Direct CP asymmetries in the decays $B \to VV$ from an effective weak Hamiltonian

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(Received 30 January 1992)

We calculate QCD coefficients for the effective operators for B decays at the m_b and m_c scale and use them to calculate branching ratios, polarization and CP asymmetries in the decay of B mesons to two vector mesons. The numerical results are compared to experimental data.

PACS number(s): 13.25.+m, 11.30.Er, 14.40.Jz

I. INTRODUCTION

In previous work [1,2], we made a systematic study of the exclusive decay of B mesons to two vector mesons [3–5] with particular attention to polarization and CP asymmetries in the decays $B \to K^*\psi$, $K^*\omega$, and $K^*\rho$ [6,2]. The renormalization-group-improved effective Hamiltonian [7,8] was evaluated in the vacuum insertion approximation [9]. Okubo-Zweig-Iizuka-(OZI) suppressed and annihilation terms were neglected. Current matrix elements were evaluated using the wave functions of Bauer, Stech, and Wirbel [4] (BSW). Branching ratios and angular correlations among subsequent decays of the vector mesons were calculated for 36 channels. As a first approximation, the calculational scheme provided a useful framework with which to organize the data. We are currently improving the form factors by using heavy quark symmetries where they are applicable [10]. In this work we will improve the effective Hamiltonian by running the QCD evolution equations for current values of the top-quark mass and correcting a numerical error in the work of Ponce [8]. We then apply this effective Hamiltonian to 34 exclusive $B \to VV$ channels, calculating branching ratios, decay rate differences caused by $B^0\bar{B}^0$ mixing, and polarization and direct CP asymmetry parameters.

In the remainder of this introduction we briefly review the calculational scheme of Refs. [1,2]. In Sec. II, we describe our method for calculating the QCD coefficients and present out results for the renormalization. In Sec. III, we apply this Hamiltonian to the decays of B into two vector mesons.

After renormalization, the effective Hamiltonian for the $\Delta c=0$, $\Delta b=\Delta s=1$ processes is, following the notation of [1],

$$H_2^{\text{eff}} = -\frac{G}{2\sqrt{2}} \left\{ V_{cs}^* V_{cb}(c_+ O_{2+}^c + c_- O_{2-}^c) + V_{ts}^* V_{tb} \sum_{i=1}^{6} c_i O_i \right\} , \quad (1)$$

where

$$\begin{aligned} O_{2\pm}^q &= [(\bar{s}u)(\bar{u}b) \pm (\bar{s}b)(\bar{u}u)] - [(\bar{s}q)(\bar{q}b) \pm (\bar{s}b)(\bar{q}q)] , \\ O_1 &= (\bar{s}b)_L(\bar{u}u)_L , \quad O_4 = (\bar{s}_i b_j)_L \sum_q (\bar{q}_j q_i)_L , \\ O_2 &= (\bar{s}_i b_j)_L(\bar{u}_j u_i)_L , \quad O_5 = (\bar{s}\lambda^a b)_L \sum_q (\bar{q}\lambda^a q)_R , \\ O_3 &= (\bar{s}b)_L \sum_q (\bar{q}q)_L , \quad O_6 = (\bar{s}b)_L \sum_q (\bar{q}q)_R . \end{aligned} \quad (2)$$

The coefficients $c_\pm$, c_1, c_2, c_3, c_4, and linear combinations of c_5 and c_6 were calculated by Ponce [8] some years ago for values of the top-quark mass and the W-boson mass that are quite different from current data, although the coefficients do not depend sensitively on these inputs. More significantly, Ponce's calculation has a numerical error which overestimates the effects of QCD corrections in c_1 and c_2. We will repeat his calculation in Sec. II.

We use the notation $H_\lambda = \langle V_1(\lambda)V_2(\lambda)|H_{\text{wk}}^{\text{eff}}|\bar{B}^0\rangle$ for the helicity matrix element, $\lambda=0,\pm1$. These can be expressed by three invariant amplitudes a,b,c, defined by the decomposition

$$H_\lambda = \epsilon_{1\mu}(\lambda)^* \epsilon_{2\nu}(\lambda)^* \left[a g^{\mu\nu} + \frac{b}{m_1 m_2} p^\mu p^\nu + \frac{ic}{m_1 m_2} \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_\beta \right] , \quad (3)$$

where $p=p_1+p_2$ is the $\bar{B}^0$ four-momentum. Thus, $H_{\pm1} = a \pm \sqrt{x^2-1}c$ and $H_0 = -ax - b(x^2-1)$, where $p^2 = (m_1^2 m_2^2/m^2)(x^2-1)$. The helicity amplitudes $\bar{H}_\lambda$ for the decay of $B^0 \to \bar{V}_1\bar{V}_2$, where $\bar{V}_1$ and $\bar{V}_2$ are the antiparticles of V_1 and V_2, respectively, have the same decomposition as (3) with $a \to \bar{a}$, $b \to \bar{b}$, and $c \to -\bar{c}$. The coefficients a, b, and c describe the s-, d-, and p-wave contribution of the two final vector particles. They have phases δ from strong interactions and weak phases ϕ originating from the CP-violating phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. When there are no strong interaction phases, $\bar{a}=a^*$, $\bar{b}=b^*$, and $\bar{c}=c^*$. Since there is a sign change in front of $\bar{c}$ in $\bar{H}_\lambda$, we have, for the case of vanishing strong phases $\delta_i^{0,1,2}$, $\bar{H}_{\pm1} = H_{\mp1}^*$, $H_0 = H_0^*$.

The angular distributions depend on the spins of the decay products of the decaying vector mesons V_1 and V_2. For $B \to K^* \psi \to (K\pi)(e^+e^-)$, the differential decay distribution is

$$\frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} = \frac{p}{16\pi^2 m^2}\frac{9}{8} \times \{ \tfrac{1}{4}\sin^2\theta_1(1+\cos^2\theta_2)(|H_{+1}|^2+|H_{-1}|^2)+\cos^2\theta_1\sin^2\theta_2|H_0|^2 - \tfrac{1}{2}\sin^2\theta_1\sin^2\theta_2[\cos2\phi\,\mathrm{Re}(H_{+1}H_{-1}^*)-\sin2\phi\,\mathrm{Im}(H_{+1}H_{-1}^*)] - \tfrac{1}{4}\sin2\theta_1\sin2\theta_2[\cos\phi\,\mathrm{Re}(H_{+1}H_0^*+H_{-1}H_0^*)-\sin2\phi\,\mathrm{Im}(H_{+1}H_0^*-H_{-1}H_0^*)]\} . \tag{4}$$

In Eq. (4), θ_1 is the polar angle of the K momentum in the rest system of the K^* meson with respect to the helicity axis, i.e., the momentum $\mathbf{p}_1$. Similarly θ_2 and ϕ are the polar and azimuthal angle of the positron e^+ in the ψ rest system with respect to the helicity axis of the ψ; i.e., ϕ is the angle between the planes of the two decays $K^* \to K\pi$ and $\psi \to e^+e^-$ (or $\mu^+\mu^-$). The ratios Γ_T/Γ and Γ_L/Γ measure the amount of transversely (longitudinally) polarized K^* (or ψ). The decay distribution is parametrized by the coefficients

$$\frac{\Gamma_T}{\Gamma} = \frac{|H_{+1}|^2+|H_{-1}|^2}{|H_0|^2+|H_{+1}|^2+|H_{-1}|^2} ,$$

$$\frac{\Gamma_L}{\Gamma} = \frac{|H_0|^2}{|H_0|^2+|H_{+1}|^2+|H_{-1}|^2} ,$$

$$\alpha_1 = \frac{\mathrm{Re}(H_{+1}H_0^*+H_{-1}H_0^*)}{|H_0|^2+|H_{+1}|^2+|H_{-1}|^2} , \tag{5}$$

$$\beta_1 = \frac{\mathrm{Im}(H_{+1}H_0^*-H_{-1}H_0^*)}{|H_0|^2+|H_{+1}|^2+|H_{-1}|^2} ,$$

$$\alpha_2 = \frac{\mathrm{Re}(H_{+1}H_{-1}^*)}{|H_0|^2+|H_{+1}|^2+|H_{-1}|^2} ,$$

$$\beta_2 = \frac{\mathrm{Im}(H_{+1}H_{-1}^*)}{|H_0|^2+|H_{+1}|^2+|H_{-1}|^2} .$$

For a description of the applicability of this distribution to a variety of processes, and another distribution that applies to the case $B \to K^*\rho \to (K\pi)(\pi)$, see Ref. [1].

In general, the dominant terms in the angular correlations are Γ_T/Γ, Γ_L/Γ, α_1, and α_2. The terms β_1 and β_2 are small since they are nonvanishing only if the helicity amplitudes H_{+1}, H_{-1}, and H_0 or the invariant amplitudes a, b, and c, respectively, have different phases. When there are no strong interaction phases, the coefficients β_1 and β_2 are nonvanishing only through the CP-violating phase of the CKM matrix under the condition that they contribute differently to a, b, and c or H_{+1}, H_{-1}, and H_0, respectively.

Let the conjugate process amplitudes be denoted $\bar{H}_+$, etc., and the invariant amplitudes $\bar{a}_i$, etc., where i denotes the independent channel or process with the final-state interaction phase δ_i and weak phase ϕ_i. Then the interesting CP differences which do not require strong phases and are proportional to weak phase differences are

$$\mathrm{Im}(H_{+1}H_{-1}^*-\bar{H}_{+1}\bar{H}_{-1}^*) = -4\sqrt{x^2-1}\sum_{i,j}\cos(\delta_{si}-\delta_{pj})\sin(\phi_{si}-\phi_{pj})|a_i c_j| \tag{6}$$

and

$$\mathrm{Im}(H_{+1}H_{-1}^*-H_{-1}H_0^*-\bar{H}_{+1}\bar{H}_0^*+\bar{H}_{-1}\bar{H}_0^*) = -4(x^2-1)^{3/2}\sum_{i,j}\cos(\delta_{pi}-\delta_{dj})\sin(\phi_{pi}-\phi_{dj})|c_i b_j| - 4x\sqrt{x^2-1}\sum_{i,j}\cos(\delta_{pi}-\delta_{sj})\sin(\phi_{pi}-\phi_{sj})|c_i a_j| . \tag{7}$$

Terms of the first type, which are numerically small in our model, can be isolated by averaging over the polar angles and looking at the ϕ dependence of the difference distribution:

$$\frac{2\pi}{\Gamma}\frac{d\Gamma}{d\phi} - \frac{2\pi}{\bar{\Gamma}}\frac{d\bar{\Gamma}}{d\phi} = -(\alpha_2-\bar{\alpha}_2)\cos2\phi - (\beta_2-\bar{\beta}_2)\sin2\phi . \tag{8}$$

Terms of the second type can be isolated by examining the ϕ dependence of the difference distribution separated according to same-hemisphere (SH) events (e.g., $0<\theta_1$, $\theta_2<\pi/2$) or opposite-hemisphere (OH) events (e.g., $0<\theta_1<\pi/2$, $\pi/2<\theta_2<\pi$):

$$\frac{2\pi}{\Gamma}\left[\frac{d\Gamma^{\mathrm{OH}}}{d\phi}-\frac{d\Gamma^{\mathrm{SH}}}{d\phi}\right] - \frac{2\pi}{\bar{\Gamma}}\left[\frac{d\bar{\Gamma}^{\mathrm{OH}}}{d\phi}-\frac{d\bar{\Gamma}^{\mathrm{SH}}}{d\phi}\right] = -\tfrac{1}{2}\{(\alpha_1-\bar{\alpha}_1)\cos\phi-(\beta_1-\bar{\beta}_1)\sin\phi\} . \tag{9}$$

A different signature for CP violation is obtained when one considers neutral B mesons only. Then it is possible to generate interference via mixing by looking at final states that can occur from B^0 and $\bar{B}^0$ decays. In the case of common final states for $\bar{B}^0$ and B^0 decays, $\Delta\Gamma$, the difference $\Gamma(B^0(t)\to f)-\Gamma(\bar{B}^0(t)\to f)$ can be written in term of the factor [11]

$$\frac{\Delta\hat{\Gamma}}{\Gamma}=\frac{\text{Im}\{(q/p)[H_0^2+2H_{+1}H_{-1}]\}}{|H_0|^2+|H_{+1}|^2+|H_{-1}|^2}\ , \qquad (10)$$

where $\Delta\Gamma=\Delta\hat{\Gamma}\sin(\Delta m t)$. This factor depends on the mixing phase q/p and the weak phase of $H_0^2+2H_{+1}H_{-1}$. The difference between $\text{Im}(q/p)$ and $\Delta\hat{\Gamma}/\Gamma$ represents the influence of spin effects in the final state on the mixing factor. In some of the considered decays, H_0, H_{+1}, and H_{-1} have equal weak phases. Then $\Delta\Gamma/\Gamma$ is maximal if $H_{+1}=H_{-1}$ [12,13].

Amplitudes for 34 $B\to VV$ decays are written down in Ref. [1] in the vacuum saturation approximation. They depend on the hadron matrix elements of quark bilinears and the coefficients of the operator product expansion which define the effective Hamiltonian, discussed in the next section.

II. EFFECTIVE HAMILTONIAN

Ponce [8] has calculated the QCD coefficients at the m_b, m_c, and $m_0=1$-GeV scales and we follow his method closely with slightly altered notation. However, he made a numerical error when evaluating the parameter ξ_1 introduced below. The error is evident because his results do not preserve the constraints $c_\pm=(c_1\pm c_2)/2$. We can reproduce his results by changing the sign of ξ_1, as shown below.

At the weak scale, the $\Delta b=\Delta s=1$, $\Delta c=0$ weak Hamiltonian can be written

$$\begin{aligned}H_2&=-\frac{G}{\sqrt{2}}\{V_{cs}^*V_{cb}[(\bar{s}u)(\bar{u}b)-(\bar{s}c)(\bar{c}b)]\\&\qquad+V_{ts}^*V_{tb}[(\bar{s}u)(\bar{u}b)-(\bar{s}t)(\bar{t}b)]\}\\&=a_2(O_{2+}^c+O_{2-}^c)+a_2'(O_{2+}^t+O_{2-}^t)\ ,\end{aligned} \qquad (11)$$

where

$$a_2=-\frac{G}{\sqrt{2}}V_{cs}^*V_{cb}\ ,\quad a_2'=-\frac{G}{\sqrt{2}}V_{ts}^*V_{tb}\ .$$

The notation $(\bar{s}u)=(\bar{s}u)_L=\bar{s}\gamma_\mu(1-\gamma_5)u$ denotes a left-handed current. After renormalization, additional operators O_i are induced and the coefficients of the old operators are changed. These changes can be expressed in terms of the factors

$$\begin{aligned}\kappa_1&=1-\frac{B_1\alpha_s}{2\pi}\ln\frac{m_c}{\mu_0}\ ,\quad \kappa_2=1-\frac{B_2\alpha_s}{2\pi\kappa_1}\ln\frac{m_b}{m_c}\ ,\\ \kappa_3&=1-\frac{B_3\alpha_s}{2\pi\kappa_1\kappa_2}\ln\frac{m_t}{m_b}\ ,\quad \kappa_4=1-\frac{B_4\alpha_s}{2\pi\kappa_1\kappa_2\kappa_3}\ln\frac{m_W}{m_t}\ ,\end{aligned} \qquad (12)$$

which arise from running the couplings from $m_W\to m_t\to m_b\to m_c\to\mu_0$. The factors B_i are

$$B_i=\tfrac{2}{3}n_i-11\ ,\quad n_1=3,\ n_2=4,\ n_3=5,\ n_4=6\ .$$

We shall take $m_W=m_t$ in all cases except when we make contact with Ponce's work [8]. Thus, for the scale m_b, we will use $\kappa_1=\kappa_2=\kappa_4=1$ and

$$\kappa_3=1-\frac{B_3\alpha_s(m_b)}{2\pi}\ln\frac{m_W}{m_b}$$

and for the scale m_c we use $\kappa_1=\kappa_4=1$ and

$$\kappa_2=1-\frac{B_2\alpha_s(m_c)}{2\pi}\ln\frac{m_b}{m_c}\ ,\quad \kappa_3=1-\frac{B_3\alpha_s(m_c)}{2\pi\kappa_2}\ln\frac{m_W}{m_t}\ .$$

When there are five active flavors, the anomalous dimensions of the operators are

$$\gamma_\pm=\frac{C_\pm\alpha_s}{4\pi}\ ,$$

$$C_+=4\ ,\quad C_-=-8\ .$$

We drop the $(\bar{s}b)(\bar{t}t)$ terms because they are unimportant for B decays. Then the renormalized Hamiltonian at the m_b scale is

$$\begin{aligned}H_2^{\text{eff}}(m_b)&=a_2\{\kappa_4^{C_+/2B_4}\kappa_3^{C_+/2B_3}O_+^c+\kappa_4^{C_-/2B_4}\kappa_3^{C_-/2B_3}O_-^c\}\\&\quad+a_2'\sum_{l=1}^{2}\sum_{i,j=1}^{6}\xi_l R_{li}^{-1}K_iR_{ij}O_j\end{aligned} \qquad (13)$$

where the operators O_j are defined by

$$\begin{aligned}O_1&=(\bar{s}b)_L(\bar{u}u)_L\ ,\quad O_4=(\bar{s}_ib_j)_L\sum_q(\bar{q}_jq_i)_L\ ,\\ O_2&=(\bar{s}_ib_j)_L(\bar{u}_ju_i)_L\ ,\quad O_5=(\bar{s}b)_L\sum_q(\bar{q}q)_R\ ,\\ O_3&=(\bar{s}b)_L\sum_q(\bar{q}q)_L\ ,\quad O_6=(\bar{s}_ib_j)_L\sum_q(\bar{q}_jq_i)_R\ ,\end{aligned} \qquad (14)$$

and $q=u,d,s,c,b$. Note the definition of O_5 and O_6 is different from that used in Eq. (2) and Refs. [1,2,6]. The coefficients c_5 and c_6 in Eq. (2) are related to Ponce's definition $c_5^{(P)}$ and $c_6^{(P)}$ by $c_5=c_6^{(P)}/2$ and $c_6=c_5^{(P)}+c_6^{(P)}/6$.

The anomalous dimension matrix M and the matrix R that diagonalizes it are given in [8]. The factors ξ_i and K_i are

$$\begin{aligned}\xi_1&=\kappa_4^{C_+/2B_4}-\kappa_4^{C_-/2B_4}\ ,\\ \xi_2&=\kappa_4^{C_+/2B_4}+\kappa_4^{C_-/2B_4}\ ,\\ K_i&=\kappa_3^{V_i/B_3}\ ,\end{aligned} \qquad (15)$$

where V_i are the eigenvalues of M, given in [8].

In proceeding to the scale m_c, these operators are again renormalized and a new Hamiltonian can be written down involving the five active flavor factors written above as well as the corresponding factors when there are only four active flavors. These are the anomalous dimensions of the operators $O_\pm$ as well as the anomalous dimension M' in the basis of a new set of operators O_i', its eigenvalues, and the matrix R' that diagonalizes M'. The anomalous dimensions of $O_\pm$ are now given by

$$C_+'=2\ ,\quad C_-'=-4\ .$$

The new operators are given by

TABLE I. QCD coefficients for $\Delta b=1$ at $\mu=m_b=5.0$ GeV.

Λ_5	c_+	c_-	c_1	c_2	c_3	c_4	c_5	c_6
0.10	0.8862	1.2732	−0.3870	2.1595	0.0167	−0.0404	0.0122	−0.0474
0.15	0.8777	1.2982	−0.4206	2.1759	0.0183	−0.0437	0.0131	−0.0520
0.20	0.8707	1.3192	−0.4485	2.1899	0.0197	−0.0464	0.0138	−0.0559
0.25	0.8646	1.3378	−0.4733	2.2024	0.0210	−0.0488	0.0144	−0.0594
0.30	0.8591	1.3549	−0.4958	2.2140	0.0221	−0.0509	0.0150	−0.0626
0.35	0.8541	1.3710	−0.5169	2.2250	0.0232	−0.0529	0.0155	−0.0656
0.40	0.8493	1.3862	−0.5369	2.2356	0.0242	−0.0548	0.0160	−0.0685

$$\begin{aligned}
&O_1'=(\bar{s}b)_L(\bar{u}u)_L\ , \quad O_4'=(\bar{s}_ib_j)_L\sum_q(\bar{q}_jq_i)_L\ ,\\
&O_2'=(\bar{s}_ib_j)_L(\bar{u}_ju_i)_L\ , \quad O_5'=(\bar{s}b)_L\sum_q(\bar{q}q)_R\ ,\\
&O_3'=(\bar{s}b)_L\sum_q(\bar{q}q)_L\ , \quad O_6'=(\bar{s}_ib_j)_L\sum_q(\bar{q}_jq_i)_R\ ,\\
&O_7'=\tfrac{1}{2}(\bar{s}\lambda^ab)_L(\bar{b}\lambda^ab)_L+\tfrac{1}{2}(\bar{s}\lambda^ab)_L(\bar{b}\lambda^ab)_R\ ,
\end{aligned} \tag{16}$$

where $q=u,d,s,c$.

In terms of these operators, the renormalized Hamiltonian at the m_c scale is

$$H_2^{\rm eff}(m_c)=a_2\{\epsilon O_+^c+\epsilon^{-1/2}O_+^c\}+a_2'\sum_{l=1}^{2}\sum_{i,j=1}^{6}\sum_{s,t=1}^{7}\xi_lR_{li}^{-1}K_iR_{ij}R_{js}'K_s'R_{st}'O_t'\ , \tag{17}$$

where

$$K_s'=\kappa_2^{V_s'/B_2}$$

and

$$\epsilon=\kappa_4^{C_-/2B_4}\kappa_3^{C_-/2B_3}\kappa_2^{C_-'/2B_2}\ .$$

The renormalization procedure preserves the constraint $c_\pm=(c_1\pm c_2)/2$. In Ref. [8], the equations are correct as written but a sign error was made in evaluating them which violates this constraint. We have solved the equations using Ponce's inputs: $m_W=100$ GeV, $m_t=30$ GeV, $m_b=5$ GeV, $m_c=2$ GeV, and $\Lambda_3=500$ MeV, using the lowest-order formula for α_s with three flavors. When the factor ξ_1 is reversed in sign, the results of [8] at the scale m_c are retrieved: $c_+=0.7737$, $c_-=1.6707$, $c_1=-0.4447$, $c_2=2.3169$, $c_3=0.0373$, $c_4=-0.0842$, $c_5=0.0244$, $c_6=-0.1061$, and $c_7=-0.0332$. When the sign error is corrected, the new results for Ponce's inputs are $c_+=0.7737$, $c_-=1.6707$, $c_1=-0.8970$, $c_2=2.4444$, $c_3=0.0384$, $c_4=-0.0870$, $c_5=0.0253$, $c_6=-0.1094$, and $c_7=-0.0350$. As one can see, the new evaluation obeys the constraints $c_\pm=(c_1\pm c_2)/2$. The small coefficients c_3–c_7 are only slightly affected by the error. Thus, Ponce's error overestimates only the QCD corrections in c_1 and c_2. When quoting results for coefficients of the operators O_i' we omit O_7' involving $\bar{s}b\bar{b}b$ quarks not relevant to this work. The coefficients, when suitably interpreted, are valid for all $\Delta b=1$ processes, as discussed by Ponce [8].

In addition to the error in Ponce's calculation, the QCD coefficient used in [1] should be modified since $m_t>m_W$ and also the QCD Λ value has changed. The more modern values of Λ come from recent data [14] from the CERN e^+e^- collider LEP. They are obtained by fitting experimental data with higher-order QCD perturbation theory [up to $O(\alpha_s^2)$]. It is clear that, to be strictly consistent, we should also evaluate the QCD coefficients $c_\pm,c_1,c_2,\ldots,c_6$ in second-order QCD. Actually, these coefficients were quite recently calculated by Buras *et al.* [15]. However, at this point we do not need to use this much more complicated formalism since $\Lambda_{\overline{\rm MS}}$ is not as yet sufficiently well known, where MS denotes the modified minimal subtraction scheme; Hebekker [14] quotes $\Lambda_{\overline{\rm MS}}=(250^{+110}_{-80})$ MeV. As one can see from the results in [15], the differences in the coefficients due to higher-order terms are comparable to the differences due to the uncertainty on $\Lambda_{\overline{\rm MS}}$.

To be specific, we evaluated new QCD coefficients with the leading-order formalism of Ponce with $m_t=m_W=81$ GeV, $m_b=5$ GeV, $m_c=1.7$ GeV, and as a function of $\Lambda_{\overline{\rm MS}}^{(5)}$ when we used m_b as the renormalization scale, and

TABLE II. QCD coefficients for $\Delta b=1$ at $\mu=m_c=1.7$ GeV.

Λ_4	c_+	c_-	c_1	c_2	c_3	c_4	c_5	c_6
0.10	0.8890	1.2654	−0.3764	2.1544	0.0189	−0.0470	0.0144	−0.0539
0.15	0.8815	1.2870	−0.4055	2.1685	0.0205	−0.0505	0.0154	−0.0585
0.20	0.8754	1.3048	−0.4293	2.1802	0.0219	−0.0534	0.0162	−0.0624
0.25	0.8703	1.3204	−0.4501	2.1907	0.0231	−0.0559	0.0169	−0.0658
0.30	0.8656	1.3346	−0.4690	2.2002	0.0242	−0.0581	0.0175	−0.0689
0.35	0.8614	1.3477	−0.4864	2.2091	0.0253	−0.0602	0.0181	−0.0718
0.40	0.8574	1.3602	−0.5027	2.2176	0.0262	−0.0622	0.0186	−0.0746

as a function of $\Lambda_{\overline{\rm MS}}^{(4)}$ for the m_c scale where α_s was always calculated with the higher-order formula of [16]. The results for the m_b scale for various values of $\Lambda_{\overline{\rm MS}}^{(5)}=\Lambda_5$ are in Table I and for the m_c scale for various values of $\Lambda_{\overline{\rm MS}}^{(4)}=\Lambda_4$ are in Table II. One can check that $c_\pm=(c_1 \pm c_2)/2$ is always satisfied; we list $c_\pm$ only as a convenience. As one can see, even going to the smaller scale at $\mu=m_c$ does not change the coefficients drastically, taking into account that $\Lambda_5=0.25$ GeV corresponds to $\Lambda_4=0.35$ GeV. $c_\pm$ are unaffected while the mixing coefficients c_3-c_6 are increased at the lower scale by about 20%. Our results in Table I may also be compared with those in Ref. [15], taking into account differences in operator definition and the difference between $\Lambda_{\overline{\rm MS}}^{(4)}$ and $\Lambda_{\overline{\rm MS}}^{(5)}$. The results of Buras *et al.* [15] at $\Lambda_{\overline{\rm MS}}^{(4)}=0.20$ closely match ours at $\Lambda_{\overline{\rm MS}}^{(5)}=0.35$. This difference is well within the error of current determinations of $\Lambda_{\overline{\rm MS}}$.

III. RESULTS FOR $B \rightarrow VV$

At this point we must specify our model by choosing the CKM matrix elements and the current form factors. For the CKM matrix we choose the "low" and "high" f_B solutions of Schmidtler and Schubert [17]: (i) $f_B=125$ MeV, $\rho=-0.41$, $\eta=0.18$, and (ii) $f_B=250$ MeV, $\rho=0.32$, $\eta=0.31$. For the current form factors we use those of BSW. Concerning the QCD coefficients and how Fierz terms are treated, it is well known that this model has problems accounting for the decays with branching ratios which are proportional to a_-^2 [18,19] because a_- has a rather small value $|a_-|=0.13$ for the QCD corrected short-distance coefficients. Here we use the notation

$$a_\pm=\tfrac{1}{2}(c_+ \pm c_-)+\frac{1}{2N_c}(c_+ \mp c_-)\,, \tag{18}$$

where N_c is the number of colors. There is a well-known analogous effect in nonleptonic D decays [4]. Therefore, several authors advocated the following modification of the short-distance QCD coefficients [4,20,21]: only terms which are dominant in the $1/N_c$ expansion are taken into account. We use this leading $1/N_c$ approximation as our model for evaluating the weak Hamiltonian. The results of this model are presented in Tables III and IV.

Much of the data for these rates has been discussed in Ref. [1]. Since then we have improved the effective Hamiltonian, corrected for the error in Ponce's calculation, and used the m_b scale with $\Lambda_5=0.25$ [14]. These changes have resulted generally in somewhat smaller branching ratios and somewhat higher *CP* asymmetries. Six of the channels have measured branching ratios which are compared to our calculation in Table V. All calculated rates are within the experimental errors, except the ψK^* channels which are a bit high. In calculating these branching

TABLE III. $\Lambda_5=0.25$ QCD coefficients *without* Fierz terms.

Channel	$B(\%)$	$\frac{\Delta\Gamma}{\Gamma}$	$\frac{\Gamma_T}{\Gamma}$	α_1 ($\cos\phi$)	α_2 ($\cos2\phi$)
		ρ positive CKM matrix			
Channels without penguin diagrams: $\Delta s=0$, $\Delta b=-\Delta c=1$ (H_1)					
$\bar{B}^0 \rightarrow \omega+D^{*0}$	0.0184	−0.586	0.284	−0.511	0.040
$\bar{B}^0 \rightarrow \rho^0+D^{*0}$	0.0188	−0.588	0.280	−0.509	0.039
$\bar{B}^0 \rightarrow \rho^-+D^{*+}$	1.26	−0.702	0.126	−0.425	0.040
$B^- \rightarrow \rho+D^{*0}$	0.875		0.102	−0.399	0.038
Channels without penguin diagrams: $\Delta b=\Delta s=-\Delta c=1$ (H_4)					
$\bar{B}^0 \rightarrow K^{*-}+D^{*+}$	0.0679	−0.694	0.161	−0.471	0.052
$\bar{B}^0 \rightarrow \bar{K}^{*0}+D^{*0}$	0.00230	−0.593	0.304	−0.537	0.055
$B^- \rightarrow K^{*-}+D^{*0}$	0.0456		0.135	−0.449	0.049
Channels without penguin diagrams: $\Delta b=\Delta c=\Delta s=1$ (H_5)					
$\bar{B}^0 \rightarrow \bar{K}^{*0}+\bar{D}^{*0}$	0.000434	0.526	0.304	−0.537	0.055
$B^- \rightarrow K^{*-}+\bar{D}^{*0}$	0.000443		0.304	−0.537	0.055
$\bar{B}^0 \rightarrow \rho^++D_s^{*-}$	0.00836	0.519	0.292	−0.518	0.043
$B^- \rightarrow \rho^0+D_s^{*-}$	0.0048		0.292	−0.518	0.043
Channels without penguin diagrams: $\Delta s=0$, $\Delta b=\Delta c=1$ (H_6)					
$\bar{B}^0 \rightarrow \rho^++\bar{D}^{*-}$	0.000407	−0.558	0.281	−0.509	0.040
$\bar{B}^0 \rightarrow \rho^0+\bar{D}^{*0}$	0.00000938	−0.558	0.280	−0.509	0.039
$\bar{B}^0 \rightarrow \omega+\bar{D}^{*0}$	0.00000916	−0.557	0.284	−0.511	0.040
$B^- \rightarrow \rho^-+\bar{D}^{*0}$	0.0000187		0.280	−0.509	0.040
$B^- \rightarrow \rho^0+D^{*-}$	0.0000169		0.281	−0.509	0.040

TABLE IV. $\Lambda_5=0.25$ QCD coefficients *without* Fierz terms.

Channel	B (%)	ρ positive CKM matrix $\frac{\Delta\Gamma}{\Gamma}$	$\frac{\Gamma_T}{\Gamma}$	α_1 ($\cos\phi$)	α_2 ($\cos2\phi$)	β_1 (10^{-4}) ($\sin\phi$)	β_2 (10^{-4}) ($\sin2\phi$)
	Channels with penguin diagrams: $\Delta c=0$, $\Delta b=\Delta s=1$ (H_2)						
$\bar{B}^0\to\bar{K}^{*0}+\omega$	0.000 883	−0.760	0.099	−0.327	0.010	10.5	−0.997
$\bar{B}^0\to\bar{K}^{*0}+\rho^0$	0.000 078 3	−0.417	0.110	−0.337	0.009	−90.7	7.91
$\bar{B}^0\to K^{*-}+\rho^+$	0.000 118	−0.910	0.106	−0.333	0.009		
$\bar{B}^0\to\bar{K}^{*0}+\psi$	0.241	−0.603	0.429	−0.621	0.123		
$\bar{B}^0\to D^{*+}+D_s^{*-}$	2.01	−0.655	0.477	−0.665	0.184		
$B^-\to K^{*-}+\omega$	0.000 348		0.094	−0.323	0.010	117	−11.2
$B^-\to K^{*-}+\rho^0$	0.000 077 9		0.103	−0.330	0.009	−89.4	7.86
$B^-\to K^{*-}+\psi$	0.242		0.428	−0.621	0.123		
$B^-\to D^{*0}+D_s^{*-}$	2.01		0.477	−0.664	0.183		
	Channels with penguin diagrams: $\Delta s=\Delta c=0$, $\Delta b=1$ (H_3)						
$\bar{B}^0\to\omega+\rho^0$	0.000 006 68	0.051	0.084	−0.300	0.007	77.5	−0.574
$\bar{B}^0\to\omega+\psi$	0.004 71	−0.592	0.394	−0.599	0.099		
$\bar{B}^0\to\rho^0+\rho^0$	0.000 091 3	−0.650	0.084	−0.298	0.007		
$\bar{B}^0\to\rho^0+\psi$	0.005 25	−0.574	0.388	−0.597	0.097		
$\bar{B}^0\to\omega+\omega$	0.000 080 2	0.360	0.087	−0.303	0.007		
$\bar{B}^0\to\rho^++\rho^-$	0.004 07	−0.696	0.084	−0.298	0.007		
$\bar{B}^0\to D^{*+}+D^{*-}$	0.105	−0.655	0.456	−0.660	0.172		
$B^-\to\omega+\rho^-$	0.001 36		0.084	−0.299	0.007	3.90	−0.029 0
$B^-\to\rho^0+\rho^-$	0.001 27		0.084	−0.299	0.007	0.015	−0.000 675
$B^-\to\rho^-+\psi$	0.010 5		0.389	−0.597	0.097		
$B^-\to D^{*0}+D^{*-}$	0.102		0.456	−0.660	0.172		

fractions we have used the LEP average for the B lifetime, 1.3 ps [22]. Unfortunately, the accuracy of the data is not good enough to get information about the QCD coefficients and/or current matrix elements. We point out, though, that the theoretical branching ratio of $B^-\to\rho^-D^{*0}$ is 30% smaller than the branching ratio for $\bar{B}^0\to\rho^-D^{*+}$. This is the effect of the destructive interference of a_+ and a_- in the B^- decay [4]. The branching ratios of the $K^*\psi$ final states are almost consistent with the data. Compared to our earlier work [1,2], this comes from the smaller value of a_- which results from the new QCD coefficients in Table I.

As remarked in Refs. [1,2], this model cannot fit the data on $B\to K^*\psi$ polarization. In the BSW form factor model, the $K^*\psi$ transverse polarization is relatively high because, while the positive helicity amplitude is small, the negative helicity amplitude is comparable in size to the longitudinal amplitude. These results test the form-factor assumptions. Data from ARGUS [23] on the exclusive decay $B\to K^*+\psi$ indicate that the best fit to angular distributions is $\Gamma_T/\Gamma=0$ with a confidence level of 95% that this ratio is less than 0.22, whereas the BSW models predict a ratio of 0.43. This prediction depends heavily on the current matrix elements and the factorization assumption but not on the QCD coefficients. The only way to accommodate this result within the factorization approximation is for the second (d-wave) axial-vector-current form factor to dominate over the first axial-vector form factor (s wave) and the vector form factor. This is in conflict with all quark models including those based on heavy quark symmetries as well as with experiments on corresponding form factors in semileptonic D decays and would indicate failure of the vacuum saturation assumptions or very significant additional contributions from annihilation terms.

Finally, let us turn to the decays $B\to K^*\omega$ and $B\to K^*\rho$ which are most interesting from the point of view of detecting direct CP violation through azimuthal

TABLE V. Comparison of calculated branching fractions (%) with data.

Channel	$\Lambda=0.25$, ρ positive CKM matrix This work	ARGUS	CLEO	Average
$\bar{B}\to\rho^-+D^{*+}$	1.26	0.7±0.4	1.9±1.6	0.8±0.4
$B^-\to\rho^-+D^{*0}$	0.875	1.0±0.7		1.0±0.7
$\bar{B}^0\to\bar{K}^{*0}+\psi$	0.241	0.11±0.05	0.14±0.06	0.12±0.04
$B^-\to K^{*-}+\psi$	0.242	0.16±0.11	0.13±0.09	0.14±0.07
$\bar{B}^0\to D^{*+}+D_s^{*-}$	2.01	2.6±1.4±0.6		2.6±1.4±0.6
$B^-\to D^{*0}+D_s^{*-}$	2.01	3.1±1.6±0.5		3.1±1.6±0.5

asymmetries. Such direct ($\Delta b = 1$) *CP* asymmetries are present in charged as well as neutral *B* decays and could clearly distinguish between a standard model and a superweak model *CP* violation. As emphasized by Winstein [24], it is very difficult to distinguish experimentally between these two models from $B^0\bar{B}^0$ mixing asymmetries. Detection of direct *CP* violation in charged *B* decays may well be the best way of ruling out a superweak model. We see from Table IV that the *CP*-odd $\sin\phi$ term may be as large as 10^{-2} for $\bar{B}^0 \rightarrow \bar{K}^{*0}\rho^0$, $B^- \rightarrow K^{*-}\omega$, and $B^- \rightarrow K^{*-}\rho^0$. The branching ratios, however, are rather small, at best of the order of 10^{-5}, but the next generation of high statistics experiments may well start testing the models. Currently the following experimental limits for branching ratios have been reported: $B(B^0 \rightarrow K^{*0}\rho^0) < 4.6 \times 10^{-4}$ [25], 6.7×10^{-4} [26]. There is also an upper limit for $B(B^+ \rightarrow K^{*+}\omega) < 1.3 \times 10^{-4}$ [25]. Clearly much better statistics will be needed to extract an angular asymmetry of 10^{-2} from the data.

ACKNOWLEDGMENTS

G.K. would like to acknowledge the hospitality of the Ohio State University. W.F.P. thanks the Desy Theory Group for its kind hospitality and the North Atlantic Treaty Organization for a Travel Grant. W.F.P. was supported in part by the U.S. Department of Energy under Contract No. DE-AC02-76ERO1545. G.K. was supported by Bundesministerium für Forschung und Technologie, 05 4HH92P/3, Bonn, FRG.

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