so useful in disentangling the form factor structure in the s.l. $D\to K^*$ decays [35] (using an event sample of ~200 events). A similar analysis for s.l. $B\to D^*$ transitions based on an event sample of ~360 events is in progress [36].

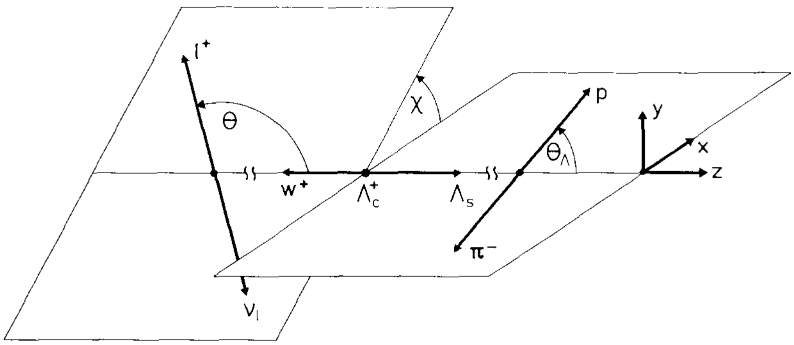

Fig. 1. Definition of polar angles Θ_Λ and Θ and azimuthal angle χ in the double "cascade" decay $\Lambda_c^+\to\Lambda_s(\to p\pi^-)+W^+(\to\ell^+\nu_\ell)$.

The angular distribution eq. (6) defines a set of eight observables which are bilinears in the four independent q^2-dependent real form factors. A measurement of these eight observables would considerably overdetermine the form factors. Note though that the complexity of the problem is reduced close to phase space boundaries as discussed earlier. At the phase space boundaries the large parentheses in eq. (6) simplify to

$$(\quad)_{q^2=0}=\tfrac{3}{4}\sin^2\Theta[\,|H_{1/2\,0}|^2(1+\alpha_\Lambda\cos\Theta_\Lambda)+|H_{-1/2\,0}|^2(1-\alpha_\Lambda\cos\Theta_\Lambda)\,] \tag{7}$$

and

$$(\quad)_{q^2_{\max}}=3(1\pm\alpha_\Lambda\cos\Theta\cos\Theta_\Lambda\mp\alpha_\Lambda\sin\Theta\sin\Theta_\Lambda\cos\chi)\,|H^{\Lambda}_{1/2\,0}|^2\,. \tag{8}$$

The relevant dynamical information contained in the three-fold angular decay distribution eq. (6) may be extracted by either one of the following methods: (i) moment analysis, (ii) analysis of suitable defined asymmetry ratios as in ref. [20], or (iii) angular fits to the data as in ref. [35] depending on the quantity and quality of the data.

Instead of analyzing the full three-fold angular dependency one can also look at two-fold and single angle decay distributions. For example, single angle decay distributions may be obtained from eq. (6) by doing two angular integrations. For the polar angle distributions of the cascade decay $\Lambda_2\to a+b$ one has

$$\frac{d\Gamma}{dq^2\,d\cos\Theta_\Lambda}\propto 1+\alpha\alpha_\Lambda\cos\Theta_\Lambda\,, \tag{9}$$

where the asymmetry parameter α is defined by

$$\alpha=\frac{|H_{1/2\,1}|^2-|H_{-1/2\,-1}|^2+|H_{1/2\,0}|^2-|H_{-1/2\,0}|^2}{|H_{1/2\,1}|^2+|H_{-1/2\,-1}|^2+|H_{1/2\,0}|^2+|H_{-1/2\,0}|^2}\,. \tag{10}$$

For the polar angle distribution in the cascade decay $W\to\ell+\nu_\ell$ one finds

$$\frac{d\Gamma}{dq^2\,d\cos\Theta}\propto 1\pm 2\alpha'\cos\Theta+\alpha''\cos^2\Theta\,, \tag{11}$$

where

$$\alpha'=\frac{|H_{1/2\,1}|^2-|H_{-1/2\,-1}|^2}{|H_{1/2\,1}|^2+|H_{-1/2\,-1}|^2+2(|H_{-1/2\,0}|^2+|H_{1/2\,0}|^2)} \tag{12}$$

and

$$\alpha''=\frac{|H_{1/2\,1}|^2+|H_{-1/2\,-1}|^2-2(|H_{-1/2\,0}|^2+|H_{1/2\,0}|^2)}{|H_{1/2\,1}|^2+|H_{-1/2\,-1}|^2+2(|H_{-1/2\,0}|^2+|H_{1/2\,0}|^2)}\,. \tag{13}$$

The upper and lower signs are for $(\ell^-\bar\nu_\ell)$ and $(\ell^+\nu_\ell)$, respectively. The azimuthal angle distribution, finally, is given by

$$\frac{d\Gamma}{dq^2d\chi}\propto 1\mp\frac{3\pi^2}{32\sqrt{2}}\gamma\alpha_\Lambda\cos\chi\,, \tag{14}$$

where the azimuthal asymmetry parameter γ is defined by

$$\gamma=\frac{2\,\mathrm{Re}(H_{-1/2\,0}H^*_{1/2\,1}+H_{1/2\,0}H^*_{-1/2\,-1})}{|H_{1/2\,1}|^2+|H_{-1/2\,-1}|^2+|H_{-1/2\,0}|^2+|H_{1/2\,0}|^2}\,. \tag{15}$$

All asymmetry parameters have been defined such that they range between -1 and $+1$.

Due to the fact that only the axial vector s-wave contribution survives in the zero recoil limit the asymmetry parameters possess well-defined limits as $q^2 \to q^2_{max}$. The polar asymmetries all vanish in this limit, and the azimuthal asymmetry γ tends to $\gamma \to \frac{2}{3}\sqrt{2}$ (see table 1). These limiting values can either be directly read from the distribution (8) or computed using the definitions (10), (12), (13) and (15). At $q^2=0$, only α', α'' and γ have a definite limiting behaviour: α' and γ tend to zero, and α'' tends to -1. The limiting behaviour of the baryon side polar asymmetry parameter α at $q^2=0$, however, involves the dynamics of the s.l. transition $\Lambda_{Q_1} \to \Lambda_{Q_2}$ and therefore is a model dependent quantity. We shall, however, see in the following that one can reliably predict $\alpha(q^2=0)=-1$ in the context of the HQET. This result of the HQET has been included in brackets in table 1.

Turning to dynamics let us briefly recapitulate the predictions of the HQET concerning current-induced transitions involving Λ-type heavy baryons. A heavy Λ-type baryon is rather simple since it is made out of a scalar light diquark system and a heavy spin $\frac{1}{2}$ quark. Thus one can identify the heavy quark spinor with the baryon's spinor when writing down transition matrix elements. For a heavy to heavy transition $\Lambda_1 \to \Lambda_2$ there is no spin interaction on the heavy quark legs in the heavy mass limit except for the current interaction itself. From Lorentz-invariance one then finds

$$\langle \Lambda_2 | J_\mu^{V+A} | \Lambda_1 \rangle = \bar{u}_2 [F(\omega) \gamma_\mu (1-\gamma_5)] u_1 , \tag{16}$$

where the single form factor $F(\omega)$ depends on the velocity transfer variable $\omega = v_1 \cdot v_2$ and is normalized to 1 at q^2_{max} or equivalently, at $\omega=1$ [9–14] [#2]. In eq. (16) we have used the conventional Bjorken–Drell state normalization with spinor normalization $\bar{u}u=2M$. In terms of the standard form factors (1) one has

$$F_1^V(q^2) = -F_1^A(q^2), \quad F_2^V(q^2) = F_2^A(q^2) = 0 , \tag{17}$$

[#2] Renormalization effects on the current vertex can easily be included via leading log resummation techniques as discussed in ref. [6] (see also ref. [37]). These renormalization effects need not concern us here since they cancel out in the polarization asymmetries that we are studying in this paper.

Table 1
Values of asymmetry parameters as defined in the main text. Column 1: allowed range; column 2: value at phase space boundary $q^2=0$. For α, α_P and γ_P the HQET result appears in brackets; column 3: value at phase space boundary q^2_{max}; Column 4: mean value of asymmetry parameters for $\Lambda_c \to \Lambda_s$ transitions for five values of $f_2/f_1 = 0.5, 0.25, 0, -0.25, -0.5$ (top to bottom); Column 5: the same for $\Lambda_b \to \Lambda_c$ transitions.

	Range	Limiting values		Mean values			Range	Limiting values		Mean values	
		$q^2=0$	q^2_{max}	$\langle\alpha,\gamma\rangle$ $\Lambda_c \to \Lambda_s$	$\langle\alpha,\gamma\rangle$ $\Lambda_b \to \Lambda_c$			$q^2=0$	q^2_{max}	$\langle\alpha,\gamma\rangle$ $\Lambda_c \to \Lambda_s$	$\langle\alpha,\gamma\rangle$ $\Lambda_b \to \Lambda_c$
α	[−1, 1]	–	0	−0.53	−0.55	γ	[−1, 1]	0	$+2\sqrt{2}/3$	0.77	0.75
		(−1)		−0.60	−0.64					0.71	0.68
				−0.70	−0.75					0.62	0.57
				−0.82	−0.87					0.47	0.39
				−0.94	−0.97					0.21	0.09
α'	[−1, 1]	0	0	0.04	0.05	α_P	[−1, 1]	–	0	0.41	0.39
				0.07	0.08			(+1)		0.40	0.38
				0.10	0.12					0.39	0.38
				0.13	0.15					0.39	0.37
				0.17	0.18					0.39	0.38
α''	[−1, 1]	−1	0	−0.41	−0.42	γ_P	[−1, 1]	–	−1/3	−0.22	−0.22
				−0.45	−0.46			(0)		−0.21	−0.21
				−0.50	−0.51					−0.18	−0.17
				−0.56	−0.57					−0.14	−0.12
				−0.62	−0.62					−0.07	−0.03

with the normalization condition $F_1^V(q^2_{max})=1$.

For a heavy to light $\Lambda_1 \to \Lambda_2$ transition there is more structure due to the fact that the light active quark q_2 can undergo spin interactions. The most general form factor structure allowing for a light side spin interaction is now [12,13]

$$\langle \Lambda_2 | J_\mu^{V+A} | \Lambda_1 \rangle = \bar{u}_2 [f_1(q^2)\gamma_\mu(1-\gamma_5) + f_2(q^2) \not{v}_1 \gamma_\mu (1-\gamma_5)] u_1 \quad (18)$$

with no normalization condition for the form factors $f_i(q^2)$ at q^2_{max}. In terms of the standard form factors (1) one finds

$$F_1^V(q^2) = -F_1^A(q^2) = f_1(q^2) + \frac{M_2}{M_1} f_2(q^2) , \quad F_2^V(q^2) = -F_2^A(q^2) = \frac{1}{M_1} f_2(q^2) . \quad (19)$$

Note that the heavy to heavy structure equation (16) is recovered by setting $f_2(q^2)=0$.

The form factors $f_i(q^2)$ can be further specified at $O(1/m_2)$ by studying the $1/m_2$ expansion on the daughter baryon's side [16]. One finds

$$f_1(\omega) = \left(1 + \frac{1}{2} \frac{\bar{\Lambda}}{m_2} \frac{1}{1+\omega}\right) G(\omega) , \quad f_2(\omega) = -\frac{1}{2} \frac{\bar{\Lambda}}{m_2} \frac{1}{1+\omega} G(\omega) , \quad (20)$$

where, quite remarkably, a normalization condition on the form factors is retained at $O(1/m_2)$ in that $G(\omega=1)=1$ [16,17,38–40]. The expansion parameter $\bar{\Lambda}$ in eq. (20) is of the order of the mass difference of the baryon and the active quark, i.e. $\bar{\Lambda} \simeq M_2 - m_2 \simeq 700$ MeV.

Returning to the asymmetry parameter α one notes that the HQET predicts $F_1^V(q^2) = -F_1^A(q^2)$ as long as the decaying Λ-type baryon is treated as heavy. Turning to the helicity amplitudes eq. (2) this means that the daughter baryon will emerge 100% negatively polarized from the decay at $q^2=0$. Equivalently, the asymmetry parameter α has the value $\alpha=-1$ at $q^2=0$. This prediction of the HQET should certainly be very reliable for the $\Lambda_b \to \Lambda_c$ transition and quite reliable for the $\Lambda_c \to \Lambda_s$ transition as it only depends on the assumption that the decaying parent Λ-type baryon is heavy [#3]. In the latter decay, the prediction $\alpha_\Lambda = -1$ would not even be spoiled by treating the decaying Λ_c at $O(1/m_c)$.

Away from the phase space boundaries the q^2-dependence of the various asymmetry parameters depend on the details of the form factor structure. However, it is quite reasonable to assume a common q^2-dependence of the two form factors $f_1(q^2)$ and $f_2(q^2)$ in eq. (19). In this case the q^2-dependence of the asymmetries depend only on the constant ratio $f_2(q^2)/f_1(q^2)$ where the ratio $r=f_2/f_1$ is expected to be smaller than 1 even for the transition $\Lambda_c \to \Lambda_s$. Let us again concentrate on the q^2-dependence of the asymmetry parameter α in $\Lambda_c \to \Lambda_s$ transitions as it will certainly be the first to be measured experimentally.

Fig. 2a shows the q^2-dependence of the asymmetry parameter α for the five form factor ratios $f_2/f_1 = 0.5, 0.25, 0, -0.25$ and -0.5. The $O(1/m_s)$ prediction eq. (20) corresponds to the form factor ratio $r \simeq -0.25$ using $m_s = 450$ MeV and $\bar{\Lambda} = 700$ MeV. All five curves slowly rise from their $q^2=0$ value of (-1) and then quickly increase to their limiting value $\alpha=0$ at the end of the physical q^2-range. In particular one has a monotonic increase of all five curves from -1 at $q^2=0$ to 0 at q^2_{max}. On the one hand this means that such a measurement is not very sensitive to the form factor ratio f_2/f_1. On the other hand this implies that one should be able to extrapolate reliably the experimental data from $q^2>0$ into $q^2=0$ where the model independent HQET prediction exists.

In fig. 2b we show mean values of the asymmetry parameter α for the above five cases. The averaging is done in the range $q^2=0$ to a given nominal maximal q^2-value $q^2(\max)$. As the averaging implies separate integration of the numerator and denominator in eq. (10) this introduces a (mild) dependence on the assumed q^2-behav-

[#3] Equivalent statements can be made about the polarization of the daughter baryon for the Cabibbo suppressed s.l. transitions $\Lambda_b \to p$ and $\Lambda_c \to n$. However, besides being rate suppressed, these transitions cannot be used to test the polarization prediction of HQET as the polarization of the daughter baryon cannot be measured,

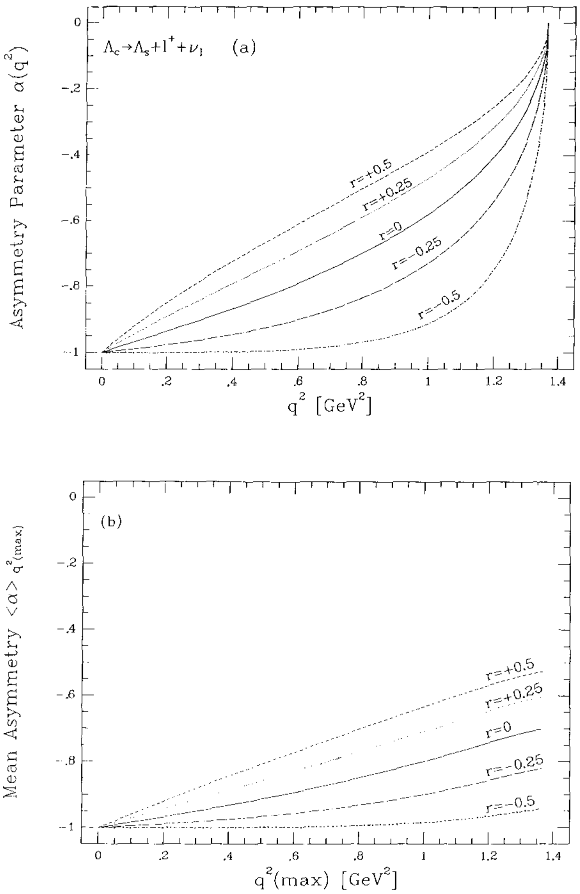

Fig. 2. Asymmetry parameter α in s.l. $\Lambda_c^+ \to \Lambda_s + \ell^+ + \nu_\ell$ transitions for five different form factor ratios $r = f_2/f_1$ (see main text): (a) asymmetry parameter α as function of q^2; (b) mean value of the asymmetry parameter as function of upper cut off value $q^2(\max)$. Averaging is done between $q^2=0$ and $q^2(\max)$.

iour of the form factors $f_1(q^2)$, $f_2(q^2)$ and $G(q^2)$. In order to be definite we have chosen a dipole form for their q^2-dependence. Thus we take

$$F(q^2) = \frac{F(q^2_{\max})}{(1 - q^2/m^2_{\mathrm{FF}})^2} \left(1 - \frac{q^2_{\max}}{m^2_{\mathrm{FF}}}\right)^2 \tag{21}$$

for $f_1(q^2)$, $f_2(q^2)$ and $G(q^2)$ where we use the mass of the D_s^* as form factor mass, i.e. $m_{\mathrm{FF}} = 2.11$ GeV. Note that the asymmetries do not depend on the normalization of the form factors $F(q^2_{\max})$ in eq. (21). Even though the differential decay rate is predicted to be weighted towards the larger q^2-values [9,14] the mean asymmetries $\langle \alpha \rangle_{q^2(\max)}$ do not significantly deviate from the $q^2=0$ value over most of the $q^2(\max)$ range. We do not show the corresponding curves for the $\Lambda_b \to \Lambda_c$ transitions as the results are very similar to the $\Lambda_c \to \Lambda_s$ case.

In table 1 we also list the mean values $\langle \alpha, \gamma \rangle$ of all the four asymmetry parameters α, α', α'' and γ where the averaging is over the full q^2-range. The sensitivity to the input form factor ratio $r = f_2/f_1$ can be judged by inspecting the variations of the means in table 1. The mean asymmetries $\langle \alpha \rangle$ and $\langle \alpha'' \rangle$ can be seen to have the least model dependence.

Table 1 also contains the corresponding mean asymmetries for $\Lambda_b \to \Lambda_c$ transitions. We have used $M_{\Lambda_b} = 5.6$ GeV for the mass of the Λ_b. The form factor mass in the dipole form factors is taken as $M_{B_c^*} = 6.34$ GeV. The mean asymmetries for $\Lambda_b \to \Lambda_c$ do not differ much from the corresponding $\Lambda_c \to \Lambda_s$ values. Note that the O(1/

m_c) prediction eq. (20) for $\Lambda_b \to \Lambda_c$ decays corresponds to the form factor ratio $r \simeq -0.10$ using $m_c = 1.45$ GeV and $\bar{\Lambda} = 700$ MeV. For this preferred value one has $\langle \alpha \rangle = -0.80$, $\langle \alpha' \rangle = 0.13$, $\langle \alpha'' \rangle = -0.54$ and $\langle \gamma \rangle = 0.51$.

Although we concentrate on predictions of the asymmetry parameters in this paper let us also list nominal rate values. For example, for $\Lambda_c \to \Lambda_s$ we obtain $\Gamma = 15.7[10^{10}\ s^{-1}]$ and $\Gamma = 19.0[10^{10}\ s^{-1}]$ for $r=0$ and our preferred value $r = -0.25$, respectively. These rates are higher than the inclusive rate $\Lambda_c^+ \to \Lambda_s X \ell^+ \nu_\ell$ measured in ref. [1] ($\Gamma = (8.0 \pm 2.6 \pm 2.6) \times 10^{10}\ s^{-1}$ for the e-mode and $\Gamma = (7.5 \pm 4.1 \pm 2.6) \times 10^{10}\ s^{-1}$ for the μ-mode using a total width $\Gamma = (0.5 \pm 0.045) \times 10^{13}\ s^{-1}$). As discussed in ref. [15] this is a common feature of all present theoretical models which all tend to overestimate the rate for exclusive s.l. $\Lambda_c \to \Lambda_s$ decays. At present this remains an unsolved puzzle which awaits resolution. Although our rate predictions tend to be too high we believe that our asymmetry predictions are still reliable. For the $\Lambda_b \to \Lambda_c$ transition we find $\Gamma = 5.34\ [10^{10}\ s^{-1}]$ using the heavy to heavy result $f_2(q^2) = 0$ and $\Gamma = 5.72\ [10^{10}\ s^{-1}]$ for our preferred value $r = -0.10$. The value for the Kobayashi–Maskawa matrix element is $V_{bc} = 0.045$.

An additional set of polarisation observables can be defined for the decay of a polarized charm baryon. For example, hadronically produced Λ's have been observed to be polarized where the polarization necessarily has to be transverse to the production plane because of parity invariance in the production process. It may well be that hadronically produced Λ_c^+'s show a similar polarization effect. Also, charm baryons from weak decays of bottom baryons are expected to be polarized. Finally, charm and bottom quarks from Z-decays are polarized. It is quite likely that there will be a polarization transfer when the charm and bottom quarks from Z-decays fragment into charm and bottom Λ-baryons.

For the density matrix of the daughter baryon one now has

$$\rho_{1/2\,1/2} = |H_{1/2\,1}|^2 (1 - P\cos\Theta_P) + |H_{1/2\,0}|^2 (1 + P\cos\Theta_P)\,,$$

$$\rho_{1/2\,-1/2} = \rho_{-1/2\,1/2} = -P\sin\Theta_P\,\mathrm{Re}(H_{1/2\,0} H^*_{-1/2\,0})\,,$$

$$\rho_{-1/2\,-1/2} = |H_{-1/2\,-1}|^2 (1 + P\cos\Theta_P) + |H_{-1/2\,0}|^2 (1 - P\cos\Theta_P)\,, \qquad (22)$$

where P denotes the degree of polarization of the parent baryon Λ_1, and Θ_P is the polar angle between the polarization direction of Λ_1 and the momentum direction of Λ_2 as shown in fig. 3 for the specific case $\Lambda_c^+ \to \Lambda_s(\to p\pi^-) + W^+(\to \ell^+ \nu_\ell)$. Again we have assumed that the helicity amplitudes are relatively real. We thus neglect possible T-odd effects in the decay distribution.

As in the unpolarized case the cascade decay $\Lambda_2 \to a(\frac{1}{2}^+) + b(0^-)$ is used to analyze the daughter baryon's polarization. One then has the four-fold decay distribution

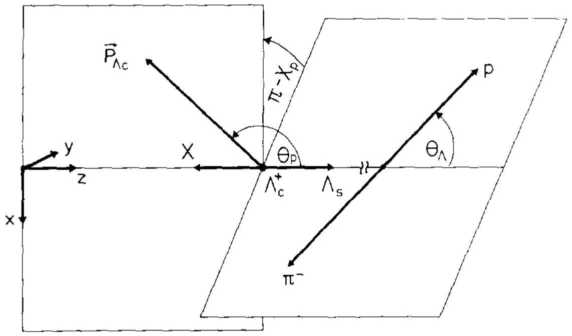

Fig. 3. Definition of polar angles Θ_Λ and Θ_P and azimuthal angle χ_P in the decay of a polarized $\Lambda_c^! \to \Lambda_s(\to p\pi^-) - +X$. The left plane is determined by the polarization vector $\boldsymbol{P}_{\Lambda_c}$ of the Λ_c.

$$\frac{d\Gamma(\Lambda_1^\uparrow \to \Lambda_2(\to \tfrac{1}{2}^+ + 0^-) + \ell + \nu_\ell)}{dq^2\, d\cos\Theta_P\, d\chi_P\, d\cos\Theta_\Lambda} = B(\Lambda_2 \to a+b)\tfrac{1}{2}\frac{G^2}{(2\pi)^4}|V_{Q_1Q_2}|^2 \frac{q^2 p}{48M_1^2}$$
$$\times \{|H_{-1/2\,0}|^2 + |H_{1/2\,0}|^2 + |H_{1/2\,1}|^2 + |H_{-1/2\,-1}|^2$$
$$+\alpha_\Lambda \cos\Theta_\Lambda(|H_{1/2\,0}|^2 - |H_{-1/2\,0}|^2 + |H_{1/2\,1}|^2 - |H_{-1/2\,-1}|^2)$$
$$+P\cos\Theta_P(|H_{1/2\,0}|^2 - |H_{-1/2\,0}|^2 - |H_{1/2\,1}|^2 + |H_{-1/2\,-1}|^2)$$
$$+P\alpha_\Lambda \cos\theta_\Lambda \cos\Theta_P(|H_{1/2\,0}|^2 + |H_{-1/2\,0}|^2 - |H_{1/2\,1}|^2 - |H_{-1/2\,-1}|^2)$$
$$-P\alpha_\Lambda \sin\Theta_\Lambda \sin\Theta_P \cos\chi_P\, 2\,\mathrm{Re}(H_{1/2\,0}H^*_{-1/2\,0})\}\,, \tag{23}$$

where the orientation angles Θ_P, Θ_Λ and χ_P are defined in fig. 3. As the lepton side angular distribution goes unanalyzed the distribution (23) holds for both the $\Lambda_b \to \Lambda_c$ and $\Lambda_c \to \Lambda_s$ transitions without any sign change. If the lepton-side information contained in the decay $W \to \ell + \nu_\ell$ were kept one would have a six-fold differential distribution.

The decay distribution (23) simplifies at the phase space boundaries. For the curly brackets in eq. (23) one finds

$$\{\ \}_{q^2=0} = (|H_{1/2\,0}|^2 + |H_{-1/2\,0}|^2)(1 + P\alpha_\Lambda \cos\Theta_P \cos\Theta_\Lambda)$$
$$+(|H_{1/2\,0}|^2 - |H_{-1/2\,0}|^2)(\alpha_\Lambda \cos\Theta_\Lambda + P\cos\Theta_P)$$
$$+2\,\mathrm{Re}(H_{1/2\,0}H^*_{-1/2\,0})(-P\alpha_\Lambda \sin\Theta_P \sin\Theta_\Lambda \cos\chi_P)\,, \tag{24}$$

which further simplifies to

$$\{\ \}_{q^2=0} = |H_{-1/2\,0}|^2(1 + P\alpha_\Lambda \cos\Theta_P \cos\Theta_\Lambda - \alpha_\Lambda \cos\Theta_\Lambda - P\cos\Theta_P) \tag{25}$$

when the HQET prediction $H_{1/2\,0}(q^2=0)=0$ is used. At q^2_{max} one has

$$\{\ \}_{q^2_{max}} = 2|H^A_{1/2\,0}|^2(3 - P\alpha_\Lambda \cos\Theta_\Lambda \cos\Theta_P + P\alpha_\Lambda \sin\Theta_P \sin\Theta_\Lambda \cos\chi_P)\,. \tag{26}$$

Integrating eq. (23) w.r.t. Θ_P and χ_P one recovers the $\cos\Theta_\Lambda$ distribution eq. (9). An integration over Θ_Λ and χ_P yields

$$\frac{d\Gamma(\Lambda_1^\uparrow)}{dq^2 d\cos\Theta_P} \propto 1 - \alpha_P P\cos\Theta_P\,, \tag{27}$$

where

$$\alpha_P = \frac{|H_{1/2\,1}|^2 - |H_{-1/2\,-1}|^2 - |H_{1/2\,0}|^2 + |H_{-1/2\,0}|^2}{|H_{1/2\,1}|^2 + |H_{-1/2\,-1}|^2 + |H_{-1/2\,0}|^2 + |H_{1/2\,0}|^2}\,. \tag{28}$$

Finally integrating eq. (23) w.r.t. Θ_Λ and Θ_P yields the distribution

$$\frac{d\Gamma(\Lambda_1^\uparrow)}{dq^2 d\chi} \propto 1 - \tfrac{1}{16}\pi^2 P\alpha_\Lambda \gamma_P \cos\chi\,, \tag{29}$$

where

$$\gamma_P = \frac{2\,\mathrm{Re}(H_{1/2\,0}H^*_{-1/2\,0})}{|H_{1/2\,1}|^2 + |H_{-1/2\,-1}|^2 + |H_{-1/2\,0}|^2 + |H_{1/2\,0}|^2}\,. \tag{30}$$

The asymmetries α_P and γ_P have been defined such that they range between +1 and −1. Their values at the phase space boundaries are listed in table 1. We have added the $q^2=0$ HQET prediction in brackets. Their mean values are listed in column 5 ($\Lambda_c \to \Lambda_s$) and column 6 ($\Lambda_b \to \Lambda_c$) again for the five form factor ratios $r=0.5$, 0.25, 0, -0.25 and -0.5. The mean value of the polar asymmetry parameter $\langle \alpha_P \rangle$ is quite stable against variations of the input form factor ratio and does not change much when going from the $\Lambda_c \to \Lambda_s$ case to the $\Lambda_b \to \Lambda_c$ case. The mean value of the azimuthal asymmetry parameter $\langle \gamma_P \rangle$, however, is quite sensitive to the input form factor ratio r. Again there is not much change going from $\Lambda_c \to \Lambda_s$ to $\Lambda_b \to \Lambda_c$. For the $\Lambda_b \to \Lambda_c$ transitions our preferred mean values are $\langle \alpha_P \rangle = 0.37$ and $\langle \gamma_P \rangle = -0.15$ using $r = -0.10$.

In conclusion we have provided the tools that are needed to analyze polarization effects in exclusive s.l. Λ-type baryon decays. We have written down joint angular decay distributions in terms of the transition form factors that describe the decay. The form factor structure is severely constrained by HQET. The suggested asymmetry measurements are an ideal instrument to test the predictions of the HQET in the baryon sector.

Decay formulae are given for the s.l. cascade decays $\frac{1}{2}^+ \to \frac{1}{2}^+ (\to \frac{1}{2}^+ + 0^-) + W(\to \ell + \nu_\ell)$ involving polarized and unpolarized decaying baryons. Corresponding distributions for $\frac{1}{2}^+ \to \frac{3}{2}^+$ transitions relevant for e.g. the s.l. decay $\Omega_c(\frac{1}{2}^+) \to \Omega^-(\frac{3}{2}^+) + \ell^+ + \nu_\ell$ can be found in ref. [41]. Ref. [41] also contains a discussion of other baryon side n.l. cascade modes as e.g. $\frac{1}{2}^+ \to \frac{1}{2}^+ + 1^-$.

The decay distributions of antibaryon decays are obtained from the baryon decay formulae discussed in this paper by noting that the transverse and longitudinally helicity amplitudes of the baryon and antibaryon transitions are related through CP-invariance. One has $H_{\lambda_2 \lambda_W}(\bar{Q}_1 \to \bar{Q}_2) = H_{-\lambda_2 - \lambda_W}(Q_1 \to Q_2)$ where again the helicity amplitudes have been taken to be real.

References

[1] ARGUS Collab., H. Albrecht et al., DESY preprint DESY 91-055 (1991).
[2] ALEPH Collab., contributed paper, Lepton Photon Symp. (Geneva, 1991).
[3] E. Eichten, Nucl. Phys. B (Proc. Suppl.) 4 (1988) 170;
G.P. Lepage and B.A. Thacker, Nucl. Phys. B (Proc. Suppl.) 4 (1988) 199;
E. Eichten and B. Hill, Phys. Lett. B 234 (1990) 511;
B. Grinstein, Nucl. Phys. B 339 (1990) 253.
[4] N. Isgur and M.B. Wise, Phys. Lett. B 237 (1990) 527.
[5] F. Hussain, J.G. Körner, K. Schilcher, G. Thompson and Y.L. Wu, Phys. Lett. B 249 (1990) 295.
[6] A.F. Falk, H. Georgi, B. Grinstein and M.B. Wise, Nucl. Phys. B 343 (1990) 1.
[7] H. Georgi, Phys. Lett. B 240 (1990) 447.
[8] J.D. Bjorken, Invited talk at Les Recontre de Physique de la Vallee de Aosta (La Thuile, Italy), to be published.
[9] F. Hussain, J.G. Körner and R. Migneron, Phys. Lett. B 248 (1990) 406.
[10] N. Isgur and M.B. Wise, Nucl. Phys. B 348 (1991) 276.
[11] H. Georgi, Nucl. Phys. B 348 (1991) 293.
[12] F. Hussain, J.G. Körner, M. Krämer and G. Thompson, Z. Phys. C 51 (1991) 321.
[13] T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. B 355 (1991) 38.
[14] F. Hussain and J.G. Körner, Z. Phys. C 51 (1991) 607.
[15] J.G. Körner and H.W. Siebert, Ann. Rev. Nucl. Part. Sci. 41 (1991) 511.
[16] H. Georgi, B. Grinstein and M.B. Wise, Phys. Lett. B 252 (1990) 456.
[17] C.G. Boyd and D.E. Brahm, Phys. Lett. B 254 (1991) 468.
[18] F. Hussain, J.G. Körner, M. Krämer, D.S. Liu and S. Tawfiq, Trieste preprint IC/91/133 (1991).
[19] J.D. Bjorken, Phys. Rev. D 40 (1989) 1513.
[20] J.G. Körner and G.A. Schuler, Z. Phys. C 46 (1990) 93.
[21] M. Krämer, diploma thesis (Mainz 1991).
[22] J.G. Körner and G. Schuler, Phys. Lett. B 231 (1989) 306;
C.A. Dominguez, J.G. Körner and K. Schilcher, Phys. Lett. B 248 (1990) 399.
[23] K. Hagiwara, A.D. Martin and M.F. Wade, Z. Phys. C 46 (1990) 299.
[24] Particle Data Group, J.J. Hernández et al., Review of particle properties, Phys. Lett. B 239 (1990) 1.
[25] S. Pakvasa, S.F. Tuan and S.P. Rosen, Phys. Rev. D 42 (1990) 3746.
[26] T. Mannel, W. Roberts and Z. Ryzak, Phys. Lett. B 255 (1991) 593.
[27] J.G. Körner and M. Krämer, Mainz preprint MZ-TH/91-07.
[28] CLEO Collab., P. Avery et al., Phys. Rev. Lett. 65 (1990) 2842.

[29] ARGUS Collab., H. Albrecht et al., DESY preprint DESY 91-091 (1991).
[30] J.D. Jackson, in: High energy physics, eds. C. de Witt and R. Gatto (Gordon and Breach, New York, 1965) p. 325;
A.D. Martin and D. Spearman, Elementary particle theory (North-Holland, Amsterdam, 1970);
H. Pilkuhn, The interactions of hadrons (North-Holland, Amsterdam, 1967).
[31] J.G. Körner, K. Schilcher and Y.L. Wu, Phys. Lett. B 242 (1990) 119.
[32] K. Hagiwara, A.D. Martin and M.F. Wade, Phys. Lett. B 228 (1989) 144; Nucl. Phys. B 327 (1989) 569.
[33] F.J. Gilman and R.L. Singleton, Phys. Rev. D 41 (1990) 142.
[34] G. Köpp, G. Kramer, W.F. Palmer and G.A. Schuler, Z. Phys. C 48 (1990) 327.
[35] J.C. Anjos et al., Phys. Rev. Lett. 65 (1990) 2630.
[36] K. Reim (ARGUS Collab.), private communication.
[37] K. Schilcher and Y.L. Wu, Mainz preprint MZ-TH/91-19 (1991).
[38] C.G. Boyd and D.E. Brahm, Phys. Lett. B 257 (1991) 393.
[39] J.K. Körner and G. Thompson, Phys. Lett. B 264 (1991) 185.
[40] R.F. Lebed and M. Suzuki, Phys. Rev. D 44 (1991) 829.
[41] J.G. Körner, M. Krämer and K. Zalewski, Mainz preprint MZ-TH/91-06.