Physics Letters B 282 (1992) 399–405
North-Holland

PHYSICS LETTERS B

Self-organizing criticality, large anomalous mass dimension and the gauge hierarchy problem

Stefan Bornholdt and Christof Wetterich
Deutsches Elektronen-Synchrotron DESY, Notkestraße 85, W-2000 Hamburg 52, FRG

Received 13 February 1992

Fermions and scalars contribute with opposite sign to the dependence of the scalar mass term m^2 on the length scale k^{-1}. We discuss the possibility that a "balance effect" between these contributions leads to an infrared stable fixed point in the evolution of the ratio $m^2(k)/k^2$. Such a behavior would result in self-organizing criticality and could naturally explain a small gauge hierarchy. It is closely related to a large anomalous mass dimension.

The masses of all elementary particles in the standard model are proportional to the vacuum expectation value (VEV) of the scalar doublet field $\langle\varphi\rangle = 174$ GeV. Why is this mass scale (the Fermi scale) much smaller than the other fundamental mass scale in our world, namely the Planck scale $M_{\mathrm{P\ell}} \approx 10^{19}$ GeV? If we believe that field theory should remain valid at length scales much shorter than the inverse of $\langle\varphi\rangle$, perhaps even up the the Planck length $M_{\mathrm{P\ell}}^{-1}$, we have to explain the appearance of a tiny dimensionless parameter given by the ratio of mass scales, e.g. $\langle\varphi\rangle / M_{\mathrm{P\ell}}$. This is the gauge hierarchy problem. In the standard model $\langle\varphi\rangle$ measures the deviation from a second order phase transition between the symmetric and the spontaneously broken phase [#1]. We may reformulate the question about the small mass ratio: Why is the standard model near the phase transition? or: Why is the system near criticality? A natural explanation of a small mass ratio would arise in a situation of "self-organizing criticality", i.e. if the system is near criticality not only for a very particular tuning of the properties of the short-distance physics. A small value of $\langle\varphi\rangle$ should result for a large range of the value of the scalar mass term at short distances.

[#1] The phase transition occurs as a function of the scalar mass term at short distances. Due to the electroweak gauge interactions it is presumably very weakly first order, but this is of no importance here.

The perturbative calculation of $\langle\varphi\rangle$ as a function of the "bare mass term" [#2] $\mu^2(\Lambda)$ typically results in a very badly converging series for any situation near the phase transition – there is in general a fine tuning problem order by order in perturbation theory. An appropriate renormalization group improved treatment is necessary. A convenient tool is the average action Γ_k [1], i.e. the effective action for averages of fields over a volume of size k^{-d}. It is closely related to the block spin action in lattice theories and is obtained by integrating out the modes with momentum squared larger than k^2 [2]. In particular, the average potential U_k allows to interpolate continuously from the bare parameters which determine the shape of the potential at $k=\Lambda$ to the usual effective potential which results in the limit $k \to 0$ and determines the VEV $\langle\varphi\rangle$. The change of the shape of the average potential as a function of the length scale k^{-1} can be described by appropriate evolution equations. Apparently, the loop expansion for these evolution equations converges for small enough couplings.

At the phase transition the curvature of U_k around its minimum is $\sim k^2$ at every scale k. If the minimum

[#2] Here Λ stands for the short-distance scale where the theory is defined, for example the inverse of the lattice size or some other momentum cutoff. The critical bare mass $\mu_0^2(\Lambda)$ corresponding to the phase transition ($\langle\varphi\rangle = 0$) is not universal whereas the range of $\mu^2(\Lambda)$ around $\mu_0^2(\Lambda)$ leading to small $\langle\varphi\rangle$ is universal up to a proportionality constant of order one.

occurs at some nonzero field value $\varphi_0(k)$ this value scales as $\varphi_0(k)\sim k$ (for $d=4$). Near the phase transition this behavior applies for all $k^2 \gg \langle\varphi\rangle^2$. Self-organizing criticality should explain the scaling $\varphi_0(k)\sim k$ over a large range in k without a particular tuning of $\mu^2(\Lambda)$. Finally, the deviation from the phase transition in the spontaneously broken phase results in a nonzero VEV $\langle\varphi\rangle = \lim_{k\to 0}\varphi_0(k)$.

A natural scaling $\varphi_0 \sim k$ would occur for an (approximate) infrared stable fixed point for the ratio φ_0^2/k^2. We investigate in this letter the possibility of such a fixed point for a scalar theory coupled to fermions. In such systems there exists a balance mechanism which could, in principle, lead to an infrared stable fixed point behavior: The contributions from scalar fluctuations tend to lower $\varphi_0^2(k)$ for decreasing k, whereas the fermion fluctuations have the opposite effect and push $\varphi_0^2(k)$ to larger values. On the other hand, for $h^2\varphi_0^2 \gg k^2$ (h is the Yukawa coupling) the fermions decouple effectively. Then only scalar loops are effective which drive $\varphi_0^2(k)$ to smaller values. For large enough Yukawa couplings $\varphi_0^2(k)$ increases for $\varphi_0^2/k^2 \lesssim h^{-2}$ and decreases for $\varphi_0^2/k^2 \gtrsim h^{-2}$. If this effect is strong enough it can lead to a stable behavior $\varphi_0^2 \sim h^{-2}k^2$ over a large range in k. We study this "balance effect" more quantitatively in the following.

In analogy to the standard model in the limit of vanishing gauge couplings and vanishing Yukawa couplings except for the top quark we start with the SU(M) symmetric action

$$S[\chi,\eta] = \int d^dx[-\bar{\mu}^2\chi^\dagger\chi + \tfrac{1}{2}\bar{\lambda}(\chi^\dagger\chi)^2 + \bar{Z}\partial_\mu\chi^\dagger\partial^\mu\chi + \bar{Z}_F(\bar{\eta}_L i\not{\partial}\eta_L + \bar{\eta}_R i\not{\partial}\eta_R) + (\bar{h}\bar{\eta}_L\chi\eta_R + \text{h.c.})] . \quad (1)$$

Here χ is an M-component complex scalar field, $\eta_L = (t, b, b', ...)_L$ an M component left-handed fermion field with N_c colors and $\eta_R = t_R$ a right-handed fermion singlet (again with N_c colors). The other light fermions are omitted here. The average action Γ_k is a functional of the average fields φ, ψ

$$\exp(-\Gamma_k[\varphi,\psi]) = \int \mathscr{D}\chi\,\mathscr{D}\eta\, P_k[\varphi,\chi]\, P_k[\psi,\eta] \times \exp(-S[\chi,\eta]) , \quad (2)$$

where the constraints P_k [3] are defined in momentum space ($\Omega = \int d^dx$)

$$P_k[\varphi,\chi] = \exp\left(-\Omega\sum_q \frac{\bar{Z}q^2}{1-f_k^2(q)} \times [\varphi^\dagger(q) - f_k(q)\chi^\dagger(q)][\varphi(q) - f_k(q)\chi(q)]\right),$$

$$P_k[\psi,\eta] = \exp\left(-\Omega\sum_q \frac{\bar{Z}_F}{1-f_k^2(q)} \times [\bar{\psi}(q) - f_k(q)\bar{\eta}(q)]^T \not{q}[\psi(q) - f_k(q)\eta(q)]\right), \quad (3)$$

and the fermionic constraint is understood as a product over all fermion species. The function

$$f_k(q) = \exp[-a(q^2/k^2)^\beta] \quad (4)$$

defines the details of the averaging scheme. The average action has the same symmetries as S and can be expanded in the number of derivatives

$$\Gamma_k[\varphi, 0] = \int d^dx[U_k(\rho) + Z_k(\rho)\partial_\mu\varphi^\dagger\partial^\mu\varphi + \tfrac{1}{4}Y_k(\rho)\partial_\mu\rho\partial^\mu\rho + ...] \quad (5)$$

with $\rho = \varphi^\dagger\varphi$. We are interested in the average potential U_k around its minimum. In the one-loop approximation it is obtained [1,3] with $N=2M$ as

$$U_k(\varphi) = -\bar{\mu}^2\rho + \tfrac{1}{2}\bar{\lambda}\rho^2 + \tfrac{1}{2}(2\pi)^{-d}\int d^dq\left(\ln\frac{P(q) - \bar{\mu}^2 + 3\bar{\lambda}\rho}{m_0^2} + (N-1)\ln\frac{P(q) - \bar{\mu}^2 + \bar{\lambda}\rho}{m_0^2} - 2^{d/2}N_c\ln\frac{P_F(q) + \bar{h}^2\rho}{m_0^2}\right), \quad (6)$$

where the inverse propagators P and P_F exhibit an infrared cutoff due to the constraints (3)

$$P(q) = \frac{\bar{Z}q^2}{1-f_k^2(q)}, \quad P_F(q) = \frac{\bar{Z}_F^2 q^2}{[1-f_k^2(q)]^2}. \quad (7)$$

We concentrate on the spontaneously broken regime where the minimum of U_k occurs for nonzero $\rho_0(k)$. Its evolution is obtained from differentiation of $U'(\rho_0(k)) = 0$ with respect to k (primes denote derivatives with respect to ρ and $t = \ln(k/m_0)$)

$$\frac{\partial}{\partial t}\rho_0 = -U_k''(\rho_0)^{-1}\frac{\partial}{\partial t}U_k'(\rho_0) . \quad (8)$$

It is convenient to use renormalized dimensionless parameters

$$\kappa(k) = k^{2-d}Z\rho_0 , \quad (9)$$

$$\lambda(k) = k^{d-4}Z^{-2}U_k''(\rho_0) , \quad (10)$$

where $Z = Z_k(\rho_0)$. The mass term for the radial scalar mode (the curvature of U_k around ρ_0) reads in terms of λ and κ

$$m^2(k) = 2k^2\lambda(k)\kappa(k) . \quad (11)$$

A situation of self-organizing criticality arises if the evolution of κ exhibits an infrared stable fixed point (at least approximately)

$$\beta_\kappa(\kappa, \lambda) = \frac{\partial}{\partial t}\kappa, \quad \beta_\kappa(\kappa_*) = 0, \quad \frac{\partial \beta_\kappa}{\partial \kappa}(\kappa_*) > 0 . \quad (12)$$

The β-functions for κ and λ can be inferred from (6) using renormalization group improvement [4]

$$\beta_\kappa = \frac{\partial \kappa}{\partial t} = (2-d-\eta)\kappa$$
$$+ 2v_d[3l_1^d s_1^d(2\lambda\kappa) + (N-1)l_1^d$$
$$- 2^{d/2}N_c(h^2/\lambda)l_{F1}^d s_{F1}^d(h^2\kappa)] , \quad (13)$$

$$\beta_\lambda = \frac{\partial \lambda}{\partial t} = (d-4+2\eta)\lambda$$
$$+ 2v_d[9\lambda^2 l_2^d s_2^d(2\lambda\kappa) + (N-1)\lambda^2 l_2^d$$
$$- 2^{d/2}N_c h^4 l_{F2}^d s_{F2}^d(h^2\kappa)] . \quad (14)$$

Here $v_d^{-1} = 2^{d+1}\pi^{d/2}\Gamma(d/2)$ and the constants l_n^d can be found in ref. [4]. They depend in general on the averaging scheme (i.e., on the value of β in (4)) except for $l_n^{2n} = 1$. The "threshold functions" s_n^d [4] depend on the ratio m^2/k^2 and similarly for the fermionic functions. We have depicted s_1^4 in fig. 1. (In (4) we choose the parameters $\beta = 3$ and $\beta = [\exp(2a) - 1]/2a$ for all numerical purposes in this letter.) The anomalous dimension $\eta = -\partial \ln Z/\partial t$ of the scalar field is computed for the average action in ref. [5]

$$\eta = 4v_d N_c j_F^d r_F^d(h^2\kappa)h^2 . \quad (15)$$

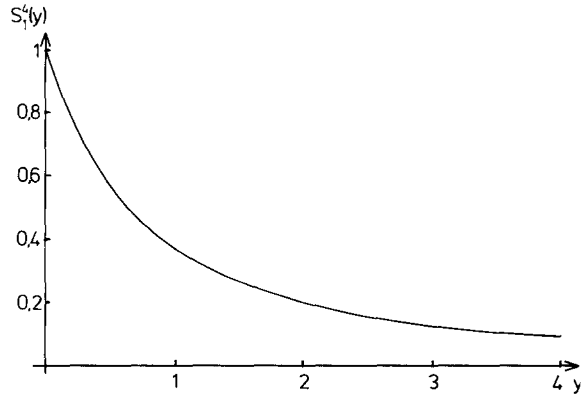

Fig. 1. Threshold function $s_1^4(y)$.

The constants j_F^d depend on the averaging scheme except for $j_F^4 = 1$ and r_F^d are "threshold functions" ($r_F^d(0) = 1$) accounting for the vanishing of the fermionic contribution to η as $h^2\kappa \to \infty$. The "balance effect" discussed above is apparent in the second term of eq. (13): For large h^2/λ this term is negative for small $h^2\kappa$ whereas it becomes positive for large $h^2\kappa$ (where the term $\sim h^2/\lambda$ drops out). Self-organizing criticality becomes possible, however, only if this balance effect is strong enough compared to the contribution from the first term in (13). This is difficult for $d > 2$.

For a slowly running λ it is convenient to discuss instead of κ the evolution of the quantity $\lambda\kappa = m^2/2k^2$. An infrared stable fixed point in $\lambda\kappa$ requires a positive derivative of $\beta_{\lambda\kappa} = \partial(\lambda\kappa)/\partial t$

$$\frac{\partial}{\partial(\lambda\kappa)}\beta_{\lambda\kappa} = -2 + \omega > 0 . \quad (16)$$

Comparison with (11) establishes ω as the anomalous mass dimension defined by

$$\omega = \frac{\partial \beta_{m^2}}{\partial m^2} ,$$

$$\beta_{m^2} = \frac{\partial m^2}{\partial t} = \left(2 + \frac{\beta_\lambda}{\lambda} + \frac{\beta_\kappa}{\kappa}\right)m^2 ,$$

$$\omega = \eta + 2v_d[(N-1)\lambda l_2^d$$
$$+ 3\lambda l_2^d s_2^d(2\lambda\kappa) - 36\lambda^2\kappa l_3^d s_3^d(2\lambda\kappa)$$
$$+ 2^{d/2+1}N_c(h^6/\lambda)\kappa l_{F3}^d s_{F3}^d(h^2\kappa)] . \quad (17)$$

Of course, ω has to be evaluated at the critical value $m_*^2(k)$. The anomalous mass dimension ω has a simple physical interpretation [4]: It governs the scale dependence of the deviation of the scalar mass term from the critical mass term $m_*^2(k)$ which is defined by $m_*^2(k\to 0)=0$

$$m^2(k)=m_*^2(k)+\delta m^2(k), \quad \frac{\partial}{\partial t}\delta m^2=\omega\delta m^2 . \tag{18}$$

Self-organizing criticality requires $\omega>2$. A sufficient condition for a natural gauge hierarchy was already discussed earlier in a somewhat different perspective [6]: A naturally small ratio $\langle\varphi\rangle/M$ arises if the average of ω between the scales M and $\langle\varphi\rangle$ equals 2:

$$\langle\omega\rangle=t_0^{-1}\int_0^{t_0} \mathrm{d}t\,\omega(t)=2, \quad t_0=\ln\frac{\langle\varphi\rangle}{M} . \tag{19}$$

The anomalous dimension may exceed 2 for k near M and be substantially smaller than 2 for k near $\langle\varphi\rangle$. A natural explanation of the gauge hierarchy could therefore arise from new physics at a high mass scale $\sim M$!

In the remainder of this letter we concentrate on the question whether $\omega\geqslant 2$ is realistic in four-dimensional models. For $h^2\kappa\ll 1$, $2\lambda\kappa\ll 1$ we can expand s_n^d

$$s^d_{(\mathrm{F})n}(y) = 1-ny\frac{l^d_{(\mathrm{F})n+1}}{l^d_{(\mathrm{F})n}}+\frac{n(n+1)}{2}y^2\frac{l^d_{(\mathrm{F})n+2}}{l^d_{(\mathrm{F})n}}+\dots , \tag{20}$$

and $\eta=N_c h^2/8\pi^2$. In lowest order we obtain the standard perturbative results

$$\omega=\frac{1}{16\pi^2}\left[(N+2)\lambda+2N_c h^2\right] , \tag{21}$$

$$\beta_\lambda=\frac{\partial\lambda}{\partial t}=\frac{1}{16\pi^2}\left[(N+8)\lambda^2-4N_c h^4+4N_c h^2\lambda\right] . \tag{22}$$

In the standard model ($N=4$, $N_c=3$) a value $\omega>2$ then requires large values of the Yukawa coupling h outside the perturbative regime. In turn, h can remain large over a large range in mass scales only if its β-function has an (approximate) fixed point for nonperturbative values of h. In view of the upper bounds on m_t derived from the analysis of radiative corrections for the LEP data this could only be relevant for a fourth generation [6] (which should not contain high neutrinos).

On the other hand, the standard model may be embedded at some mass scale M_x into a unified model with a substantially larger number of degrees of freedom, e.g. grand unified models or string theories. It is conceivable that ω is considerably larger than two for mass scales above M_x and drops to a rather small value for the standard model below M_x, but nevertheless obeys (19) with $M>M_x$. As a first check if a large anomalous mass dimension becomes possible for a large number of degrees of freedom we consider the above scalar-fermion model for arbitrary (large) N and N_c. Even though in realistic grand unified models the large number of scalar and fermion degrees of freedom arises from large representations rather than from a large flavor group SU($N/2$), our toy model may reflect some general features of systems with many particles. Here we consider a fixed value of the Yukawa coupling $h=4(h^2/16\pi^2\approx 0.1)$ and we will argue later that this is not unrealistic for suitable models. For definiteness we choose $N_c=5$, $N=60$.

We have to investigate the solutions of the coupled system of evolution equations (13), (14) for κ and λ (inserting (15)). For fixed κ and h^2 the evolution of λ is always determined by an infrared fixed point since β_λ ($\lambda=0$) is negative and β_λ ($\lambda\to\infty$) is positive. In our case we also find an infrared fixed point for κ. Numerically the simultaneous fixed point is given by $\kappa_*=0.011$, $\lambda_*=5.911$. In fig. 2 we have depicted β_κ for $\lambda=\lambda_*$. We observe a considerable range of attraction in κ. (This range actually continues in the symmetric regime where the minimum of the short-distance potential occurs at the origin $\rho=0$.) The anomalous dimension at the fixed point can be read off from the derivative of β_κ (16) at the fixed point. One finds $\omega=3.28$, a value slightly smaller than the approximation (21) for $\lambda=\lambda_*$. (Note that $h^2\kappa_*=0.18$.) Our toy model implements self-organizing criticality provided the one-loop evolution equations are reliable and a value $h=4$ can be sustained over a sufficient range in scales.

In order to demonstrate that fairly large Yukawa couplings can be stable in suitable models we enlarge our toy model. We also include QCD like gauge interactions with fermions in N_g generations of the fun-

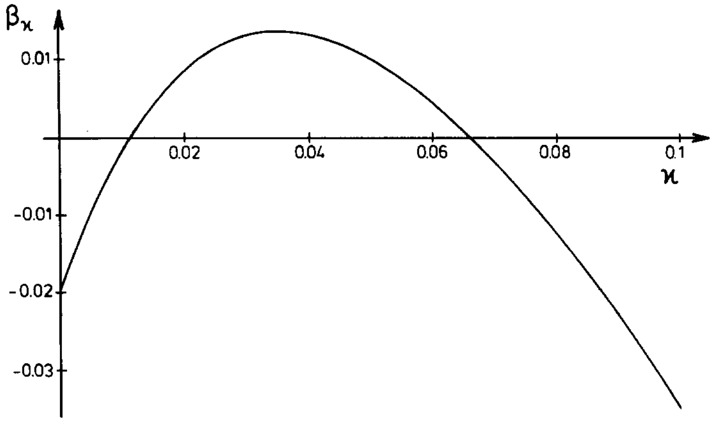

Fig. 2. The function $\beta_\kappa(\kappa)$ for $N_c=5$, $N=60$, $h=4$, $\lambda=5.911$.

damental representation of the gauge group $SU(N_c)$ and simultaneously in the $N/2$ component and singlet representations of the global symmetry $SU(N/2)$. (As in the standard model we assume $SU(N_c)$ to be vectorlike with $N/2$ singlets in each generation. The Yukawa couplings of all singlets except for one are set to zero. The generalization of the leptons plays no role here.) For $h^2\kappa \ll 1$, $2\lambda\kappa \ll 1$ we can employ the standard one-loop β-functions for h and the gauge coupling g [7]

$$\beta_{h^2}=\left((\tfrac{3}{2}+N_c)h^2-3\frac{N_c^2-1}{N_c}g^2\right)\frac{h^2}{8\pi^2}, \tag{23}$$

$$\beta_{g^2}=-(\tfrac{11}{3}N_c+\tfrac{1}{3}NN_g)\frac{g^4}{8\pi^2}. \tag{24}$$

The evolution equations for the ratios h^2/g^2 [8,6] and λ/h^2 [6,9] exhibit infrared stable fixed points (for large enough N)

$$\left(\frac{h^2}{g^2}\right)_* = \frac{NN_gN_c-2N_c^2-9}{3N_c(N_c+\frac{3}{2})}, \tag{25}$$

$$\left(\frac{\lambda}{h^2}\right)_* = \frac{1}{2(N+8)}\Bigg[-2N_c+3-6\frac{N_c^2-1}{N_c}\left(\frac{g^2}{h^2}\right)_* + \left\{\left[2N_c-3+6\frac{N_c^2-1}{N_c}\left(\frac{g^2}{h^2}\right)_*\right]^2 + 16N_c(N+8)\right\}^{1/2}\Bigg]. \tag{26}$$

In the following we will consider situations where g^2 runs very slowly, in particular for g^2 near an infrared stable fixed point g_*^2. In this case the fixed point for h^2 is given by the vanishing of β_{h^2} (23)

$$\left(\frac{h^2}{g^2}\right)_* = \frac{3(N_c^2-1)}{N_c(N_c+\frac{3}{2})}. \tag{27}$$

Inserting (25) or (27) and (26) into (21) we obtain ω_* as a function of g^2. The value of $\alpha=g^2/4\pi$ for which ω_* reaches 2 is depicted as $\hat{\alpha}$ in fig. 3 for $N_c=5$, $N_g=1$ and using (27) ($\omega_*(\hat{\alpha})=2$). For large N this value is considerably smaller than one.

The question if α can remain larger than $\hat{\alpha}$ over a substantial range of scales depends on $\beta_\alpha=\partial\alpha/\partial t$. Since the one loop contribution to β_α vanishes for a critical ratio N/N_c we display here the two-loop result [10]

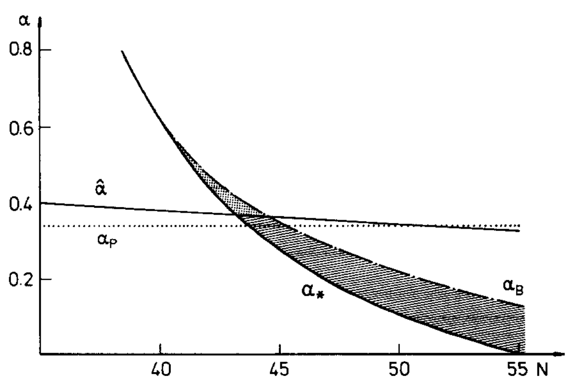

Fig. 3. Infrared fixed point α_* and critical value $\hat{\alpha}$ (see text) as a function of N, for $N_c=5$.

$$\frac{\beta_\alpha}{\alpha}=\frac{\alpha}{2\pi}B+\frac{\alpha^2}{8\pi^2}C_1+\frac{\alpha h^2}{32\pi^3}C_2\,, \tag{28}$$

$$B=-\tfrac{11}{3}N_c+\tfrac{1}{3}NN_g\,,$$

$$C_1=\tfrac{34}{3}N_c^2+\tfrac{5}{3}N_cNN_g+\frac{N_c^2-1}{2N_c}NN_g\,,$$

$$C_2=-\tfrac{1}{2}N\,. \tag{29}$$

Inserting the fixed point (27) we can combine the two loop terms in (28)

$$C=C_1+C_2\left(\frac{h^2}{g^2}\right)_*=-\tfrac{34}{3}N_c^2+\tfrac{5}{3}N_cNN_g$$

$$+\frac{1}{2}\frac{N_c^2-1}{N_c}NN_g-\frac{N}{2}\frac{3(N_c^2-1)}{N_c(N_c+\frac{3}{2})}\,. \tag{30}$$

As is well known [11] the evolution equation (28) is characterized by an infrared stable fixed point α_* if $B<0$ and $C>0$

$$\alpha_*=-\frac{4\pi B}{C}\,. \tag{31}$$

We have plotted α_* as a function of N in fig. 3 and observe $\alpha_*>\hat{\alpha}$ for a range in N for which $\alpha\gtrsim 0.4$. If the perturbative β-functions (23), (24), (28) can be trusted in this range our toy model has a large anomalous mass dimension $\omega>2$! (This would hold for all scales between M and M_x in the scenario described above.) We note that a fixed-point value $h_*>4$ occurs in this model for $N_c=5$, $N\leqslant 48$, thereby qualitatively justifying our assumption above with constant $h_*=4$.

An estimate of the validity of the perturbative β-functions requires, in principle, knowledge of the higher order terms in the loop expansion. From an inspection of the structure of the terms in (28) we make a rough estimate that perturbation theory seems to be valid for

$$\alpha<\alpha_p=\frac{6\pi}{11N_c}\,. \tag{32}$$

A comparison with fig. 3 shows $\alpha_*>\hat{\alpha}$ for α in the vicinity of or somewhat larger than α_p. We conclude that $\omega>2$ may occur at the boundary of validity of perturbation theory in this model. This picture does not change qualitatively if we use instead of the fixed point α_* the upper bound α_B of the "infrared interval" [9] for $M/M_x=10^4$ (also shown in fig. 3). (Here α_B is defined as the maximum value of $\alpha(M_x)$ if the gauge coupling runs between M and M_x according to (28).) We also note that we have done the quantitative analysis of infrared stable fixed points with Yukawa couplings and gauge couplings only for $h^2\kappa\ll 1$. This is not a good approximation for $N_c=5$, $N<45$ as may be seen by using the linear approximation for β_κ in order to compute the fixed point κ_*. One typically obtains $h^2\kappa_*$ of order one half. For $h^2\kappa$ near one all β-functions are modified and the corresponding fixed points are shifted. A new possibility to obtain large values of ω may open up for small values of λ. The last term in eq. (17) $\sim\lambda^{-1}$ becomes dominant for sufficiently small values of λ/h^4 and similarly in eq. (13). On the other hand the growth of λ due to the negative term $\sim h^4$ in (14) is slowed down by the function $s_{F2}^4(h^2\kappa)$. It remains to be investigated if an infrared fixed point behavior for $\lambda\kappa$ can be induced by λ remaining very small over a sufficient range in t.

In conclusion, it does not seem unreasonable that suitable models with a high number of degrees of freedom lead to an anomalous mass dimension ω exceeding 2, thereby providing a possible mechanism for a natural gauge hierarchy. In our example a value $\omega>2$ could not be firmly established within the limit of validity of perturbation theory. No good reason is known to us, however, why $\omega>2$ should not be realized for slowly running couplings either at the boundary of validity of perturbation theory or even within perturbation theory in other, appropriately chosen models. We also emphasize that our mechanism may lead to the phenomenon of self-organizing criticality in two- and three-dimensional models where large values of the couplings λ and h^2 occur rather naturally as a result of the infrared fixed-point behavior.

References

[1] A. Ringwald and C. Wetterich, Nucl. Phys. B 334 (1990) 506;
C. Wetterich, Nucl. Phys. B 352 (1991) 529.
[2] L.P. Kadanoff, Physics 2 (1966) 263;
K.G. Wilson, Phys. Rev. B 4 (1971) 3174, 3184;
K.G. Wilson and I.G. Kogut, Phys. Rep. 12 (1974) 75;
F.J. Wegner, in: Phase transitions and critical phenomena,

Vol. 6, eds. C. Domb and M. S. Greene (Academic Press, New York, 1976);
J. Polchinski, Nucl. Phys. B 231 (1984) 269.
[3] C. Wetterich, Z. Phys. C 48 (1990) 693.
[4] C. Wetterich, DESY preprint DESY 91-088 (1991).
[5] S. Bornholdt and C. Wetterich, to appear.
[6] C. Wetterich, Phys. Lett. B 104 (1981) 269.
[7] T.P. Cheng, E. Eichten and L.F. Li, Phys. Rev. D 9 (1974) 2259.
[8] B. Pendleton and G.G. Ross, Phys. Lett. B 98 (1981) 291.
[9] C. Wetterich, in: Superstrings, unified theories and cosmology 1987, eds. G. Furlan, J.C. Pati, D.W. Sciama, E. Sezgin and Q. Shafi (World Scientific, Singapore, 1988) p. 403.
[10] M.E. Machacek and M.T. Vaughn, Nucl. Phys. B 222 (1983) 83.
[11] W.E. Caswell, Phys. Rev. Lett. 33 (1974) 244;
T. Appelquist and L.C.R. Wijewardhana, Phys. Rev. D 36 (1987) 568.