

Heavy quark symmetry at large recoil

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Received 11 November 1991; revised manuscript received 13 June 1992

We analyze the large recoil behaviour of heavy meson transition form factors using the Brodsky–Lepage hard scattering formalism. At the leading order of the heavy mass scale the large recoil form factors exhibit a new type of heavy quark symmetry. We discuss next-to-leading mass effects and present explicit $1/M_Q$ expressions for the form factors in the peaking approximation.

The properties of QCD in the infinite quark mass limit are currently the subject of intense study [1–7]. In the infinite quark mass limit, the heavy quark sector of QCD becomes independent of quark masses, and the effective lagrangian of the heavy quark effective theory (HQET) exhibits new spin and flavour symmetries [1–7]. Of particular interest are semi-leptonic (s.l.) transitions between heavy mesons. In the formal limit of infinite quark masses, all mesonic form factors can be expressed in terms of a single universal function $\xi_0(\omega=v_1 \cdot v_2)$, the Isgur–Wise function [2].

The new symmetries are expected to be rather good close to the zero recoil point where not much momentum is transferred to the spectator system. However, as one moves away from the zero recoil point and more momentum gets transferred to the spectator system, hard gluon exchange including spin flip interactions becomes more important and the heavy quark symmetry can be expected to break down.

In the large recoil limit the limiting behaviour of

the form factors can be conveniently studied in the Brodsky–Lepage formalism [8]. As it turns out the form factors exhibit a new heavy quark symmetry in the large recoil limit which is reminiscent but not identical to the heavy quark symmetry at low recoil. We find that the transition form factors have the correct large momentum transfer power behaviour as expected from dimensional counting rules. We analyze the structure of the $1/M_Q$ contributions in the large recoil limit and present explicit $1/M_Q$ expressions for the form factors in an approximation where the quark partons all move with the same velocity as the heavy meson (peaking approximation).

Let us begin our discussion by defining the six independent form factors that describe the current-induced $B \rightarrow D$ and $B \rightarrow D^*$ transitions. Following the convention of ref. [9] one has

$$\begin{aligned} \langle D(v_2) | V_\mu | B(v_1) \rangle &= \sqrt{M_1 M_2} \\ &\times [\xi_+(\omega)(v_1 + v_2)_\mu + \xi_-(\omega)(v_1 - v_2)_\mu], \\ \langle D^*(v_2) | V_\mu | B(v_1) \rangle & \\ &= i\sqrt{M_1 M_2} \xi_V(\omega) \epsilon_{\mu\nu\alpha\beta} \epsilon_2^{*\nu} v_2^\alpha v_1^\beta, \end{aligned} \quad (1)$$

¹ Supported in part by the Bundesministerium für Forschung und Technologie, FRG, under contract 06MZ760.

² Supported in part by the Bundesministerium für Forschung und Technologie, FRG, under contract 06WU765.

$$\langle D^*(v_2) | A_\mu | B(v_1) \rangle = \sqrt{M_1 M_2} [\xi_{A_1}(\omega)(\omega+1)\epsilon_{2\mu}^* - \xi_{A_2}(\omega)\epsilon_2^* \cdot v_1 v_{1\mu} - \xi_{A_3}(\omega)\epsilon_2^* \cdot v_1 v_{2\mu}]. \quad (1 \text{ cont'd})$$

The velocities and the masses of the initial and final state mesons are denoted by v_1 , M_1 and v_2 , M_2 , respectively. We have also introduced the velocity transfer variable $\omega = v_1 \cdot v_2$. In the heavy quark limit and close to zero recoil $\omega \approx 1$, the functions $\xi_i(\omega)$ become related to a single universal form factor, the Isgur–Wise function [2]

$$\begin{aligned} \xi_+(\omega) &= \xi_V(\omega) = \xi_{A_1}(\omega) = \xi_{A_3}(\omega) = \xi_0(\omega), \\ \xi_-(\omega) &= \xi_{A_2}(\omega) = 0. \end{aligned} \quad (2)$$

The HQET result eq. (2) can most easily be derived using the trace formula ^{#1}

$$\langle D(D^*) | V_\mu - A_\mu | B \rangle = -\frac{1}{4} \sqrt{M_1 M_2} \text{Tr} \{ [(\psi_2 - 1)\gamma_5 + (\psi_2 - 1)\not{\epsilon}_2^*] \gamma_\mu (1 - \gamma_5) (\psi_1 + 1)\gamma_5 \} \xi_0(\omega). \quad (3)$$

In eq. (3) the spin wave functions of the final state mesons D and D^* are written as

$$\frac{1}{2\sqrt{2}} (\psi_2 - 1)\gamma_5, \quad \frac{1}{2\sqrt{2}} (\psi_2 - 1)\not{\epsilon}_2^*,$$

respectively, and the spin wave function of the initial state B meson is written as

$$\frac{1}{2\sqrt{2}} (\psi_1 + 1)\gamma_5.$$

Eq. (3) contains the unrenormalized weak transition current. Renormalization effects on the current vertex can easily be included via leading log resummation techniques as discussed in refs. [6,15] (see also refs. [16,17]). Note though that extra care has to be taken in the analysis of the renormalization group result when there are several large scales as in the large recoil limit [16].

We are interested in the large recoil limit of the form factors, i.e. in their large ω - or q^2 -behaviour, where

$$\omega = \frac{M_1^2 + M_2^2 - q^2}{2M_1 M_2}. \quad (4)$$

The s.l. transitions $B \rightarrow D(D^*)$ are a border-line case

^{#1} Early applications of the trace formula to the $B \rightarrow D(D^*)$ sector can be found in refs. [10,11] (see also refs. [12–14]).

in this regard as the maximal value that ω acquires in these transitions is $\omega_{\max} \approx 1.5$ ($\omega_{\max} = M_1^2 + M_2^2 / 2M_1 M_2$). However, when one increases the mass of the decaying heavy meson, larger values of ω can be obtained as e.g. for s.l. top to bottom meson decays. Also, large space-like values of ω can be reached in principle in the reaction $\ell + D \rightarrow B + \nu$ or $\nu + D \rightarrow B + \ell$.

Let us now turn to the discussion of the large ω -behaviour of the form factors. According to Brodsky and Lepage [8] the large ω -behaviour of the form factors is obtained by convoluting the initial and final state hadron's distribution amplitude with a hard scattering amplitude. The hard scattering amplitude is computed in perturbative QCD in the collinear approximation, whereas the distribution amplitudes (DA) contain the nonperturbative long distance dynamics. The DAs are obtained by integrating the hadron wave functions over their transverse momentum.

To be specific one writes for the $B \rightarrow D(D^*)$ transition

$$\begin{aligned} \langle D(D^*) | V_\mu - A_\mu | B \rangle &= 2\sqrt{m_b m_c} \epsilon f_B f_{D(D^*)} \\ &\times \int dx_1 dy_1 \Phi_{D(D^*)}^*(y_1) T_\mu(x_1, y_1, \omega) \Phi_B(x_1), \end{aligned} \quad (5)$$

where x_1 and y_1 are the longitudinal momentum fractions of the bottom and charm quark. The distribution amplitudes are defined such that they are normalized to one, i.e.

$$\int_0^1 \Phi_B(x_1) dx_1 = \int_0^1 \Phi_{D(D^*)}(y_1) dy_1 = 1. \quad (6)$$

The wave function factors f_B and $f_{D(D^*)}$ are proportional to the usual B and $D(D^*)$ decay constants as given by their respective wave functions at the origin. The heavy quark masses and the effective light spectator quark masses are denoted by (m_b, m_c) and ϵ , respectively. Their explicit appearance in eq. (5) is due to our normalization convention of the meson bound states. The hard scattering amplitude expression $T^\mu(x_1, y_1, \omega)$ in eq. (5) contains the projections on the meson states, i.e. $T_\mu(x_1, y_1, \omega)$ can be written as a trace of the bare hard scattering amplitude with the spin wave functions defined after eq. (3). To leading order in the strong coupling constant α_s the hard scattering amplitude is given by the one-gluon

exchange diagrams figs. 1a and 1b. Finally, in order to switch to a more generic notation, in the following we replace the bottom label by 1 and the charm label by 2, i.e. $m_b \rightarrow m_1$, $m_c \rightarrow m_2$, $\Phi_B \rightarrow \Phi_1$, etc.

For the internal off-shell momenta in fig. 1a and 1b one has $q_2 = M_2 v_2 - x_{II} M_1 v_1$, $q_1 = M_1 v_1 - y_{II} M_2 v_2$ and $q_G = x_{II} M_1 v_1 - y_{II} M_2 v_2$. For the heavy quarks' spin sum propagators one then finds

$$\begin{aligned} \not{q}_1 + m_1 &= M_1 (\not{\psi}_1 + 1) - \epsilon (\not{\psi}_2 + 1) + O\left(y_{II} - \frac{\epsilon}{M_2}\right), \\ \not{q}_2 + m_2 &= M_2 (\not{\psi}_2 + 1) - \epsilon (\not{\psi}_1 + 1) + O\left(x_{II} - \frac{\epsilon}{M_1}\right), \end{aligned} \tag{7}$$

where we have used $M_1 = m_1 + \epsilon$ and $M_2 = m_2 + \epsilon$ (zero binding approximation).

The DAs for heavy hadrons are known to have a pronounced peak at $x_1 = 1 - \epsilon/M_Q$ with a width which shrinks with increasing quark (or hadron) mass (proportional to $1/M_Q$ in most present models). It is clear that only regions close to the peak position contribute to eq. (5) to any degree of significance. This implies that, to a very good approximation, the quarks and the hadron made from them have the same velocity. This is of course trivially true at the peak position. One can thus replace $x_{II} M_1$ and $y_{II} M_2$ by the spectator mass parameter ϵ , safely staying within the theoretical uncertainties of the parton model assumptions. By dropping the $O(x_{II} - \epsilon/M_1)$ and $O(y_{II} - \epsilon/M_2)$ terms in (7) using the above identification one will incur errors in the $1/M_Q$ terms whose exact form depends on the choice of the DA. We shall return to this point later on. In the above

approximation the inverse heavy quark propagators take the rather convenient form of sums of positive energy projectors, eq. (7).

Using the form eq. (7) for the heavy quark spin sum propagators one finally obtains

$$\begin{aligned} T_\mu(x_1, y_1, \omega) &= -4\pi\alpha_s C_F \\ &\times \frac{1}{8} \text{Tr}[(\not{\psi}_2 - 1)\gamma_5 + (\not{\psi}_2 - 1)\not{\psi}_2^*] \\ &\times \left(\gamma_\alpha [M_2(\not{\psi}_2 + 1) - \epsilon(\not{\psi}_1 + 1)] \frac{1}{q_2^2 - m_2^2} \gamma_\mu (1 - \gamma_5) \right. \\ &\left. + \gamma_\mu (1 - \gamma_5) [M_1(\not{\psi}_1 + 1) - \epsilon(\not{\psi}_2 + 1)] \gamma_\alpha \frac{1}{q_1^2 - m_1^2} \right) \\ &\times (\not{\psi}_1 + 1) \gamma_5 \gamma^\alpha \frac{1}{q_G^2}, \end{aligned} \tag{8}$$

where C_F is the usual colour factor $C_F = \frac{4}{3}$.

The trace eq. (8) can easily be worked out. Inserting the result into the integral eq. (5) one has the following results for the mesonic form factors ^{#2}:

$$\begin{aligned} \xi_+ / f_1 f_2 &= \mathcal{H}_3 - (2 - \omega)(\mathcal{H}_1 + \mathcal{H}_2), \\ \xi_- / f_1 f_2 &= -(2 - \omega)(\mathcal{H}_1 - \mathcal{H}_2), \\ \xi_V / f_1 f_2 &= \xi_{A3} / f_1 f_2 = \mathcal{H}_3, \\ \xi_{A1} / f_1 f_2 &= \mathcal{H}_3 - 2 \frac{1}{\omega + 1} \mathcal{H}_1 - 2 \frac{1 - \omega}{\omega + 1} \mathcal{H}_2, \\ \xi_{A2} / f_1 f_2 &= -2 \mathcal{H}_1, \end{aligned} \tag{9}$$

where we have introduced the three independent form factor integrals ^{#3, #4}

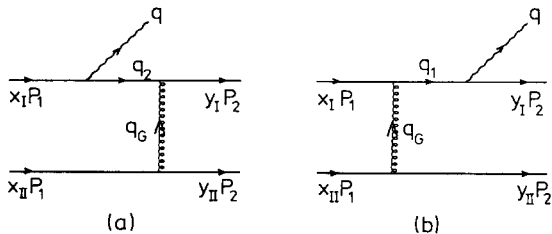


Fig. 1. Hard scattering contributions to mesonic weak transition form factors. In the case of bottom to charm transition the label 1 stands for bottom and label 2 for charm.

^{#2} We do not agree with the large recoil structure of the mesonic form factors as written down in ref. [19].

^{#3} We have dropped a factor $\sqrt{m_1 m_2 / M_1 M_2}$ in order to be consistent with our heavy quark approximation.

^{#4} When one attempts to numerically integrate the form factor integrals (10) one may encounter singularities at the endpoints of the integration range. For example, the fermion poles eq. (11) have singularities at $x_1 = 1 - \epsilon/M_2 \omega$ and at $y_1 = 1 - \epsilon/M_2 \omega$ inside the integration range. These endpoint singularities would have to be dealt with by e.g. introducing cut-offs at the endpoints. However, as these endpoint singularities are associated with the soft confinement aspect of the Brodsky-Lepage picture they have no bearing on the heavy mass structure of the hard scattering integrals that we are discussing here (see also the discussion in ref. [20]).

$$\begin{aligned} \mathcal{H}_1 &= -8\pi\alpha_s C_F \int dx_1 dy_1 \Phi_2^* \Phi_1 \frac{\epsilon^2}{q_G^2(q_2^2 - m_2^2)}, \\ \mathcal{H}_2 &= -8\pi\alpha_s C_F \int dx_1 dy_1 \Phi_2^* \Phi_1 \frac{\epsilon^2}{q_G^2(q_1^2 - m_1^2)}, \\ \mathcal{H}_3 &= -8\pi\alpha_s C_F \\ &\times \int dx_1 dy_1 \Phi_2^* \Phi_1 \frac{\epsilon}{q_G^2} \left(\frac{M_2}{q_2^2 - m_2^2} + \frac{M_1}{q_1^2 - m_1^2} \right). \end{aligned} \quad (10)$$

It is instructive to study the case where the x - and y -dependence of the hard scattering amplitude is ignored relative to that of the DAs, i.e. evaluating the hard scattering amplitude at the positions of the maxima of the two DAs. We shall refer to this approximation as the peaking approximation. The peaking approximation numerically is quite reliable in many cases and allows one to discuss the qualitative features of the model results in a rather simple fashion. We mention that the quality of the peaking approximation has been investigated numerically in the heavy baryon sector [18]. The form factor integrals can be evaluated by using the peaking approximation as follows. For the pole propagators in (10) one has

$$\begin{aligned} q_1^2 - m_1^2 &= 2\epsilon M_1(1 - \omega) + O\left(y_{II} - \frac{\epsilon}{M_2}\right), \\ q_2^2 - m_2^2 &= 2\epsilon M_2(1 - \omega) + O\left(x_{II} - \frac{\epsilon}{M_1}\right), \\ q_G^2 &= 2\epsilon^2(1 - \omega) + O\left(x_{II} - \frac{\epsilon}{M_1}; y_{II} - \frac{\epsilon}{M_2}\right). \end{aligned} \quad (11)$$

Eqs. (10) and (11) show that the leading contribution comes from the form factor integral \mathcal{H}_3 . The form factor integral \mathcal{H}_3 can be evaluated using the normalization conditions eq. (6). One obtains

$$\mathcal{H}_3 = -\frac{4\pi\alpha_s C_F}{\epsilon^2(1 - \omega)^2} \equiv \zeta_{BL}(\omega). \quad (12)$$

In (12) we have introduced a mass scale independent universal form factor function $\zeta_{BL}(\omega)$ which we shall refer to as the Brodsky–Lepage (BL) form factor function. The form factor integrals \mathcal{H}_1 and \mathcal{H}_2 can

seen to be related to \mathcal{H}_3 in the peaking approximation. One finds

$$M_2 \mathcal{H}_1 = M_1 \mathcal{H}_2 = \frac{1}{2} \epsilon \mathcal{H}_3. \quad (13)$$

Collecting all our results eqs. (9)–(13) we finally obtain

$$\begin{aligned} \xi_+/f_1 f_2 &= \zeta_{BL} \left[1 + \left\{ \frac{\omega-1}{-1} \right\} \frac{1}{2} \epsilon \left(\frac{1}{M_1} + \frac{1}{M_2} \right) \right], \\ \xi_-/f_1 f_2 &= \zeta_{BL} \left[0 + \left\{ \frac{\omega-1}{-1} \right\} \frac{1}{2} \epsilon \left(-\frac{1}{M_1} + \frac{1}{M_2} \right) \right], \\ \xi_V/f_1 f_2 &= \zeta_{BL}, \\ \xi_{A_1}/f_1 f_2 &= \zeta_{BL} \left(1 - \frac{\epsilon}{M_2} \frac{1}{1+\omega} + \left\{ \frac{\omega-1}{-1} \right\} \frac{\epsilon}{M_1} \frac{1}{1+\omega} \right), \\ \xi_{A_2}/f_1 f_2 &= \zeta_{BL} \left(0 - \frac{\epsilon}{M_2} \frac{1}{\omega+1} \left\{ \frac{-1}{\omega+2} \right\} \right), \\ \xi_{A_3}/f_1 f_2 &= \zeta_{BL} \left(1 - \frac{\epsilon}{M_2} \frac{1}{\omega+1} \left\{ \frac{-1}{1} \right\} \right). \end{aligned} \quad (14)$$

We emphasize that (14) follows directly from the peaking approximation without having done a $1/M$ expansion. In the peaking approximation the range of validity of the perturbative approach is obvious: ω has to be sufficiently large for the exchanged gluon to be sufficiently hard, cf. eq. (11).

In order to exhibit the spin non-flip (NF) and spin flip (F) content of the hard scattering one-gluon exchange contribution we have introduced a curly bracket notation in eq. (14). The upper entry gives the non-flip (NF; longitudinal gluon) and the lower entry gives the flip (F; transverse gluon) contributions. Terms with no curly brackets are non-flip. Technically the NF–F content can be conveniently projected out by separating the gluon’s spin coupling to the light quarks into its non-flip and flip contributions:

$$\begin{aligned} \gamma_\alpha &= \frac{1}{\omega+1} (v_2 + v_2)_\alpha \quad (\text{NF}), \\ &= \gamma_\alpha - \frac{1}{\omega+1} (v_1 + v_2)_\alpha \quad (\text{F}). \end{aligned} \quad (15)$$

One notes that the leading term in eq. (14) has a well defined heavy mass scale dependence which is given by the wave function at the origin factors f_i .

These are well known to scale as $1/\sqrt{M_i}$ [15]. After removal of the wave function factors one remains with a mass scale independent leading contribution with a structure identical to the low recoil form factor result (2). In fact the leading contribution in (10) can easily be seen to collapse into the trace eq. (3) using the Bloch–Nordsieck type NF replacement given in eq. (15). Quite remarkably also the $1/M_Q$ contributions in eq. (14) have the $1/M_Q$ structure of the low recoil results [9,21] after removal of the wave function factors. For example, in the notation of ref. [9], the $1/M_Q$ terms in eq. (15) can be written as

$$\begin{aligned}\rho_1 &= \frac{1}{2}(\omega - 2)\epsilon, \\ \rho_2 &= \rho_3 = -\frac{1}{2}\epsilon, \\ \rho_4 &= \frac{1}{2}(\omega - 1)\epsilon,\end{aligned}\quad (16)$$

where the common form factor function (in our case ξ_{BL}) has been factored out as in ref. [9].

The structure of the leading and next-to-leading $1/M_Q$ contributions in eq. (14) can be understood by an inspection of the $1/M_Q$ structure of the effective lagrangian of the HQET as e.g. written down in refs. [7,21]. The leading term in the effective HQET lagrangian has no mass scale dependence and therefore induces only non-flip contributions. On the contrary, the $1/M_Q$ pieces in the effective lagrangian contain an explicit spin flip term as well as non-flip terms that lead to the momentum type $1/M_Q$ non-flip insertions in (14).

The results (14) hold in a more general setting even when the peaking approximation is not used. Suppose that the two integrals in \mathcal{H}_3 (cf. eq. (10)) have a well-defined limit for $M_1, M_2 \rightarrow \infty$. Then it follows that the heavy quark symmetry structure of the leading term in (14) still obtains. We mention that for DAs presently in use [20,22] the above integration limits exist even when moderate changes are applied to the DAs [20,22]^{#5}. We emphasize that the heavy quark symmetry structure of the leading term in (14) is model independent for the DAs presently in use [20,22]. This means that in the infinite mass limit, with ω held fixed and large, one will have the sym-

metry relations $\xi_+ = \xi_V = \xi_{A_1} = \xi_{A_3}$ and $\xi_- = \xi_{A_2} = 0$. The limiting form of the function $\mathcal{H}_3(\omega)$, however, does depend on the choice of the DAs and may differ from that of eq. (12) except, of course, in its power dependence.

We emphasize that the structure of the $1/M_Q$ corrections at large recoil do not in general possess the low recoil $1/M_Q$ structure. The large recoil structure of the $1/M_Q$ corrections depend on the detailed structure of the DAs, on the way the propagators are treated and would also be changed by the inclusion of $1/M_Q$ corrections to the spin wave function itself. Note, however, that the leading large recoil behaviour of the form factors in the heavy mass expansion is not affected by any of these changes. Thus, for instance, the large recoil behaviour of ξ_+ sets in when $\omega\epsilon/M_Q$ becomes large. In spite of $\omega\epsilon/M_Q$ becoming large a $1/M_Q$ expansion is still meaningful since, as can be easily seen from eq. (8), there are no contributions of order ω^2/M_Q^2 or higher order. The maximal power of ω that can appear in the numerator is one.

A numerical estimate of the size of the hard scattering contribution, say for $B \rightarrow D, D^*$ transitions can be obtained from eqs. (12) and (14). Taking account of the large uncertainties of the (unmeasured) meson decay constants f_B and f_D our prediction for the hard scattering contribution lies at the lower range of theoretical estimates for the non-perturbative contributions [9,23] for values of ω around $\omega \simeq 2.5$. For large values of ω the perturbative contribution is dominant as it is power behaved compared to the exponential decrease of the non-perturbative contributions. Much below $\omega \simeq 2.5$ the perturbative contributions cease to be reliable because of the small virtuality of the gluon as mentioned before. We emphasize that our prime concern in this paper is not the description of the form factors in the decay region but their large recoil behaviour. As mentioned before, large values of ω can be reached in principle in semi-leptonic decays of top mesons (if they exist) and in the scattering reactions $\ell D \rightarrow B\nu$.

Concerning the large ω -behaviour of the form factors eq. (14) one notes that they have the correct power counting behaviour as known from dimensional counting rules [8,12]. Thus one has $\xi_+, \xi_- \sim \omega^{-1}$ and $\xi_V, \xi_{A_1}, \xi_{A_2}, \xi_{A_3} \rightarrow \omega^{-2}$. In the case of

^{#5} We emphasize that this is not necessarily true for every DA. For example, multiplying the DA of ref. [18] by $(1-x)^2$ is at first glance a reasonable heavy meson DA but the limits of the above two integrals in \mathcal{H}_3 do not exist.

the pseudoscalar to pseudoscalar transition as in $B \rightarrow D$ the leading monopole power behaviour is generated by transverse spin flip gluon exchange as eq. (14) shows. The leading ω -behaviour is thus mass suppressed. In the pseudoscalar to vector case the leading ω -behaviour is generated by longitudinal non-flip gluon exchange and therefore is not mass suppressed (excepting ζ_{A_2}). The on-set of the asymptotic ω -power behaviour can thus be expected to be rather slow in the $B \rightarrow D$ case. This is different in the $B \rightarrow D^*$ case where the asymptotic power behaviour would come on faster. Looking at elastic form factor data of light hadrons one has gained the experience that the asymptotic power behaviour sets in quite early. In this contest it is interesting to note that a recent fit to the experimental data on $B \rightarrow D^*$ transitions suggests a dipole type behaviour for the Isgur-Wise form factor function [24].

In conclusion, we have analyzed the large recoil behaviour of current-induced transitions among heavy mesons using the Brodsky-Lepage hard scattering formalism. To leading order in the heavy mass the form factors show a model independent spin-flavour symmetry structure which is identical to the spin-flavour symmetry at low recoil after having removed mass scale dependent wave function factors. The structure of the next-to-leading contributions is in general model dependent. We have evaluated the $1/M_Q$ contributions to the form factors in the peaking approximation. The next-to-leading contributions in the peaking approximation have a structure similar to the low recoil $1/M_Q$ results.

We have calculated the explicit ω -dependence of the various form factors in the peaking approximation. Now that the limiting structure of the heavy meson transition form factors is known at low recoil ($\omega \rightarrow 1$) and at large recoil ($\omega \rightarrow \infty$) it would be interesting to find a form factor representation which interpolates between the two limiting regions.

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