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$Z^0 \to d_i\bar{d}_j$ *CP* asymmetry in a model with an extra vector-like singlet quark

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We study the *CP* asymmetry $Z^0 \to d_i\bar{d}_j, d_jd_i$ in the context of models with an extra heavy vector-like singlet D quark. We show that values for the asymmetry much larger than the standard model predictions can be obtained and possibly checked at LEP with upgraded luminosity or next future high energy colliders.

Recently much theoretical effort has been devoted to understand the origin and mechanism of *CP* violation. From the experimental point of view the experiments NA31 at CERN, and E731 at Fermilab, have not yet given an illuminating result on the K^0–$\bar{K}^0$ system – the only one where *CP* violation has been detected up to now.

Looking for *CP* effects out of the K^0–$\bar{K}^0$ system – maybe in the B^0–$\bar{B}^0$ system, with data from B-factories, or through experiences at LEP or other high energy machines – is becoming crucial to check the standard model (SM). Predictions of models beyond the standard one must then be compared with data in order to search for new sources of *CP* violation.

In this work we will study *CP* violation at the Z^0 peak, looking at its flavour changing neutral current (FCNC) decay products $Z^0 \to d_i\bar{d}_j$.

In the standard model (SM), due to GIM mechanism, this kind of processes are forbidden at tree level, so that looking for *CP* violation in FCNC channels [1,2] turns out to be a doubly suppressed observable [#1]. From the diagrammatic point of view, one can get a *CP* violating observable in these processes, from the interference of two one-loop amplitudes. Consequently the effect will be suppressed by α^2, by the usual GIM factors and by the necessary and sufficient conditions (NSCs) to have *CP* violation operating in this process [1,3]; in our case the NSCs imply the presence of a complex phase, some angles and the non degeneracy of the masses of any pair of up-type quarks. Thus, in the SM, the effects are outrageously small.

One possibility to increase the effect is to introduce FCNC at tree level. This can be implemented in several ways, but specially interesting are those models that can be viewed as an effective low energy limit of a supersymmetric or string unification model. Following this suggestion one can add, for instance, extra doublets of Higgs, as suggested by supersymmetric SM models, but fermions tend to couple proportional to their masses and this could end up in a big suppression. Another possibility – naturally realized in E_6 and string extended models – is to extend the fermion sector. Obviously, in any case, one has to put severe restrictions on the tree level FCNC couplings according to present experimental data.

In this letter we follow the last strategy. We will accommodate a new heavy vector-like $SU(2)_L$ singlet in the quark sector, in addition to the minimal content of fermions of the SM. Left and right-handed

[#1] Nevertheless, from the experimental point of view, sometimes it is better to start with a rare process to look for tiny effects.

components of vector-like fermions transform in the same way under the $SU(2)_L \otimes U(1)$ gauge group of the theory, so Dirac mass terms are allowed and their corresponding masses are basically unbounded.

Models with vector-like fermions have been profusely studied [4] and it has been shown that, in addition, they can introduce new CP phases [5] associated with the extra fermions. These new CP phases and FCNC – also present in the model – are usually suppressed by factors of order m/M, being m the scale of the standard fermions and M the mass of the new vector-like fermion, in such a way that predictions of the model do not upset present experimental data but they can still leave place for effects of "new physics" at present e^+e^- collider energies.

We will concentrate on models with an extra vector-like singlet D-quark with quantum number of a d-type quark. From the phenomenological point of view, one of the most economical ansatz for the structure of the mass matrix in such models was proposed in ref. [6]. The simple assumption that, in absence of mixing with the new D-quark, the mass eigenstates of the three standard quark generations must coincide with the electroweak ones, has turned out to be quite attractive, leading to very interesting phenomenological predictions both at high and low energies (inversion of the empirical regularity $m_t > m_b$, $m_c > m_s$ for the u and d quarks and new structure of the mixing matrix for light quarks, among others).

Nevertheless we will study the most general model where one extra vector-like singlet D-quark has been added to the SM. Without loss of generality [5], *up* and *down* quark mass matrices (M_u and M_d) can be chosen as

$$M_d = \begin{array}{c} \\ \bar{d}_L \\ \\ \bar{D}_L \end{array} \begin{array}{c} \begin{array}{cccc} & d_R & & D_R \end{array} \\ \begin{pmatrix} m_d^0 & 0 & 0 & m'_1 \\ 0 & m_s^0 & 0 & m'_2 \\ 0 & 0 & m_b^0 & m'_3 \\ 0 & 0 & 0 & M \end{pmatrix} \end{array}, \qquad (1)$$

where M is the mass of the heavy D quark, m'_i give the mixing with the light quarks; and M_u is a general complex 3×3 matrix $M_u = (m_{ij})$ $(i,j=1,2,3)$.

Diagonalization can be then carried out in two steps:

– Using standard procedures we first diagonalize the 3×3 M_u matrix. As in the SM, one of the matrices needed to put M_u in a diagonal form will be the usual Cabibbo–Kobayashi–Maskawa matrix $(\mathrm{CKM})_{ij}$, present in the charged current sector of the SM lagrangian.

– Next, diagonalization of the 4×4 M_d mass matrix (1) is performed perturbatively (in powers of $m'_i/M \ll 1$) assuming that m'_i terms – which break SU(2) – must be suppressed with respect to the mass M of the singlet D-quark. In this way, only mass ratios appear in the expressions and agreement with present data can be obtained for a wide range of scales in (1).

Applying the described procedure, the relevant pieces of the light sector of the lagrangian are

$$\mathcal{L} = \frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_{Li}\gamma^\mu d_{Lj} V_{ij} + \text{h.c.}$$
$$+ \frac{g}{2\cos\theta_w} Z_\mu (\bar{u}_{Li}\gamma^\mu u_{Li} - B_{ij}\bar{d}_{Li}\gamma^\mu d_{Lj}$$
$$- 2\sin^2\theta_w J^\mu_{\text{e.m.}}) . \qquad (2)$$

The mixing matrix V_{ij} $(i,j=1,2,3)$, which is no longer unitary, is given by

$$V_{ij} = \sum_{k=1}^{3} (\mathrm{CKM})_{ik} U^{\dagger}_{kj} , \qquad (3)$$

with U_{kj} being the 3×3 sector of the 4×4 unitary matrix U, which diagonalizes M_d (D(diagonal)$= UM_dU'^{+}$). The standard phenomenology involving charged currents can be immediately reproduced by the model just noticing that the 3×3 sector of the U matrix is diagonal up to first order in powers of (m'_i/M) [5]:

$$U_{kj} = \delta_{kj} + \mathrm{O}(m'_k m'^*_j/M^2) \quad (i,j=1,2,3) .$$

The FCNC couplings B_{ij} $(i\neq j)$ are also given as a function of the same diagonalizing matrix as

$$B_{ij} = \delta_{ij} - U_{i4}U^*_{j4} , \qquad (4)$$

with $U_{i4} = -m'_i/M$, and satisfy

$$(V^+V)_{ij} = B_{ij} . \qquad (5)$$

FCNC B_{ij} couplings have been constrained combining data on K^0–$\bar{K}^0$, B^0–$\bar{B}^0$ systems, ..., for a wide m_t range, and following different strategies [7]. Here we address the problem of CP violation in the way of ref. [8]: the bounds on the B_{ij} couplings are obtained requiring that their contribution to FCNC processes is not larger than the experimental values. Then, using the most recent results from UA1 [9] and BNL [#2], and taking into account the bounds on mixing angles, one can get the following results [11]:

$$|B_{ds}| \leqslant 2.3\times 10^{-4}, \quad |B_{db}| \leqslant 1.8\times 10^{-3},$$
$$|B_{sb}| \leqslant 1.8\times 10^{-3}. \tag{6}$$

Let us consider the process $Z^0 \to d_i \bar{d}_j$ with $i \neq j$. In the minimal SM one has to compute ten one-loop diagrams (in the 't Hooft–Feynman gauge) [1,12] as responsible for the flavour-changing decay of the Z^0. From this calculation, the values of the BRs for the different channels are

$$\mathrm{BR}^{(\mathrm{SM})}(Z^0 \to d\bar{s}) \sim (0.2\text{–}4)\times 10^{-12},$$
$$\mathrm{BR}^{(\mathrm{SM})}(Z^0 \to b\bar{d}) \sim (0.8\text{–}18)\times 10^{-10},$$
$$\mathrm{BR}^{(\mathrm{SM})}(Z^0 \to b\bar{s}) \sim (0.2\text{–}4)\times 10^{-8}, \tag{7}$$

for m_t in the range [90, 210] GeV [13].

In the model (2), the decay is allowed at tree level – due to the FCNC term B_{ij} (fig. 1a) – and the width due to this term can be then immediately calculated from (6):

$$\mathrm{BR}^{(0)}(Z^0 \to d\bar{s}) \leqslant 1.0\times 10^{-8},$$
$$\mathrm{BR}^{(0)}(Z^0 \to b\bar{d}) \leqslant 6.2\times 10^{-7},$$
$$\mathrm{BR}^{(0)}(Z^0 \to b\bar{s}) \leqslant 6.2\times 10^{-7}. \tag{8}$$

We see therefore that present data are still compatible with the assumption that flavour-changing decay of the Z^0 to d-type quarks can be dominated by tree level FCNC.

Going back to the *CP* problem, we know that to get a non-zero value for *CP* asymmetry,

$$a_{ij}^{CP} = \frac{\Gamma(Z^0 \to d_i \bar{d}_j) - \Gamma(Z^0 \to \bar{d}_i d_j)}{\Gamma(Z^0 \to d_i \bar{d}_j) + \Gamma(Z^0 \to \bar{d}_i d_j)}, \tag{9}$$

one has to compute the Z^0 flavour-changing decay, at one-loop (at least) in perturbation theory, in order to get an absorptive piece. In this way, the *CP* non-conserving effects will arise from the interference of amplitudes with different weak phases and absorptive parts.

[#2] The BNL E-791 experiment reports the following rare K_L decay result: $\mathrm{BR}(K_L \to \mu\mu) = (7.0 \pm 0.82)\times 10^{-9}$ [10].

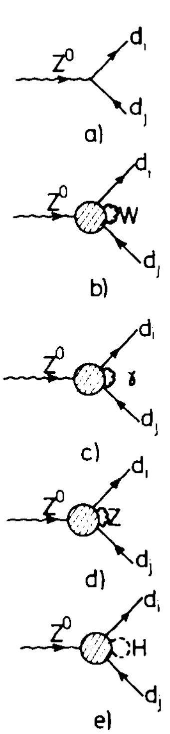

Fig. 1. Diagrams contributing to $Z^0 \to d_i \bar{d}_j$. The diagram 1a represents the tree level contribution.

In our case, the diagrams we have to consider are shown in fig. 1. Contrary to the SM case we have a tree diagram (fig. 1a) and a huge amount of one loop diagrams: fig. 1b represents all possible one loop diagrams with an internal W interchanged in all possible ways, the same happens for figs. 1c, 1d and 1e interchanging the role of the W by Z, γ and H respectively. In the limit that the mass of the vector-like quarks go to infinity, the new D-quark decouples [14] and we get the standard model. In fig. 1 this decoupling is realized in the following way: in the limit $M \to \infty$ all flavour changing neutral vertices go to zero, so diagrams 1a, 1c, 1d and 1e vanishes at the same time that diagram 1b reduces to the wellknown SM contribution.

In order to explain our calculation let us fix the order of magnitude of all the diagrams. Looking at eqs. (7) and (8) it is clear that diagram 1a will dominate at least over the SM contribution in diagram 1b. Therefore, provided we have appropriate weak phases we will get an asymmetry from the interference of a tree level with a one loop diagram, contrary to the

SM where the CP asymmetry is obtained from the interference of two one loop amplitudes. So the CP asymmetry at leading order in α will arise from the interference of diagram 1a with the other diagrams. The weak phase of 1a is given by B_{ij} and it is obvious that diagram 1c has a weak phase $B_{ij}\alpha$, so interference of 1a with 1c gives zero contribution to a_{ij}^{CP}. The same happens with diagram 1d being of order $B_{ij}\alpha[1+O(|U_{4k}|^2)]$. Finally, diagram 1e is gauge independent due to Higgs mass presence and model dependent (we need to specify the Higgs sector of the theory). In addition, with one Higgs doublet, the weak phase comes from the Higgs flavour changing coupling proportional to B_{ij} and consequently does not contribute to the CP asymmetry. In conclusion, *in order to calculate* (9) we can start including in the amplitude just figs. 1a and 1b, nevertheless it must be stressed that this amplitude is not gauge invariant and even worse not finite, but we are interested not in the amplitude itself but in the *CP violating interference* and as we will show later on, the result for a_{ij}^{CP} is finite and gauge invariant.

Keeping these remarks in mind, the decay amplitude can be written now as the sum of two terms:

$$T(Z^0 \to d_i \bar{d}_j) \equiv T_{ij} = T_{ij}^{(a)} + T_{ij}^{(b)} . \qquad (10)$$

The first one is the tree level contribution,

$$T_{ij}^{(a)} = -\frac{g}{2\cos\theta_w} B_{ij}\bar{u}(p_i)\gamma^\mu L v(p_j)\epsilon_\mu(p) , \qquad (11)$$

with $L=\frac{1}{2}(1-\gamma_5)$.

The second one takes into account all diagrams included in fig. 1b. So diagrams contributing to this piece are topologically equivalent to the SM one-loop diagrams. In the limit of zero external masses, it reduces [1] to an effective V–A vertex:

$$T_{ij}^{(b)} = \frac{g}{2\cos\theta_w} A_{ij}\bar{u}(p_i)\gamma^\mu L v(p_j)\epsilon_\mu(p) , \qquad (12)$$

with

$$A_{ij} = \frac{\alpha}{4\pi}\sum_{k=1}^{3}\lambda_{ij}^k I(x_k) \quad \text{and } \alpha \equiv \frac{g^2}{4\pi} . \qquad (13)$$

In the previous formula $\lambda_{ij}^k \equiv V_{ki}^* V_{kj}$, V_{ij} being the 3×3 mixing matrix (2) and $x_k \equiv (m_k/M_w)^2$. The index k identifies the up quark running into the loops in fig. 1b and $I(x_k)$ is the flavour-changing form factor arising from the sum of all diagrams included in fig. 1b.

As we have previously explained the diagrams included in fig. 1b are the same as in the SM, the only difference in the calculation is that, in our model, GIM cancellation ($\sum_{k=1}^{3}\lambda_{ij}^k=\delta_{ij}$) does not operate. Instead we have

$$\sum_{k=1}^{3}\lambda_{ij}^k = B_{ij} , \qquad (14)$$

due to the unitarity of the complete 4×4 $U_{\alpha\beta}$ matrix.

Therefore, we can put

$$A_{ij} = \frac{\alpha}{4\pi}\sum_{k=1}^{3}\lambda_{ij}^k I(x_k)$$
$$= \frac{\alpha}{4\pi}\left(B_{ij}I(0) + \sum_{k=c,t}\lambda_{ij}^k F(x_k)\right) , \qquad (15)$$

where we have set $m_1 = m_u = 0$, and

$$F(x_i) \equiv I(x_i) - I(0) , \qquad (16)$$

are the finite and gauge invariant (for Z^0 on shell) form factors of the SM [1].

From eq. (15) we get the following result: the second term on the RHS is the SM result up to minor changes in the matrix elements that we will neglect. The first one on the RHS is the new contribution, that is divergent and gauge dependent, as it should, because it is proportional to B_{ij}, as diagrams 1c and 1d that have been disregarded. Being this dangerous piece proportional to B_{ij} it will not contribute to the asymmetry and also this part of A_{ij} can be neglected as diagrams 1c and 1d.

Finally, we must say that diagrams coming from renormalization counterterms will not contribute to the asymmetry because they do not have absorptive parts. So we arrive, as we promised, to the following finite and gauge invariant expression for the asymmetry:

$$a_{ij}^{CP} \simeq 2\frac{\alpha}{4\pi}\frac{\sum_{k=c,t}\mathrm{Im}(B_{ij}^*\lambda_{ij}^k)\,\mathrm{Im}\,F(x_k)}{|B_{ij}|^2+\ldots} . \qquad (17)$$

In the denominator, the dots have been included to remind us that the calculation is valid provided the FCNC tree level contribution is bigger than the one loop SM one. From (17) we see that we only need the absorptive part of the form factor. This will notably simplify the calculation, in particular in the

Feynman–'t Hooft gauge we have only the two diagrams of fig. 2. We found

$$\begin{aligned}\operatorname{Im} I(x) = 2\pi \Bigg\{ & \left[a\left(\frac{3}{2} + \frac{1-x}{s}\right) + \tfrac{1}{3}bx\left(\frac{1}{2} - \frac{1-x}{s}\right)\right] \sqrt{1-\frac{4x}{s}} \\ & + \frac{1}{s}\left[a\left(s + 2(1-x) + \tfrac{1}{2}x^2 + \frac{(1-x)^2}{s}\right) - \tfrac{2}{3}bx\left(1 + \frac{(1-x)^2}{2s}\right)\right] \\ & \times \log\left(\frac{2(1-x)+s-\sqrt{s(s-4x)}}{2(1-x)+s+\sqrt{s(s-4x)}}\right)\Bigg\}\, \theta(s-4x), \end{aligned} \tag{18}$$

where

$$a \equiv \tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_w, \quad b \equiv \sin^2\theta_w, \quad s = \frac{M_Z^2}{M_W^2}.$$

For illustrative purposes we consider the s$\bar{\text{b}}$ channel. Present data on the standard 3×3 mixing matrix V_{ij} gives $\lambda^c_{bs} \approx V_{cb} \approx |V_{ts}|$ and $\lambda^t_{bs} \approx V^*_{ts} \approx -|V_{ts}| \approx -0.044$. In addition from eq. (18) we have

$$\begin{aligned} &\operatorname{Im} F(x_c) = 8.1\times10^{-4} \quad \text{for } m_c = 1.5 \text{ GeV}, \\ &\operatorname{Im} F(x_t) = 0.73 \quad \text{for } m_t > \tfrac{1}{2}M_Z, \end{aligned} \tag{19}$$

so in the numerator of eq. (17) we have $F(x_t)$ dominance.

Not having any constraint on the relative phase of B_{bs} and λ^t_{bs} we will take this relative phase $\frac{1}{2}\pi$ in order to show an upper bound of the asymmetry. Note that in this case we can substitute the dots of the denominator of eq. (17) by the SM contribution to A_{sb}, that we rename A'_{sb}, that is also dominated by the top contribution ($|F(x_c)| \sim 6\times10^{-4}$, $1 \leqslant |F(x_t)| \leqslant 4$ for $90 \leqslant m_t \leqslant 210$ GeV [13]). So using

$$|A'_{sb}| \simeq \frac{\alpha}{4\pi} |V_{ts}|\,|F(x_t)|, \tag{20}$$

we get our final result

$$\begin{aligned} |a^{CP}_{sb}| &\leqslant \frac{\alpha}{4\pi}\frac{2|B_{sb}|}{|B_{sb}|^2 + |A'_{sb}|^2} |V_{ts} \operatorname{Im} F(x_t)| \\ &\approx 1.8\times10^{-4} \frac{|B_{sb}|}{|B_{sb}|^2 + |A'_{sb}|^2}. \end{aligned} \tag{21}$$

The asymmetry (21) has its maximum value when $|A'_{sb}| \approx |B_{sb}|$ as we would expect, decreasing when one of the amplitudes becomes greater than the other one (remember that $|B_{sb}| \leqslant 1.8\times10^{-3}$ and $1\times10^{-4} \leqslant |A'_{sb}| \leqslant 4\times10^{-4}$). If we assume that FCNC coupling dominates the width, we can reasonably accept $|B_{sb}| \approx 1\times10^{-3}$, which gives a CP asymmetry:

$$|a^{CP}_{sb}| \leqslant 2\times10^{-1}. \tag{22}$$

Finally, we would like to make a comment about the numbers of events needed in this case to get a signal for the asymmetry. At 1σ level this is given by

$$N_Z = \frac{1}{\text{BR}(Z^0 \to s\bar{b})}\frac{1}{2a^2_{sb}}. \tag{23}$$

One can become convinced that if we have the interference of two amplitudes f_0 and f_1, and one of them (f_0) is dominant, the number N_Z is controlled by the intensity of the suppressed amplitude f_1 since, essentially

$$N_Z \sim \frac{1}{|f_1|^2}\frac{|f_0|^2}{\text{BR}},$$

and BR $\propto |f_0|^2$.

Now, considering that BR($Z^0 \to$ bs) and $1/a^2_{sb}$ are proportional to $|B_{sb}|^2$, one finds that the number of events is roughly independent of the magnitude of the FCNC coupling:

$$N_Z \approx \frac{4\times10^7}{|\operatorname{Im} F(x_t)|^2} \sim 7\times10^7. \tag{24}$$

Values for N_Z of this order can be not so far from the capabilities of future e^+e^- colliders or even from LEP, upgraded with higher luminosity.

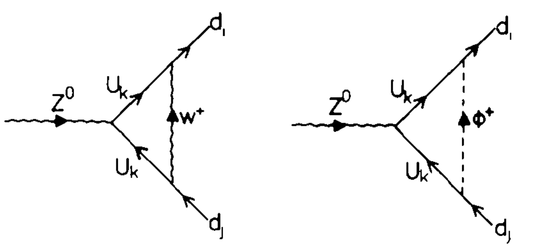

Fig. 2. One loop Feynman diagrams contributing to the absorptive part of the amplitude $Z^0 \to d_i\bar{d}_j$ in the Feynman–'t Hooft gauge.

In conclusion, we have shown that with the addition of vector-like fermions to the SM one can get spectacular results for CP observables. In particular, a_{sb}^{CP} can be several orders of magnitude bigger than in the SM model [1,2]. Needless to say that the discovery of flavour changing neutral currents at the level of eq. (8) would be the most important goal but our result in eq. (22) clearly indicates that it is worthwhile a harder search for CP violating effects in the kind of models here discussed.

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