

## Polarization and $CP$ asymmetries in the decays $B \rightarrow K^* \psi$ , $K^* \omega$ , and $K^* \rho$

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We calculate branching ratios, polarization and  $CP$  asymmetries for the final exclusive states  $K^* \psi$ ,  $K^* \omega$ ,  $K^* \rho$ , make comparisons with experimental data, and suggest further tests of the underlying theoretical framework.

In previous work [1] we made a systematic study of the exclusive decay of  $B$  mesons to two vector mesons [2–4]. The renormalization group improved effective hamiltonian [5,6] was evaluated in the vacuum insertion approximation [7]. OZI suppressed and annihilation terms were neglected. Current matrix elements were evaluated using the wave functions of Bauer, Stech and Wirbel [3]. Branching ratios and angular correlations among subsequent decays of the vector mesons were calculated for 34 channels. As a first approximation, the calculational scheme provided a useful framework with which to organize the data. We are currently improving the form factors in this work by using heavy quark symmetries where they are applicable [8]. We are also improving the effective hamiltonian by running the QCD coefficients for current values of the top quark mass. In this short work we will report new results on: (1) The polarization in  $K^* \psi$  final states and (2) Direct  $CP$  asymmetries in  $K^* \omega$  and  $K^* \rho$  final states [9], which are excellent probes of penguin [10] term influence on decay amplitudes [1].

After renormalization, the effective hamiltonian for  $\Delta c = 0$ ,  $\Delta b = \Delta s = 1$  processes is

$$H_2^{\text{eff}} = -\frac{G}{2\sqrt{2}} \left( V_{cs}^* V_{cb} (c_+ O_{2+}^c + c_- O_{2-}^c) + V_{ts}^* V_{tb} \sum_{i=1}^6 c_i O_i \right), \quad (1)$$

where

$$O_{2\pm}^q = [(\bar{s}u)(\bar{u}b) \pm (\bar{s}b)(\bar{u}u)] - [(\bar{s}q)(\bar{q}b) \pm (\bar{s}b)(\bar{q}q)], \quad (2)$$

$$O_1 = (\bar{s}b)_L (\bar{u}u)_L, \quad O_2 = (\bar{s}_i b_j)_L (\bar{u}_j u_i)_L, \quad O_3 = (\bar{s}b)_L \sum_q (\bar{q}q)_L,$$

$$O_4 = (\bar{s}_i b_j)_L \sum_q (\bar{q}_j q_i)_L, \quad O_5 = (\bar{s} \lambda^a b)_L \sum_q (\bar{q} \lambda^a q)_R, \quad O_6 = (\bar{s}b)_L \sum_q (\bar{q}q)_R.$$

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The coefficients  $c_{\pm}$ ,  $c_1$ ,  $c_2$ ,  $c_3$ ,  $c_4$ ,  $c_5$  and  $c_6$  were calculated by Ponce [6] some years ago for values of the top quark mass and the  $W$ -boson mass that are quite different from current data, although the coefficients do not depend sensitively on these inputs. More significantly, this calculation has a numerical error which over estimates the effects of the QCD corrections in  $c_1$  and  $c_2$ . We have repeated the calculation and find for  $\Lambda_{\text{QCD}} = 0.2 \text{ GeV}$  and  $m_t = m_W = 81 \text{ GeV}$  that at the scale  $m_b$  we have  $c_+ = 0.8499$ ,  $c_- = 1.3844$ ,  $c_1 = -0.5344$ ,  $c_2 = 2.2343$ ,  $c_3 = 0.0241$ ,  $c_4 = -0.0548$ ,  $c_5 = 0.0159$ ,  $c_6 = -0.0681$ .

We use the notation  $H_{\lambda} = \langle V_1(\lambda) V_2(\lambda) | H_{\text{wk}}^{\text{eff}} | \bar{B}^0 \rangle$  for the helicity matrix element,  $\lambda = 0, \pm 1$ . These can be expressed by three invariant amplitudes  $a, b, c$ , defined by the decomposition

$$H_{\lambda} = \epsilon_{1\mu}(\lambda)^* \epsilon_{2\nu}(\lambda)^* \left( a g^{\mu\nu} + \frac{b}{m_1 m_2} p^{\mu} p^{\nu} + \frac{ic}{m_1 m_2} \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} \right), \quad (3)$$

where  $p = p_1 + p_2$  is the  $\bar{B}^0$  four momentum. Thus  $H_{\pm 1} = a \pm \sqrt{x^2 - 1} c$ ,  $H_0 = -ax - b(x^2 - 1)$  where  $p^2 = (m_1^2 m_2^2 / m^2)(x^2 - 1)$ . The helicity amplitudes  $\bar{H}_{\lambda}$  for the decay of  $B^0 \rightarrow \bar{V}_1 \bar{V}_2$ , where  $\bar{V}_1$  and  $\bar{V}_2$  are the antiparticles of  $V_1$  and  $V_2$  respectively, have the same decomposition as (3) with  $a \rightarrow \bar{a}$ ,  $b \rightarrow \bar{b}$  and  $c \rightarrow -\bar{c}$ . The coefficients  $a, b$  and  $c$  describe the s-, d- and p-wave contribution of the two final vector particles. They have phases  $\delta$  from strong interactions and weak phases  $\phi$  originating from the  $CP$  violating phase in the CKM matrix. When there are no strong interaction phases,  $\bar{a} = a^*$ ,  $\bar{b} = b^*$  and  $\bar{c} = c^*$ . Since there is a sign change in front of  $\bar{c}$  in  $\bar{H}_{\lambda}$  we have for the case of vanishing strong phases  $\delta_i^{0,1,2}$ :  $\bar{H}_{\pm 1} = H_{\mp 1}^*$ ,  $\bar{H}_0 = H_0^*$ .

The angular distributions depend on the spins of the decay products of the decaying vector mesons  $V_1$  and  $V_2$ . For  $B \rightarrow K^* \psi \rightarrow (K\pi)(e^+e^-)$  the differential decay distribution is

$$\begin{aligned} \frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} &= \frac{p}{16\pi^2 m^2} \cdot \frac{9}{8} \left\{ \frac{1}{4} \sin^2\theta_1 (1 + \cos^2\theta_2) (|H_{+1}|^2 + |H_{-1}|^2) + \cos^2\theta_1 \sin^2\theta_2 |H_0|^2 \right. \\ &\quad \left. - \frac{1}{2} \sin^2\theta_1 \sin^2\theta_2 [\cos 2\phi \text{Re}(H_{+1}H_{-1}^*) - \sin 2\phi \text{Im}(H_{+1}H_{-1}^*)] \right. \\ &\quad \left. - \frac{1}{4} \sin 2\theta_1 \sin 2\theta_2 [\cos\phi \text{Re}(H_{+1}H_0^* + H_{-1}H_0^*) - \sin\phi \text{Im}(H_{+1}H_0^* - H_{-1}H_0^*)] \right\}. \end{aligned} \quad (4)$$

In eq. (4),  $\theta_1$  is the polar angle of the  $K$  momentum in the rest system of the  $K^*$  meson with respect to the helicity axis, i.e. the momentum  $p_1$ . Similarly  $\theta_2$  and  $\phi$  are the polar and azimuthal angle of the positron  $e^+$  in the  $\psi$  rest system with respect to the helicity axis of the  $\psi$ , i.e.  $\phi$  is the angle between the planes of the two decays  $K^* \rightarrow K\pi$  and  $\psi \rightarrow e^+e^-$  (or  $\mu^+\mu^-$ ). The ratios  $\Gamma_{\text{T}}/\Gamma$  and  $\Gamma_{\text{L}}/\Gamma$  measure the amount of transversely (longitudinally) polarized  $K^*$  (or  $\psi$ ). The decay distribution is parameterized by the coefficients

$$\begin{aligned} \frac{\Gamma_{\text{T}}}{\Gamma} &= \frac{|H_{+1}|^2 + |H_{-1}|^2}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}, & \frac{\Gamma_{\text{L}}}{\Gamma} &= \frac{|H_0|^2}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}, \\ \alpha_1 &= \frac{\text{Re}(H_{+1}H_0^* + H_{-1}H_0^*)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}, & \alpha_2 &= \frac{\text{Re}(H_{+1}H_{-1}^*)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}, \\ \beta_1 &= \frac{\text{Im}(H_{+1}H_0^* - H_{-1}H_0^*)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}, & \beta_2 &= \frac{\text{Im}(H_{+1}H_{-1}^*)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}. \end{aligned}$$

For the case  $B \rightarrow K^* \rho \rightarrow (K\pi)(\pi\pi)$  the decay angular distribution is

$$\begin{aligned} \frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} &= \frac{p}{16\pi^2 m^2} \cdot \frac{9}{4} \left\{ \frac{1}{4} \sin^2\theta_1 \sin^2\theta_2 (|H_{+1}|^2 + |H_{-1}|^2) + \cos^2\theta_1 \cos^2\theta_2 |H_0|^2 \right. \\ &\quad \left. + \frac{1}{2} \sin^2\theta_1 \sin^2\theta_2 [\cos 2\phi \text{Re}(H_{+1}H_{-1}^*) - \sin 2\phi \text{Im}(H_{+1}H_{-1}^*)] \right. \\ &\quad \left. + \frac{1}{4} \sin 2\theta_1 \sin 2\theta_2 [\cos\phi \text{Re}(H_{+1}H_0^* + H_{-1}H_0^*) - \sin\phi \text{Im}(H_{+1}H_0^* - H_{-1}H_0^*)] \right\}. \end{aligned} \quad (5)$$

The angular distribution of  $B \rightarrow K^* \omega$  is also given by eq. (5) independent of whether one defines the direction  $(\theta_2, \phi)$  by the momentum of one of the outgoing pions, for example, the momentum of the  $\pi^+$ , or by the normal of the decay plane formed by the momenta of  $\pi^+$ ,  $\pi^-$  and  $\pi^0$  in the  $\omega$  rest system.

In general the dominant terms in the angular correlations are  $\Gamma_T/\Gamma$ ,  $\Gamma_L/\Gamma$ ,  $\alpha_1$  and  $\alpha_2$ . The terms  $\beta_1$  and  $\beta_2$  are small since they are nonvanishing only if the helicity amplitudes  $H_{+1}$ ,  $H_{-1}$  and  $H_0$  or the invariant amplitudes  $a$ ,  $b$  and  $c$ , respectively have different phases. When there are no strong interaction phases the coefficients  $\beta_1$  and  $\beta_2$  are nonvanishing only through the  $CP$  violating phase of the CKM matrix under the condition that they contribute differently to  $a$ ,  $b$  and  $c$  or  $H_{+1}$ ,  $H_{-1}$  and  $H_0$  respectively.

Let the conjugate process amplitudes be denoted  $\bar{H}_{+1}$ , etc and the invariant amplitudes  $\bar{a}_i$ , etc where  $i$  denotes the independent channel or process with final state interaction phase  $\delta_i$  and weak phase  $\phi_i$ . Then the interesting  $CP$  differences which do not require strong phases and are proportional to weak phase differences are

$$\text{Im}(H_{+1}H_{-1}^* - \bar{H}_{+1}\bar{H}_{-1}^*) = -4\sqrt{x^2 - 1} \sum_{i,j} \cos(\delta_{si} - \delta_{pj}) \sin(\phi_{si} - \phi_{pj}) |a_i c_j| \quad (6)$$

and

$$\begin{aligned} & \text{Im}(H_{+1}H_{-1}^* - H_{-1}H_0^* - H_{+1}H_0^* + \bar{H}_{-1}\bar{H}_0^*) \\ &= -4(x^2 - 1)^{3/2} \sum_{i,j} \cos(\delta_{pi} - \delta_{dj}) \sin(\phi_{pi} - \phi_{dj}) |c_i b_j| - 4x\sqrt{x^2 - 1} \sum_{i,j} \cos(\delta_{pi} - \delta_{sj}) \sin(\phi_{pi} - \phi_{sj}) |c_i a_j|. \end{aligned} \quad (7)$$

Terms of the first type, which are numerically small in our model, can be isolated by averaging over the polar angles and looking at the  $\phi$  dependence of the difference distribution:

$$\frac{2\pi}{\Gamma} \frac{d\Gamma}{d\phi} - \frac{2\pi}{\bar{\Gamma}} \frac{d\bar{\Gamma}}{d\phi} = -(\alpha_2 - \bar{\alpha}_2) \cos 2\phi - (\beta_2 - \bar{\beta}_2) \sin 2\phi. \quad (8)$$

Terms of the second type can be isolated by examining the  $\phi$  dependence of the difference distribution separated according to same hemisphere (SH) events (e.g.  $0 < \theta_1, \theta_2 < \frac{1}{2}\pi$ ) or opposite hemisphere (OH) events (e.g.  $0 < \theta_1 < \frac{1}{2}\pi, \frac{1}{2}\pi < \theta_2 < \pi$ ):

$$\frac{2\pi}{\Gamma} \left( \frac{d\Gamma^{\text{OH}}}{d\phi} - \frac{d\Gamma^{\text{SH}}}{d\phi} \right) - \frac{2\pi}{\bar{\Gamma}} \left( \frac{d\bar{\Gamma}^{\text{OH}}}{d\phi} - \frac{d\bar{\Gamma}^{\text{SH}}}{d\phi} \right) = -\frac{1}{2} [(\alpha_1 - \bar{\alpha}_1) \cos \phi - (\beta_1 - \bar{\beta}_1) \sin \phi]. \quad (9)$$

A different signature for  $CP$  violation is obtained when one considers neutral  $B$  mesons only. Then it is possible to generate interference via mixing by looking at final states that can occur from  $B^0$  and  $\bar{B}^0$  decays. In the case of common final states for  $\bar{B}^0$  and  $B^0$  decays,  $\Delta\Gamma$ , the difference  $\Gamma(B^0(t) \rightarrow f) - \Gamma(\bar{B}^0(t) \rightarrow f)$  is [11]

$$\frac{\Delta\Gamma}{\Gamma} = \frac{\text{Im}\{(q/p)[H_0^2 + 2H_{+1}H_{-1}]\}}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}. \quad (10)$$

This quantity depends on the mixing phase  $q/p$  and the weak phase of  $H_0^2 + 2H_{+1}H_{-1}$ . The difference between  $\text{Im}(q/p)$  and  $\Delta\Gamma/\Gamma$  represents the influence of spin effects in the final state on the mixing factor. In some of the considered decays  $H_0$ ,  $H_{+1}$  and  $H_{-1}$  have equal weak phases. Then  $\Delta\Gamma/\Gamma$  is maximal if  $H_{+1} = H_{-1}$  [12,13].

Using vacuum saturation and neglecting terms like  $\langle V_1 V_2 | j_\mu | 0 \rangle \langle 0 | \bar{j}^\mu | B \rangle$  and  $\langle V_1 V_2 | S | 0 \rangle \langle 0 | P | B \rangle$  which are suppressed by factors  $m_p^2/(m_B^2 - m_p^2)$  where  $m_p \ll m_B$  is the form factor mass, the matrix element for the decay  $B^0 \rightarrow K^* \psi$  is

$$\begin{aligned} \langle K^{*0} \psi | H_2^{\text{eff}} | \bar{B}^0 \rangle = & -\frac{G}{\sqrt{2}} \{ \langle \bar{K}^{*0} | (\bar{s}b)_\mu | \bar{B}^0 \rangle \langle \bar{\psi} | (\bar{c}c)^\mu | 0 \rangle [-a_- A_c + (a_3 + c_6) A_t] \\ & + \langle \psi | (\bar{d}b)_\mu | \bar{B}^0 \rangle \langle \bar{K}^{*0} | (\bar{s}d)^\mu | 0 \rangle a_4 A_t \}, \end{aligned} \quad (11)$$

where we have introduced the following combinations of the QCD coefficients:  $c_+, c_-, c_1, \dots, c_6$ :  $a_\pm = \frac{2}{3}c_+ \pm \frac{1}{3}c_-$ ,  $a_1 = \frac{1}{6}c_1 + \frac{1}{2}c_2$ ,  $a_2 = \frac{1}{2}c_1 + \frac{1}{6}c_2$ ,  $a_3 = \frac{1}{2}c_3 + \frac{1}{6}c_4$ , and  $a_4 = \frac{1}{6}c_3 + \frac{1}{2}c_4$ .  $A_c = V_{cs}^* V_{cb}$  and  $A_t = V_{ts}^* V_{tb}$ .

In eq. (11) the current matrix element of the second term is OZI forbidden and will be neglected. Therefore all helicity matrix elements will have the same phase for this case so that the  $CP$  asymmetries  $\beta_1$  and  $\beta_2$  vanish. Penguin effects are small in this case since  $(a_3 + c_6)$  is small compared to  $a_-$ . Thus the branching ratio is determined essentially by the factor  $a_- A_c$ . The QCD coefficient  $a_-$  is non-dominant,  $|a_-| \ll a_+$ . The matrix element for the corresponding charged decay is

$$\begin{aligned} \langle K^{*-} \psi | H_2^{\text{eff}} | B^- \rangle = & -\frac{G}{\sqrt{2}} \{ \langle K^{*-} | (\bar{s}b)_\mu | B^- \rangle \langle \psi | (\bar{c}c)^\mu | 0 \rangle [-a_- A_c + (a_4 + c_6) A_t] \\ & + \langle \psi | (\bar{u}b)_\mu | B^- \rangle \langle K^{*-} | (\bar{s}u)^\mu | 0 \rangle [a_+ A_c + (a_1 + a_4) A_t] \}. \end{aligned} \quad (12)$$

The second term is OZI forbidden and is in addition suppressed in zeroth order QCD because  $A_c + A_t = -A_u$ . Eq. (11) and eq. (13) below have different penguins but are numerically almost the same because the penguin effects are small.

The matrix element for  $\bar{B}^0 \rightarrow \bar{K}^{*0} \omega$  is

$$\begin{aligned} \langle \bar{K}^{*0} \omega | H_2^{\text{eff}} | \bar{B}^0 \rangle = & -\frac{G}{\sqrt{2}} \{ \langle \bar{K}^{*0} | (\bar{s}b)_\mu | \bar{B}^0 \rangle \langle \omega | (\bar{u}u)^\mu | 0 \rangle (a_- A_c + a_2 A_t) \\ & + \langle \bar{K}^{*0} | (\bar{s}b)_\mu | \bar{B}^0 \rangle \langle \omega | (\bar{u}u + \bar{d}d)^\mu | 0 \rangle (a_3 + c_6) A_t + \langle \omega | (\bar{d}b)_\mu | \bar{B}^0 \rangle \langle \bar{K}^{*0} | (\bar{s}d)^\mu | 0 \rangle a_4 A_t \} \end{aligned} \quad (13)$$

and for  $\bar{B}^0 \rightarrow K^{*0} \rho^0$  is

$$\begin{aligned} \langle K^{*0} \rho^0 | H_2^{\text{eff}} | \bar{B}^0 \rangle = & -\frac{G}{\sqrt{2}} \{ \langle K^{*0} | (\bar{s}b)_\mu | \bar{B}^0 \rangle \langle \rho^0 | (\bar{u}u)^\mu | 0 \rangle (a_- A_c + a_2 A_t) + \langle \rho^0 | (\bar{d}b)_\mu | \bar{B}^0 \rangle \langle \bar{K}^{*0} | (\bar{s}d)^\mu | 0 \rangle a_4 A_t \}. \end{aligned} \quad (14)$$

Apart from very small mass difference effects, these two amplitudes differ only because of QCD effects arising from the coefficients  $a_3, a_4, c_6$ . These terms interfere with the first term and produce non-vanishing  $\beta_1$  and  $\beta_2$ . We shall see that the penguins of  $\bar{K}^{*0} \omega$  are much stronger than those of  $K^{*0} \rho$ . Without higher order QCD corrections we have  $a_- = a_2 (c_+ = c_- = 1, c_1 = 0, c_2 = 2)$  so that the first term in eq. (13) and (14) is proportional to  $A_c + A_t = -A_u = -V_{us}^* V_{ub}$  due to the unitarity of the CKM matrix. This term is also small when QCD corrections are made because the operator mixing preserves the relations  $c_1 = c_+ - c_-$  and  $c_2 = c_+ + c_-$ . (This is not the case for the early solution [6].) This means that the first terms are strongly suppressed since  $V_{us}^* V_{ub}$  is very small. With QCD corrections including penguins this suppression is lifted and larger branching fractions are possible.

The matrix elements for the corresponding  $B^-$  decays,  $B^- \rightarrow K^{*-} \omega$  and  $B^- \rightarrow K^{*-} \rho^0$  are

$$\begin{aligned} \langle K^{*-} \omega | H_2^{\text{eff}} | B^- \rangle = & -\frac{G}{\sqrt{2}} \{ \langle K^{*-} | (\bar{s}b)_\mu | B^- \rangle \langle \omega | (\bar{u}u)^\mu | 0 \rangle (a_- A_c + a_2 A_t) \\ & + \langle K^{*-} | (\bar{s}b)_\mu | B^- \rangle \langle \omega | (\bar{u}u + \bar{d}d)^\mu | 0 \rangle (a_3 + c_6) A_t + \langle \omega | (\bar{u}b)_\mu | B^- \rangle \langle K^{*-} | (\bar{s}u)^\mu | 0 \rangle [a_+ A_c + (a_1 + a_4) A_t] \}, \end{aligned} \quad (15)$$

$$\begin{aligned} \langle K^{*-} \rho^0 | H_2^{\text{eff}} | B^- \rangle = & -\frac{G}{\sqrt{2}} \{ \langle K^{*-} | (\bar{s}b)_\mu | B^- \rangle \langle \rho^0 | (\bar{u}u)^\mu | 0 \rangle (a_- A_c + a_2 A_t) \\ & + \langle \rho^0 | (\bar{u}b)_\mu | B^- \rangle \langle K^{*-} | (\bar{s}u)^\mu | 0 \rangle [a_+ A_c + (a_1 + a_4) A_t] \} \end{aligned} \quad (16)$$

Eqs. (15) and (16) differ even when there are no penguins because the  $a_+$  and  $a_-$  amplitudes interfere destructively for the  $K^{*-}\omega$  final state and constructively for the  $K^{*-}\rho$  final state, as we shall see in the numerical results below.

At this point we must specify our model by choosing the CKM [14] matrix elements, the current form factors, and the coefficients that define the effective weak hamiltonian. For the CKM matrix we choose the “low” and “high”  $f_B$  solutions of Schmidtler and Schubert [15]: (i)  $f_B = 125$  MeV,  $\rho = -0.41$ ,  $\eta = 0.18$  and (ii)  $f_B = 250$  MeV,  $\rho = 0.32$ ,  $\eta = 0.31$ . For the current form factors we use (i) those of BSW and (ii) for comparison as an alternative a modification suggested by the data of Anjos et al. [16] for  $D \rightarrow K^* l \nu$  decay, which is related by heavy quark symmetries [17] to the corresponding  $B$  decay, as measured by the  $B$  to  $K^*$  current matrix element. For this comparison we maintain the form factor  $a$  as given by BSW but choose  $b = 0$  and  $c$  is multiplied by  $2 A_1/V$  where  $A_1$  and  $V$  are the BSW form factors [3]. These are denoted “alternative” form factors in the tables and the text. For the weak hamiltonian coefficients we use the results quoted after eq. (2), presenting results for two cases: (i) with penguins and (ii) without penguins.

Concerning the QCD coefficients and how Fierz terms are treated, it is well known that this model has problems accounting for the decays with branching ratios which are proportional to  $a_-^2$  [18,19] because  $a_-$  has a rather small value  $|a_-| = 0.105$  for the QCD corrected short distance coefficients. There is a well known analogous effect in nonleptonic  $D$  decays [3]. Therefore several authors advocated the following modification of the short-distance QCD coefficients [3,26,21]: only terms which are dominant in the  $1/N_c$  expansion are taken into account. We use this leading  $1/N_c$  approximation as our model for evaluating the weak hamiltonian.

The results of our calculation are tabulated in tables 1 and 2. Except for the  $B^0 B^0$  mixing parameter  $\Delta F/\Gamma$  the positive and negative  $\rho$  solutions are quite similar. Penguin effects are very small in  $K^* \psi$  channels but very important in  $K^* \rho$  and  $K^* \omega$  channels, where they significantly enhance the rates and induce  $CP$  asymmetries. Penguins can interfere constructively or destructively with the other terms in these channels and have different effects in the  $\rho$  and  $\omega$  final states. The alternative form factors, which have a vanishing second axial vector form factor ( $b$ ) actually have *larger* rates because of a strong destructive interference for these amplitudes between the  $a$  and  $b$  ( $s$ -wave and  $d$ -wave) terms in the BSW form factors which contribute to the rate in the form  $(a + xb)$  where the kinematical variable  $x$  was defined below eq. (3). When, for instance, the BSW form factors are modified by the substitution  $b \rightarrow -b$ , the  $K^* \psi$  rate increases by a factor of three and the  $K^* \omega$  rate increases by a factor of 30.

The value of  $\Delta F/\Gamma$  should be compared with the  $\text{Im}(q/p) = -0.25$  for the negative  $\rho$  solution and  $\text{Im}(q/p) = -0.74$  for the positive  $\rho$  solution as a measure of the influence of spin effects on the mixing parameter. These spin effects are rather unimportant except for the very small rates without penguins in the  $\rho$  negative solution. In the latter case, the  $q/p$  phase is very different from the large phases of the helicity amplitudes, so that they have strong effects on the mixing asymmetry. This is no longer the case when QCD effects lift the suppression of this amplitude, producing helicity amplitudes which have relatively weak complex phases. It is also not the case in the negative  $\rho$  solution where the helicity amplitudes and  $q/p$  have approximately the same phase.

The branching ratio of  $\bar{B} \rightarrow K^* \psi$  in these models varies between 0.25% and 0.67%. Recent experimental results are [22]:  $(0.16 \pm 0.11)\%$  (ARGUS) and  $(0.13 \pm 0.09)\%$  (CLEO) for  $K^{*-} \psi$  and  $(0.11 \pm 0.05)\%$  (ARGUS) and  $0.14 \pm 0.06\%$  (CLEO) for  $\bar{K}^{*0} \psi$ . Thus our models are somewhat high when compared to central values but the BSW form factors are within experimental error. To be specific we continue to quote results in terms of the original model.

In the BSW form factor model, the  $K^* \psi$  transverse polarization is relatively high because, while the positive helicity amplitude is small, the negative helicity amplitude is comparable in size to the longitudinal amplitude.

Table 1

*B* meson decay parameters using CKM matrix with  $\rho$  positive solution;  $m_b$  scale solution for  $H_{\text{eff}}$ ,  $A_{\text{QCD}} = 0.2$ .

Channel	Br (%)	$\Delta\Gamma/\Gamma$	$\Gamma_T/\Gamma$	$\alpha_1$ [cos $\phi$ ]	$\alpha_2$ [cos $2\phi$ ]	$\beta_1$ ( $\times 10^{-4}$ ) [sin $\phi$ ]	$\beta_2$ ( $\times 10^{-4}$ ) [sin $2\phi$ ]
<i>BSW form factors – no penguins</i>							
$B^0 \rightarrow \bar{K}^{*0} + \psi$	0.248	-0.602	0.429	-0.621	0.123	-	-
$B^- \rightarrow K^{*-} + \psi$	0.249	-	0.428	-0.621	0.123	-	-
$B^0 \rightarrow \bar{K}^{*0} + \omega$	0.00000674	-0.652	0.0902	-0.319	0.0109	-	-
$B^0 \rightarrow \bar{K}^{*0} + \rho^0$	0.00000670	-0.653	0.0876	-0.315	0.0106	-	-
$B^- \rightarrow K^{*-} + \omega$	0.0000538	-	0.113	-0.339	0.00832	-	-
$B^- \rightarrow K^{*-} + \rho^0$	0.000159	-	0.102	-0.329	0.00929	-	-
<i>BSW form factors – with penguins</i>							
$B^0 \rightarrow \bar{K}^{*0} + \psi$	0.291	-0.603	0.429	-0.621	0.123	-	-
$B^- \rightarrow K^{*-} + \psi$	0.292	-	0.428	-0.621	0.123	-	-
$B^0 \rightarrow \bar{K}^{*0} + \omega$	0.00137	-0.753	0.0974	-0.326	0.0102	7.64	-0.738
$B^0 \rightarrow \bar{K}^{*0} + \rho^0$	0.0000896	-0.415	0.110	-0.337	0.00869	-90.9	7.94
$B^- \rightarrow K^{*-} + \omega$	0.000656	-	0.0934	-0.322	0.0105	85.4	-8.21
$B^- \rightarrow K^{*-} + \rho^0$	0.0000786	-	0.103	-0.330	0.00917	-99.9	8.78
<i>Alternative form factors – no penguins</i>							
$B^0 \rightarrow \bar{K}^{*0} + \psi$	0.571	-0.552	0.272	-0.462	0.0108	-	-
$B^- \rightarrow K^{*-} + \psi$	0.574	-	0.271	-0.461	0.0105	-	-
$B^0 \rightarrow \bar{K}^{*0} + \omega$	0.0000684	-0.685	0.0162	-0.104	-0.00257	-	-
$B^0 \rightarrow \bar{K}^{*0} + \rho^0$	0.0000685	-0.686	0.0156	-0.102	-0.00248	-	-
$B^- \rightarrow K^{*-} + \omega$	0.000640	-	0.0171	-0.104	-0.00310	-	-
$B^- \rightarrow K^{*-} + \rho^0$	0.00183	-	0.0162	-0.102	-0.00283	-	-
<i>Alternative form factors – with penguins</i>							
$B^0 \rightarrow \bar{K}^{*0} + \psi$	0.670	-0.553	0.272	-0.462	0.0108	-	-
$B^- \rightarrow K^{*-} + \psi$	0.674	-	0.271	-0.461	0.0105	-	-
$B^0 \rightarrow \bar{K}^{*0} + \omega$	0.0146	-0.796	0.0165	-0.104	-0.00275	1.16	-0.0615
$B^0 \rightarrow \bar{K}^{*0} + \rho^0$	0.00108	-0.477	0.0167	-0.102	-0.000301	-12.6	0.656
$B^- \rightarrow K^{*-} + \omega$	0.00687	-	0.0162	-0.104	-0.00265	13.0	-0.686
$B^- \rightarrow K^{*-} + \rho^0$	0.000908	-	0.0162	-0.102	-0.00286	-14.4	0.743

However, in the alternative form factor model the negative helicity amplitude is only half the longitudinal, reducing the polarization accordingly. The transitions to light mesons are all much more longitudinal but the effect persists that the alternative form factors reduce the transversality still further. Because all amplitudes are dominantly longitudinal, with the possible exception of  $K^*\psi$ ,  $\alpha_1$  and  $\beta_1$  dominate over  $\alpha_2$  and  $\beta_2$ . These results test the form factor assumptions

Preliminary data from ARGUS [22] on the exclusive decay  $B \rightarrow K^* + \psi$  indicate that the best fit to angular distributions is  $\Gamma_T/\Gamma = 0$  with a confidence level of 95% that this ratio is less than 0.22, whereas the BSW models predict a ratio of 0.43. This prediction depends heavily on the current matrix elements and not on the QCD coefficients. If this experimental result persists it will show that the BSW wave functions are not valid here and different matrix elements are needed, perhaps similar to those found by Anjos et al. [16] for  $\langle D|j^\mu|K^* \rangle$  matrix elements.

Let us finally turn in more detail to the decays  $B \rightarrow K^*\omega$  and  $B \rightarrow K^*\rho$  which are most interesting from the point of view of detecting direct  $CP$  violation through azimuthal asymmetries. We see from tables 1 and 2 that the sin  $\phi$ -term may be as large as  $10^{-2}$  (for  $B^- \rightarrow K^{*-}\omega$ ). Generally the asymmetries are at least as high as  $10^{-3}$ . The branching ratios however are modest, at best of the order of  $10^{-4}$  but the next generation of high statistics experiments may well start testing these asymmetries. Indeed, current experimental information

Table 2

*B* meson decay parameters using CKM matrix with  $\rho$  negative solution;  $m_b$  scale solution for  $H_{\text{eff}}$ ,  $A_{\text{QCD}} = 0.2$ .

Channel	Br (%)	$\Delta\Gamma/\Gamma$	$\Gamma_T/\Gamma$	$\alpha_1$ [cos $\phi$ ]	$\alpha_2$ [cos $2\phi$ ]	$\beta_1$ ( $\times 10^{-4}$ ) [sin $\phi$ ]	$\beta_2$ ( $\times 10^{-4}$ ) [sin $2\phi$ ]
<i>BSW form factors - no penguins</i>							
$B^0 \rightarrow \bar{K}^{*0} + \psi$	0.248	-0.202	0.429	-0.621	0.123	-	-
$B^- \rightarrow K^{*-} + \psi$	0.249	-	0.428	-0.621	0.123	-	-
$B^0 \rightarrow \bar{K}^{*0} + \omega$	0.00000681	0.509	0.0902	-0.319	0.0109	-	-
$B^0 \rightarrow \bar{K}^{*0} + \rho^0$	0.00000677	0.510	0.0876	-0.315	0.0106	-	-
$B^- \rightarrow K^{*-} + \omega$	0.0000544	-	0.113	-0.339	0.00832	-	-
$B^- \rightarrow K^{*-} + \rho^0$	0.000161	-	0.102	-0.329	0.00929	-	-
<i>BSW form factors - with penguins</i>							
$B^0 \rightarrow \bar{K}^{*0} + \psi$	0.290	-0.203	0.429	-0.621	0.123	-	-
$B^- \rightarrow K^{*-} + \psi$	0.291	-	0.428	-0.621	0.123	-	-
$B^0 \rightarrow \bar{K}^{*0} + \omega$	0.000988	-0.304	0.0984	-0.327	0.0101	6.24	-0.593
$B^0 \rightarrow \bar{K}^{*0} + \rho^0$	0.000174	-0.0951	0.103	-0.330	0.00933	-27.2	2.37
$B^- \rightarrow K^{*-} + \omega$	0.00131	-	0.0993	-0.328	0.00991	24.8	-2.39
$B^- \rightarrow K^{*-} + \rho^0$	0.000526	-	0.104	-0.331	0.00912	-8.66	-0.762
<i>Alternative form factors - no penguins</i>							
$B^0 \rightarrow \bar{K}^{*0} + \psi$	0.571	-0.185	0.272	-0.462	0.0108	-	-
$B^- \rightarrow K^{*-} + \psi$	0.574	-	0.271	-0.461	0.0105	-	-
$B^0 \rightarrow \bar{K}^{*0} + \omega$	0.0000691	0.535	0.0161	-0.104	-0.00256	-	-
$B^0 \rightarrow \bar{K}^{*0} + \rho^0$	0.0000692	0.535	0.0156	-0.102	-0.00248	-	-
$B^- \rightarrow K^{*-} + \omega$	0.000646	-	0.0171	-0.104	-0.00312	-	-
$B^- \rightarrow K^{*-} + \rho^0$	0.00185	-	0.0162	-0.102	-0.00283	-	-
<i>Alternative form factors - with penguins</i>							
$B^0 \rightarrow \bar{K}^{*0} + \psi$	0.667	-0.186	0.272	-0.462	0.0108	-	-
$B^- \rightarrow K^{*-} + \psi$	0.670	-	0.271	-0.461	0.0105	-	-
$B^0 \rightarrow \bar{K}^{*0} + \omega$	0.0106	-0.321	0.0166	-0.104	-0.00277	0.928	-0.0491
$B^0 \rightarrow \bar{K}^{*0} + \rho^0$	0.00200	-0.111	0.0164	-0.102	-0.00286	-3.95	0.205
$B^- \rightarrow K^{*-} + \omega$	0.0143	-	0.0165	-0.104	-0.00279	3.63	-0.191
$B^- \rightarrow K^{*-} + \rho^0$	0.00611	-	0.0163	-0.102	-0.00287	-1.24	0.0642

is on the threshold of testing the branching ratios, which differ considerably from model to model. Currently the following experimental limits for branching ratios have been reported:  $\text{Br}(B^0 \rightarrow K^{*0}\rho^0) < 4.6 \times 10^{-4}$  [23],  $6.7 \times 10^{-4}$  [24], compared to our values ranging as high as  $10^{-5}$  (alternative form factor, with QCD corrections.) There is also an upper limit for  $\text{Br}(B^+ \rightarrow K^{*+}\omega) < 1.3 \times 10^{-4}$  [23] to be compared with  $\text{Br}(B^- \rightarrow K^{*-}\omega) = 0.6 \times 10^{-4}$  (alternative form factors with QCD corrections.) Current experiments are clearly on the verge of testing form factor models and QCD models of the effective weak hamiltonian.

Better branching ratio limits for  $K^*\omega$  and  $K^*\rho$  as well as a verification of the reported low polarization in  $K^*\psi$  will provide very useful constraints on weak interaction parameters. The ultimate test, angular distributions, seem in the realm of possibility in the next generation of precision *B* meson physics.

#### Note added

After we had submitted this work it was brought to our attention that Grinstein [25] had calculated the QCD coefficients correctly and that Buras et al. [26] have recently also calculated the the next-to-leading order. Our results agree with those of Grinstein when we use the same input parameters. The higher order corrections of Buras et al. are for all practical purposes not needed at this point because of the uncertainty in the determination

of  $A_{\overline{MS}}$ . If these coefficients are evaluated with the same  $A_{\overline{MS}}$  as in this work the coefficients of the penguin operators are enhanced leading to larger  $CP$  asymmetries.

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