Nuclear Physics B392 (1993) 428–460
North-Holland

NUCLEAR
PHYSICS B

Lagrangians and anomaly candidates of $D=4$, $N=1$ rigid supersymmetry

Friedemann Brandt [1]

Institut für Theoretische Physik, Universität Hannover, Appelstraße 2, W-3000 Hannover 1, Germany

Received 2 June 1992
Accepted for publication 28 September 1992

We investigate how to construct the most general invariant action and whether there are anomaly candidates in $D=4$, $N=1$ globally supersymmetric theories. The importance of the representation theory of the supersymmetry algebra for these questions is discussed. For a class of theories with special supersymmetric multiplet structure complete answers are given. Examples of more general cases are discussed.

1. Introduction

This paper deals with the questions of how to construct the most general $N=1$ globally supersymmetric local action in four dimensions (renormalizable or not) and whether there are up-to-now unknown candidates of anomalies in rigid $D=4$, $N=1$ supersymmetry (SUSY). I consider pure SUSY, i.e. I do not require further symmetries apart from SUSY and Poincaré invariance.

A well-known method to construct $N=1$ supersymmetric actions W is given by

$$W=\int \mathrm{d}^4x\, \bar{D}^2\{D^2 g(\phi)+h(\phi_c)\}+\text{c.c.},\qquad D^2=D^\alpha D_\alpha,\quad \bar{D}^2=\bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha},\quad (1.1)$$

where ϕ denotes collectively the finite set of elementary fields φ^i present in a given theory and their partial derivatives,

$$\phi\in\{\varphi^i,\ \partial_a\varphi^i,\ \partial_a\partial_b\varphi^i,\dots\}.\qquad (1.2)$$

[1] Supported by Deutsche Forschungsgemeinschaft.

ϕ_c denotes antichiral linear combinations of the ϕ ($D_\alpha \phi_c = 0$) and D_α, $\bar{D}_{\dot{\alpha}}$ denote the spinor derivatives (SUSY generators) which are assumed to be represented linearly on the ϕ according to the SUSY algebra *

$$\{D_\alpha, D_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = [D_\alpha, \partial_a] = [\bar{D}_{\dot{\alpha}}, \partial_a] = [\partial_a, \partial_b] = 0,$$

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i\sigma^a{}_{\alpha\dot{\alpha}}\partial_a. \tag{1.3}$$

Using (1.3) and the BRS operator introduced below one easily verifies that actions of the form (1.1) are supersymmetric. However it is not known yet whether or in what cases (1.1) gives the most general supersymmetric action.

To investigate this question and the anomaly problem I shall use a BRS operator δ for the algebra (1.3) which is constructed analogously to the Yang–Mills case by introducing a ghost C^A for each of the operators appearing in (1.3):

$$C^A \in \{C^a, \xi^\alpha, \bar{\xi}^{\dot{\alpha}}\}.$$

By means of δ the integrands of supersymmetric actions and anomalies can be characterized as solutions of the so-called consistency equation (cf. refs. [1,2]) which in terms of differential forms reads

$$\delta\omega_4^G + \mathrm{d}\omega_3^{G+1} = 0, \qquad \omega_4^G \neq \delta\eta_4^{G-1} + \mathrm{d}\eta_3^G. \tag{1.4}$$

Here

$$\mathrm{d} = \mathrm{d}x^a\,\partial_a \tag{1.5}$$

is the exterior derivative and ω_p^g, η_p^g are p-forms with ghost number g depending locally, i.e. polynomially ** on the ϕ, on the ghosts and may also depend explicitly on the coordinates. The general solution ω_4^0 of (1.4) with ghost number 0 gives the integrand of the most general invariant classical action, solutions ω_4^1 with ghost number 1 are integrands of candidates of anomalies,

$$\omega_4^0 = \mathrm{d}^4x\ \mathscr{L}(\phi), \qquad \omega_4^1 = \mathrm{d}^4x\ C^A\mathscr{A}_A(\phi).$$

Since rigid SUSY is considered the ghosts are constant and (1.4) is equivalent to

$$\delta\mathscr{L} = C^A\partial_a\mathscr{V}_A^a, \qquad \mathscr{L} \neq \partial_a\mathscr{W}^a,$$

$$\delta(C^A\mathscr{A}_A) = C^AC^B\partial_a\mathscr{V}_{AB}^a, \quad C^A\mathscr{A}_A \neq \delta\mathscr{W} + C^A\partial_a\mathscr{W}_A^a,$$

* The σ^a-matrices and further conventions used in this paper are listed in appendix C.

** This definition of locality may be generalized by admitting forms which are infinite series in the undifferentiated φ^i but still polynomials in the derivatives of the φ^i. The results of this paper are valid also under this weaker assumption.

where the $\mathscr{V}$'s and $\mathscr{W}$'s are arbitrary local functions of the ϕ and explicit coordinates x^a (forms and differentials dx^a serve to hide indices).

The paper is organized as follows: Sect. 2 collects assumptions underlying the investigation, especially the approach to SUSY. In sect. 3 the BRS operator is defined. In sect. 4 I give the representation theory of the subalgebra $\{D_\alpha, D_\beta\} = 0$ of (1.3) which turns out to be essential for the classification of the solutions of (1.4). In this section I also define a class of theories which I call QDS theories by the property to contain only special representations of this subalgebra. For this class of theories (1.4) is solved completely and the result is given in sect. 5. Sect. 6 sketches the proof of this result. In sect. 7 examples for the non-QDS case are discussed which show that generally there are solutions of (1.4) with ghost number 1, i.e. candidates for pure SUSY anomalies. Sect. 8 is a brief conclusion. It is followed by three appendices. The first appendix gives the proof of one of the theorems used in sect. 6 and of a generalization of this theorem, the second sketches the derivation of the results given in sect. 4, the third contains the conventions used in this paper.

Remark (i) The question whether (1.1) gives the most general form of a supersymmetric action is equivalent to asking whether each supersymmetric action can be written as a superspace integral of the form

$$W = \int d^4x \left\{ d^2\bar{\theta}\, d^2\theta\, g(\Psi) + d^2\bar{\theta}\, h(\Psi_c) + \text{c.c.} \right\},$$

where $\Psi = \Psi(\theta, \bar{\theta}, \phi)$ are superfields and Ψ_c are superfields whose lowest component fields ϕ_c are D_α-invariant.

Remark (ii) The question whether there are solutions of (1.4) with ghost number 1 in globally supersymmetric theories has been investigated at the beginning of the preceding decade [13]. The result obtained there was the absence of solutions ω_4^1 of (1.4) in the pure supersymmetric case (i.e. if no symmetries are considered apart from SUSY and Poincaré invariance). Recently the authors of ref. [13] remarked [14] that by tacit assumption they investigated only special supersymmetric theories in ref. [13], namely theories containing only "vector supermultiplets", i.e. SUSY multiplets which can be described by means of unconstrained scalar superfields. Such theories can be proved to have QDS structure, i.e. the results for QDS theories listed in sect. 5 generalize those of ref. [13]. I note that the anomaly problem has been taken up again also in ref. [9].

2. Variables and approach to SUSY

First of all I stress an important point underlying my investigation and my approach to SUSY: throughout this paper the ϕ, eq. (1.2), are treated as a set of infinitely many variables * which are independent apart from the algebraic relations

$$\partial_{a_1}\dots\partial_{a_n}\varphi^i=\partial_{(a_1}\dots\partial_{a_n)}\varphi^i$$

following from $[\partial_a,\partial_b]=0$. A basis for this set of variables is denoted by $\{\phi^r\}$ and may be chosen e.g. as

$$\{\phi^r\}=\left\{\partial_{a_1}\dots\partial_{a_n}\varphi^i:\quad n\geqslant 0,\ a_k\geqslant a_{k+1}\right\}. \tag{2.1}$$

All equations have to hold identically in these variables. In particular the SUSY algebra is assumed to be represented linearly on these variables and the partial derivatives act algebraically on them (∂_a maps the variable φ^i to the variable $\partial_a\varphi^i$, etc.)

Since I consider global SUSY the ghosts C^A are constant,

$$\partial_a C^A=0. \tag{2.2}$$

Differentials $\mathrm{d}x^a$ and explicit coordinates x^a are treated as additional variables on which ∂_a acts according to

$$\partial_a\,\mathrm{d}x^b=0,\qquad \partial_a x^b=\delta_a^b. \tag{2.3}$$

The reasons for using this approach are:

(i) The locality of the forms appearing in (1.4) can be easily guaranteed.

(ii) (1.4) is required to hold on off-shell fields, i.e. integrands of actions and anomalies have to solve the consistency equation (1.4) irrespectively of the x-dependence of the φ^i. Since the φ^i depend arbitrarily on x, the same holds for their partial derivatives and this justifies to regard (2.1) as a set of independent variables.

The locality of the forms appearing in (1.4) of course is an important requirement. In particular for $G=1$ it originates in renormalization theory [1,2] that anomalies (in lowest loop order) correspond to local functionals of the fields. The usefulness of the approach to regard the ϕ^r as infinitely many independent variables in connection with locality becomes obvious from the following facts:

Each volume form $\omega_4^G(\mathrm{d}x,x)=\mathrm{d}^4x\ \mathscr{L}(C^A,\phi(x))$ is exact due to the Poincaré lemma for forms of x in contractible coordinate patches:

$$\begin{aligned} p\neq 0:&\quad \mathrm{d}\omega_p\,(\mathrm{d}x,x,C^A)=0 &&\Leftrightarrow\quad \omega_p=\mathrm{d}\eta_{p-1}\,(\mathrm{d}x,x,C^A),\\ p=0:&\quad \mathrm{d}\omega_0\,(x,C^A)=0 &&\Leftrightarrow\quad \omega_0=\omega_0(C^A). \end{aligned} \tag{2.4}$$

* In the mathematical literature this approach is treated in the framework of jet bundles [12].

This however does not imply that (1.4) has no solution since this lemma does not contain any information about the locality of the forms η_{p-1}. Treating the ϕ^r as a set of infinitely many independent variables allows us to keep control of the locality. Namely a lemma can be proved which is analogous to the ordinary Poincaré lemma and gives the cohomology of $\mathrm{d} = \mathrm{d}x^a\partial_a$ where ∂_a acts algebraically on the ϕ^r and according to (2.2) and (2.3) on the C^A, x^a and $\mathrm{d}x^a$. It is called the algebraic Poincaré lemma [3,12] and reads in this case (four dimensions, constant ghosts):

$$\begin{aligned}
&0<p<4: && \mathrm{d}\omega_p\,(\mathrm{d}x,\, x,\, C^A,\, \phi)=0 && \Leftrightarrow \omega_p=\mathrm{d}\eta_{p-1}\,(\mathrm{d}x,\, x,\, C^A,\, \phi),\\
&p=0: && \mathrm{d}\omega_0\,(x,\, C^A,\, \phi)=0 && \Leftrightarrow \omega_0=\omega_0(C^A),\\
&p=4: && \mathrm{d}^4x\ \mathscr{L}(x,\, C^A,\, \phi) && \\
& && \quad =\mathrm{d}\eta_3(\mathrm{d}x,\, x,\, C^A,\, \phi) && \Leftrightarrow \frac{\hat\partial\mathscr{L}}{\hat\partial\varphi^i}=0\quad\forall\varphi^i.
\end{aligned}\tag{2.5}$$

In (2.5) all forms depend locally on the ϕ and $\hat\partial/\hat\partial\varphi^i$ denotes the Euler derivative with respect to φ^i,

$$\frac{\hat\partial}{\hat\partial\varphi^i}=\sum_{n\geq 0}\ \sum_{a_{j+1}\geq a_j}(-)^n\partial_{a_1}\ldots\partial_{a_n}\frac{\partial}{\partial\left(\partial_{a_1}\ldots\partial_{a_n}\varphi^i\right)}.$$

According to (2.5) volume forms are exact in the space of local forms if and only if their Euler derivative with respect to each elementary field vanishes. In particular there may be solutions of (1.4) while a naive application of (2.4) would lead to the conclusion that there are no solutions of (1.4) even for ghost number 0.

3. BRS operator

Apart from the occurrence of anticommutators (1.3) has the form of a Lie algebra,

$$[D_A,\, D_B]=D_AD_B-(-)^{|A||B|}D_BD_A=f_{AB}{}^CD_C,$$

where $|A|$ denotes the grading of D_A (cf. appendix C)

$$D_A\in\left\{\partial_a,\, D_\alpha,\, \bar D_{\dot\alpha}\right\},\quad |a|=0,\quad |\alpha|=|\dot\alpha|=1,$$

and the only nonvanishing structure constants $f_{AB}{}^C$ are given by

$$f_{\alpha\dot\alpha}{}^a=f_{\dot\alpha\alpha}{}^a=-2i\sigma^a{}_{\alpha\dot\alpha}.$$

The BRS operator δ for the algebra (1.3) is constructed analogously to the Yang–Mills case [2]:

$$\delta\phi^r = C^A D_A \phi^r, \qquad \delta C^A = \tfrac{1}{2}(-)^{|B|} C^B C^C f_{CB}{}^A \tag{3.1}$$

which reads more explicitly ($C_{\alpha\dot\alpha} = \sigma^a{}_{\alpha\dot\alpha} C_a$)

$$\delta\phi^r = \left(C^a \partial_a + \xi^\alpha D_\alpha + \bar\xi^{\dot\alpha} \bar D_{\dot\alpha}\right)\phi^r, \qquad \delta C^{\alpha\dot\alpha} = 4i\xi^\alpha \bar\xi^{\dot\alpha}, \qquad \delta\xi^\alpha = \delta\bar\xi^{\dot\alpha} = 0.$$

The ghosts C^A have opposite grading compared to the corresponding generators D_A, the translation ghosts are chosen to be real:

$$C^A \in \{C^a, \xi^\alpha, \bar\xi^{\dot\alpha}\}, \quad |C^a| = 1, \quad |\xi^\alpha| = |\bar\xi^{\dot\alpha}| = 0, \quad C^a = (C^a)^*, \quad \bar\xi^{\dot\alpha} = (\xi^\alpha)^*.$$

On differentials and explicit coordinates δ is defined trivially,

$$\delta\, \mathrm{d}x^a = \delta x^a = 0.$$

On functions of the variables ϕ^r, C^A, x^a, $\mathrm{d}x^a$, the BRS operator is defined as antiderivation (fermionic first-order differential operator). By construction the BRS operator is real, fermionic, nilpotent and commutes with the partial derivatives,

$$\delta^2 = [\partial_a, \delta] = 0, \quad |\delta| = 1, \quad \delta = \delta^*.$$

4. D_α representations and QDS structure

I mentioned already that the SUSY algebra (1.3) is similar to a Lie algebra. However, apart from the occurrence of anticommutators there is another difference compared to the Yang–Mills case. Namely the representations of the SUSY algebra on the variables ϕ^r have infinite dimension since the partial derivatives occur in the algebra. But (1.3) contains two subalgebras, $\{D_\alpha, D_\beta\} = 0$ and $\{\bar D_{\dot\alpha}, \bar D_{\dot\beta}\} = 0$, whose representations on the ϕ^r decompose into finite-dimensional representations which are indecomposable (reducible but not completely reducible). This holds due to the assumption there are only finitely many elementary fields φ^i. It turns out that the general solution of (1.4) for a given theory depends on the kinds of indecomposable representations of these subalgebras present in the theory. In appendix B the derivation of the representation theory of $\{D_\alpha, D_\beta\} = 0$ is sketched. The multiplets transforming according to representations of $\{D_\alpha, D_\beta\} = 0$ are called D_α-multiplets. The possible indecomposable representations are given in the following on irreducible SL(2, C) tensors built of the ϕ^r, i.e.

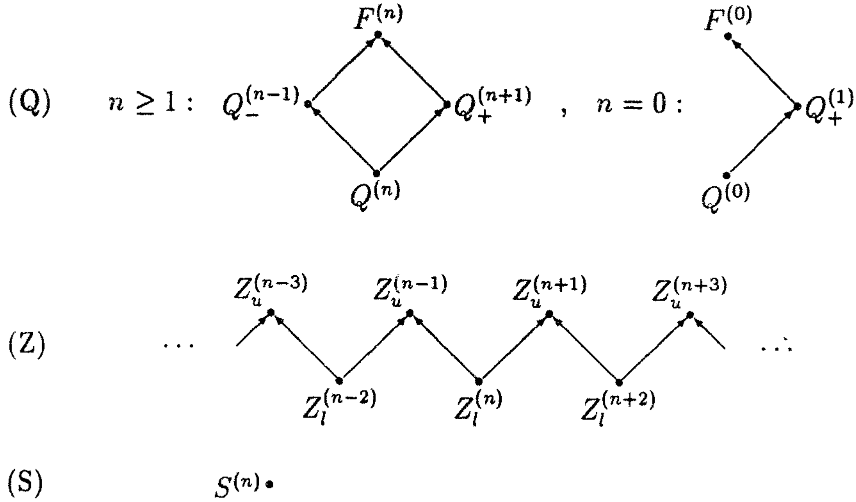
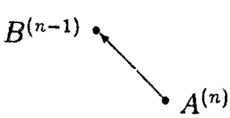


Fig. 1.

the component fields of these D_α-multiplets are irreducible SL(2, $\mathbb{C}$) tensors whose components are totally symmetric in their undotted and dotted spinor indices respectively and are linear combinations $\hat{\phi}^r = a_s^r \phi^s$ of the ϕ^r. There are 3 types of indecomposable D_α-multiplets on irreducible SL(2, $\mathbb{C}$) tensors. These types are denoted by (Q), (Z) and (S) which can be represented by the simple diagrams given in fig. 1 (the indecomposable $\bar{D}_{\dot\alpha}$-multiplets of course are analogous). Here $Q^{(n)}, \ldots, S^{(n)}$ denote the component fields of the D_α-multiplets and the arrows indicate how D_α acts on them (see fig. 2: an arrow pointing to the right denotes symmetrized D_α-transformation, an arrow pointing to the left denotes antisymmetrized D_α-transformation). All component fields of a particular multiplet carry the same number of dotted indices and the number of undotted indices is denoted by the superscript, e.g. $Q^{(n)}$ denotes an SL(2, $\mathbb{C}$) tensor whose components are totally symmetric in their n undotted indices and carry a definite number of dotted indices in which they are totally symmetric as well.

$$:\Leftrightarrow \qquad B^{(n+1)}{}_{\alpha_{n+1}\ldots\alpha_1} = \tfrac{1}{n+1} D_{(\alpha_{n+1}} A^{(n)}{}_{\alpha_n\ldots\alpha_1)}$$



$$:\Leftrightarrow \qquad B^{(n-1)}{}_{\alpha_{n-1}\ldots\alpha_1} = (-)^n \tfrac{n}{n+1} D^{\alpha_n} A^{(n)}{}_{\alpha_n\ldots\alpha_1}$$

Fig. 2.

The (Q) multiplets are quartets of SL(2, ℂ) tensors unless the lowest component field carries only dotted (but no undotted) spinor indices (in this case they are triplets).

The (S) multiplets are singlets, i.e. the components of the tensors $S^{(n)}$ are D_α-invariant linear combinations of the ϕ^r which cannot be written as linear combination of any $D_\alpha \phi^s$.

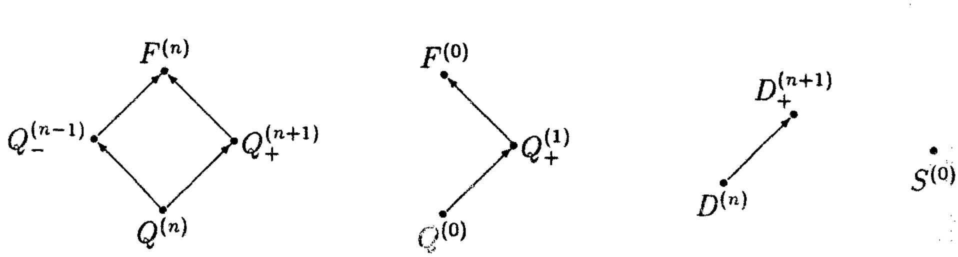
The zig–zag diagrams representing the (Z) multiplets can have arbitrary lengths, i.e. there are D_α-multiplets with an arbitrary number of component fields. However it can be shown that a particular theory can contain only (Z) multiplets whose number of component fields does not exceed some maximal value.

One can easily construct SUSY multiplets (multiplets of the complete SUSY algebra (1.3)) which contain an arbitrary D_α-multiplet of fig. 1 without imposing differential equations for the elementary fields making up these multiplets (cf. remark at the end of sect. 7). In other words: all D_α-multiplets represented by the diagrams of fig. 1 in fact may occur in supersymmetric theories.

However most of the prominent supersymmetric theories have a simple D_α-multiplet structure: the only indecomposable D_α-multiplets occurring in them are (Q) multiplets (with arbitrary number of indices), singlets $S^{(0)}$ without undotted indices and special (Z) multiplets represented by very short zig–zag diagrams containing only one arrow pointing to the right. For these special (Z) multiplets I introduce the notation (D) (see fig. 3). Theories with this simple D_α-multiplet structure are called QDS theories in the following.

Definition. A theory is called QDS theory if the D_α-representation decomposes into a sum of (Q) and (D) multiplets and singlets which have only dotted indices.

To prove the QDS structure of a theory one has to decompose its D_α-representation into indecomposable parts, i.e. one has to prove the existence of a basis $\{\hat{\phi}^r = a^r_s \phi^s\}$ consisting of components of irreducible SL(2, ℂ) tensors $T^{(n)}$ such that each $T^{(n)}$ is a component field of one and only one D_α-multiplet represented by



(Q) (Q) (D) (S)

Fig. 3.

one of the diagrams given in fig. 3. Note that a theory contains an infinite number of D_α-multiplets since there are infinitely many independent variables ϕ^r. Thus it may seem to be difficult to prove the QDS structure of a given theory. In practice however this turns out to be easier than it sounds since usually one can guess (and then prove) the complete D_α-multiplet structure of a theory after one has determined the "lowest" indecomposable D_α-multiplets containing the undifferentiated fields and their derivatives of lowest orders. Moreover, if one is interested only in solutions of (1.4) whose (power counting) dimension does not exceed a certain value $d_{\max}$ (e.g. for reasons of renormalizability) then it is sufficient to determine only those indecomposable D_α-multiplets which consist of the undifferentiated fields and their partial derivatives up to some maximal order which depends on $d_{\max}$.

Note that if a theory has QDS structure then the $\bar{D}_{\dot\alpha}$-multiplets have QDS structure as well since $\bar{D}_{\dot\alpha}$ is the complex conjugate of D_α which implies that $\{(\hat{\phi}^r)^*\}$ which is a basis of the ϕ as well makes up indecomposable $\bar{D}_{\dot\alpha}$-multiplets. However in general $\{(\hat{\phi}^r)^*\} \neq \{\hat{\phi}^r\}$ and in fact generally there is no basis consisting of components of indecomposable D_α-multiplets *and* indecomposable $\bar{D}_{\dot\alpha}$-multiplets. This complicates the determination of the representation theory of the complete SUSY algebra (1.3) which in fact is not known on the variables ϕ^r. (The chiral multiplet discussed below has the unusual property that one can choose $\{(\hat{\phi}^r)^*\} = \{\hat{\phi}^r\}$.)

To demonstrate how one determines the D_α-multiplet structure of a given SUSY multiplet I treat a simple example of a SUSY multiplet with QDS structure, namely a (scalar) chiral multiplet consisting of the elementary fields φ, ψ_α, F and the complex conjugate fields $\bar\varphi$, $\bar\psi_{\dot\alpha}$, $\bar F$ on which D_α acts according to

$$\begin{aligned} &D_\alpha\varphi = \psi_\alpha, \quad D_\alpha\psi_\beta = \epsilon_{\alpha\beta}F, \qquad D_\alpha F = 0, \\ &D_\alpha\bar\varphi = 0, \qquad D_\alpha\bar\psi_{\dot\alpha} = -2i\partial_{\alpha\dot\alpha}\bar\varphi, \quad D_\alpha\bar F = -2i\partial_{\alpha\dot\alpha}\bar\psi^{\dot\alpha}, \end{aligned} \tag{4.1}$$

where

$$\partial_{\alpha\dot\alpha} = \sigma^a{}_{\alpha\dot\alpha}\partial_a.$$

The complete list of indecomposable D_α-multiplets arising from (4.1) is given by

(i) (Q) multiplets whose lowest component fields are given by φ, $\bar F$ and the following derivatives of these fields * ($m, n \geq 0$, $\Box = \partial_a\partial^a$):

$$\Box^m\partial^{(\dot\alpha_n}_{(\alpha_n}\cdots\partial^{\dot\alpha_1)}_{\alpha_1)}\varphi, \qquad \Box^m\partial^{(\dot\alpha_n}_{(\alpha_n}\cdots\partial^{\dot\alpha_1)}_{\alpha_1)}\bar F;$$

* A set of independent components of these derivatives is a basis for the derivatives of φ and $\bar F$.

(ii) (D) multiplets which in the notation $(X, D_\beta X)$ read

$$\left(\partial^{(\dot\alpha_n}_{(\alpha_n}\ldots\partial^{\dot\alpha_1}_{\alpha_1)}\bar\psi^{\dot\beta)}, -2i\partial^{(\dot\alpha_n}_{(\alpha_n}\ldots\partial^{\dot\alpha_1}_{\alpha_1}\partial^{\dot\beta)}_{\beta)}\bar\varphi\right);$$

(iii) precisely one singlet (S) given by $\bar\varphi$.

One can check that the (Q) multiplets whose lowest components fields are given by $\bar F$ and its derivatives contain all derivatives of $\bar\varphi$ and $\bar\psi$ apart from those which make up the (D) multiplets. ψ, F and their derivatives group into the (Q) multiplets whose lowest component fields are given by φ and its derivatives. This proves the QDS structure of a chiral multiplet whose lowest component field is a Lorentz scalar. Further examples of SUSY multiplets which have QDS structure are "vector multiplets". A SUSY multiplet whose QDS structure is less obvious than that of chiral and vector multiplets is given by table 2 in sect. 7.

5. Results for QDS theories

For QDS theories the consistency equation (1.4) has been solved completely for each ghost number. The result is remarkably simple: for $G \neq 0$ there are only trivial solutions of $\delta\omega_4^G + \mathrm{d}\omega_3^{G+1} = 0$:

$$G \geqslant 1: \quad \delta\omega_4^G + \mathrm{d}\omega_3^{G+1} = 0 \quad \Leftrightarrow \quad \omega_4^G = \delta\eta_4^{G-1} + \mathrm{d}\eta_3^G. \tag{5.1}$$

In particular there are only trivial solutions with $G = 1$ and therefore there are no pure SUSY anomalies in QDS theories.

The nontrivial real solutions with $G = 0$ can always be written in the form of the integrand of (1.1):

$$\delta\omega_4^0 + \mathrm{d}\omega_3^1 = 0, \quad \omega_4^0 = \left(\omega_4^0\right)^* \quad \Leftrightarrow \quad \omega_4^0 = \mathrm{d}^4x\,\left\{\bar D^2\left[D^2 g(\phi) + h(S)\right] + \mathrm{c.c.}\right\} + \mathrm{d}\eta_3^0, \tag{5.2}$$

where $S = S^{(0)}$ are the D_α-singlets appearing in the theory.

Remark. Eq. (5.2) in fact contains more information about the structure of supersymmetric actions in the QDS case than (1.1) since it implies that in the QDS case each contribution $\bar D^2 h(\phi_c)$ to the lagrangian can be written also in the form $\bar D^2 D^2 h_1(\phi) + \bar D^2 h_2(S)$ for appropriate h_1, h_2 (note that $S \in \{\phi_c\}$ but $\{S\} \neq \{\phi_c\}$). This originates in (6.8).

6. Proof of the results for QDS theories

This section gives the proof of the results stated in sect. 5. Thereby a method is described how one can investigate (1.4) systematically also in the general (non-QDS) case and it is pointed out where the simple structure of the QDS result originates and where more general cases differ.

First I list two theorems which will be used within the proof. They treat the cohomologies of two operators δ_- and b which appear in the following decomposition of the BRS operator (3.1):

$$\delta = \delta_- + \delta_0 + \delta_+, \tag{6.1}$$

$$\delta_- = 4i\xi^\alpha\bar{\xi}^{\dot{\alpha}}\frac{\partial}{\partial C^{\alpha\dot{\alpha}}}, \tag{6.2}$$

$$\delta_0 = b + \bar{b}, \qquad b = \xi^\alpha D_\alpha, \qquad \bar{b} = \bar{\xi}^{\dot{\alpha}}\bar{D}_{\dot{\alpha}}, \tag{6.3}$$

$$\delta_+ = C^a\partial_a. \tag{6.4}$$

Theorem. δ_--cohomology:

$$\delta_- f(C^A) = 0 \quad \Leftrightarrow \quad f = P(\bar{\vartheta}, \xi) + Q(\vartheta, \bar{\xi}) + \Theta R + \delta_- Y(C^A), \quad Q|_{\bar{\xi}=0} = 0, \tag{6.5}$$

where R does not depend on the ghosts and

$$\vartheta^\alpha = \bar{\xi}_{\dot{\alpha}}C^{\alpha\dot{\alpha}}, \qquad \bar{\vartheta}^{\dot{\alpha}} = C^{\alpha\dot{\alpha}}\xi_\alpha, \qquad \Theta = \bar{\xi}_{\dot{\alpha}}C^{\alpha\dot{\alpha}}\xi_\alpha. \tag{6.6}$$

This theorem states that the nontrivial * parts of δ_--invariants have degree 0, 1 and 2 in the C^a since ϑ and $\bar{\vartheta}$ anticommute. Furthermore there is precisely one nontrivial contribution to δ_--invariants which depends both on ξ and $\bar{\xi}$, namely ΘR. In (6.5) the condition $Q|_{\bar{\xi}=0} = 0$ fixes arbitrarily the ghost number-0-part of f to appear in P (without this condition P and Q both can contain a part depending only on the ϕ). This guarantees that $f = P + Q + \Theta R + \delta_- Y$ is a direct sum:

$$P(\bar{\vartheta}, \xi) + Q(\vartheta, \bar{\xi}) + R\Theta + \delta_- Y(C^A) = 0, \quad Q|_{\bar{\xi}=0} = 0 \Leftrightarrow P = Q = R = \delta_- Y = 0. \tag{6.7}$$

Eqs. (6.5)–(6.7) can in principle be proved by inspecting all polynomials in the ghosts (separately for each ghost number). A proof can be found in ref. [6].

The cohomology of b (respectively, $\bar{b}$), eq. (6.3), depends on the special supersymmetric structure of a given theory. Here it becomes important which of the indecomposable D_α-representations represented by fig. 1 of sect. 4 appear in the theory. It turns out that one especially needs the b-cohomology on $l_{\alpha\beta}$-in-

* f is called (δ_-)-trivial if $f = \delta_- g$ for some g. Analogously f is called (b)-trivial if $f = bg$.

variant functions $f(\xi, \phi)$ where $l_{\alpha\beta} = l_{\beta\alpha}$ denote the generators of SL(2, $\mathbb{C}$) transformations of undotted indices (cf. appendix C). In the QDS case this cohomology is remarkably simple.

Theorem. b-cohomology on $l_{\alpha\beta}$-invariant functions $f(\xi, \phi)$ in QDS theories:

$$bf(\xi, \phi) = l_{\alpha\beta} f(\xi, \phi) = 0 \quad \Leftrightarrow \quad f(\xi, \phi) = D^2 g(\phi) + h(S) + bY(\xi, \phi) \tag{6.8}$$

where $S = S^{(0)}$ denote the D_α-singlets (which are $l_{\alpha\beta}$-invariant by the definition of QDS theories) and $l_{\alpha\beta} g = l_{\alpha\beta} Y = 0$. The proof of this theorem is given in appendix A.

Eq. (6.8) is the key to the solutions of (1.4) for QDS theories. (6.8) states that b- and $l_{\alpha\beta}$-invariant functions of the ξ and ϕ are b-trivial unless they are independent of ξ and that the only b-invariant functions $f(\phi)$ are precisely those which depend only on the D_α-singlets and those which can be written as $D^2 g(\phi)$. These b-invariants are obviously nontrivial since they do not depend on ξ. Generally (6.8) does not hold for theories which do not possess QDS structure (cf. sect. 7) and this is the reason for the fact that the results valid for QDS theories do not hold generally. In particular, candidates for pure SUSY anomalies arise from nontrivial b-invariants which depend on ξ. The absence of such b-invariants implies the absence of pure SUSY anomalies in the QDS case. I remark that (6.8) arises from a more general result given in eq. (A.24) of appendix A.

The proof of the results stated in sect. 5 proceeds in several steps (i)–(v) which are first described briefly and then explained in some more detail. The first three steps (i)–(iii) do not make any assumption on the supersymmetric multiplet structure and sketch the method used to solve (1.4) in the general case. (iv) and (v) are valid only for the QDS case.

(i) (a) Each Lorentz-invariant solution ω_4^G of (1.4) is related to a Lorentz-invariant zero form ω_0^g with ghost number $g = G + 4$ which does not depend explicitly on the coordinates x^a and solves

$$\delta\omega_0^g(C^A, \phi^r) = 0, \qquad \omega_0^g \neq \delta\eta_0^{g-1}(C^A, \phi^r), \qquad g = G + 4. \tag{6.9}$$

(b) ω_4^G is obtained from the corresponding solution of ω_0^g of (6.9) by taking the 4-form-part $[\omega]_4$ of a function $\omega(\mathrm{d}x^a, C^A, \phi^r)$ which arises from ω_0^g by replacing each translation ghost C^a contained in ω_0^g with $C^a + \mathrm{d}x^a$:

$$\omega_4^G = [\omega]_4, \quad \omega(\mathrm{d}x^a, C^A, \phi^r) = \omega_0^g(C^a + \mathrm{d}x^a, \xi^\alpha, \bar{\xi}^{\dot\alpha}, \phi^r). \tag{6.10}$$

(ii) (6.9) is decomposed into parts of definite translation ghost number (degree in the C^a)

$$\begin{gathered}\delta_- f_{\underline{n}} = 0, \quad \delta_0 f_{\underline{n}} + \delta_- f_{\underline{n}+1} = 0, \\ \delta_+ f_{\underline{n}} + \delta_0 f_{\underline{n}+1} + \delta_- f_{\underline{n}+2} = 0, \dots, \delta_+ f_{\bar{n}} = 0,\end{gathered} \tag{6.11}$$

where f_n denotes the part of ω_0^{G+4} which has translation ghost number n. $f_{\underline{n}}$ ($f_{\bar{n}}$) denotes the part of lowest (highest) translation ghost number contained in ω_0^g:

$$\omega_0^g = \sum_{n=\underline{n}}^{\bar{n}} f_n, \qquad C^a \frac{\partial f_n}{\partial C^a} = n f_n. \tag{6.12}$$

(iii) (6.11) is investigated starting from the first of these equations. This gives according to (6.5) due to $g \geqslant 4$:

$$f_{\underline{n}} = \delta_- Y_{\underline{n}+1} + P(\bar{\vartheta}, \xi, \phi) + Q(\vartheta, \bar{\xi}, \phi), \tag{6.13}$$

where P and Q have translation ghost number $\underline{n}$. Inserting (6.13) into the second of eqs. (6.11) yields (up to a δ_--trivial contribution) $f_{\underline{n}+1}$ in terms of the functions appearing in $f_{\underline{n}}$ and shows that the parts P and Q of $f_{\underline{n}}$ satisfy

$$bP = \bar{b}Q = 0. \tag{6.14}$$

To solve (6.14) one has to compute the b-cohomology. Having done this one proceeds by inserting $f_{\underline{n}+1}$ into the third of eqs. (6.11) etc.

(iv) In the QDS case eq. (6.14) implies

$$\begin{aligned} &\underline{n} \neq 2 \text{ or } g \geqslant 5: && P = b g_{\underline{n}}(\bar{\vartheta}, \xi, \phi), && Q = \bar{b} h_{\underline{n}}(\vartheta, \bar{\xi}, \phi), \\ &\underline{n} = 2, \quad g = 4: && P = \bar{\vartheta}_{\dot{\alpha}} \bar{\vartheta}^{\dot{\alpha}} p(\phi), && Q = \vartheta^{\alpha} \vartheta_{\alpha} q(\phi), \\ & && p = D^2 p_1(\phi) + p_2(S), && q = \bar{D}^2 q_1(\phi) + q_2(\bar{S}). \end{aligned} \tag{6.15}$$

(v) In the QDS case eq. (6.15) implies

$$g \geqslant 5: \quad \omega_0^g = \delta \eta_0^g, \tag{6.16}$$

$$g = 4: \quad \omega_0^4 = \delta \eta_0^3 + \hat{P} + \hat{Q}, \quad \hat{P} = \frac{i}{4} \bar{D}'^2 \{C^{\#} p\}, \quad \hat{Q} = -\frac{i}{4} D'^2 \{C^{\#} q\}, \tag{6.17}$$

where $C^{\#}$, $\bar{D}'_{\dot{\alpha}}$ and D'_{α} are given by

$$C^{\#} = -\tfrac{1}{24} \epsilon_{abcd} C^a C^b C^c C^d, \quad \bar{D}'_{\dot{\alpha}} = \left(\bar{D}_{\dot{\alpha}} \phi^r\right) \frac{\partial}{\partial \phi^r} + 4 i \xi^{\alpha} \frac{\partial}{\partial C^{\alpha\dot{\alpha}}}, \quad D'_{\alpha} = \left(\bar{D}'_{\dot{\alpha}}\right)^*. \tag{6.18}$$

The result of sect. 5 follows from (6.10), (6.16) and (6.17).

(i) has been proved in refs. [5,7] in a more general version valid not only for supersymmetric theories or four dimensions. I do not repeat the derivation of part (a) of the statement (i) but I note that it can be derived by applying δ to (1.4) by means of the algebraic Poincaré lemma (2.5). Part (b) follows simply from the fact that the operator (s + d) acts on the variables $(C^a + \mathrm{d}x^a)$, ξ^α, $\bar{\xi}^{\dot\alpha}$, ϕ^r exactly like s acts on the variables C^a, ξ^α, $\bar{\xi}^{\dot\alpha}$, ϕ^r. This implies

$$\mathrm{s}\omega_0^g = 0 \quad \Leftrightarrow \quad (\mathrm{s}+\mathrm{d})\omega = 0,$$

where ω has been given in (6.10). Denoting the p-form-part contained in ω by $[\omega]_p$, the identity $(\mathrm{s}+\mathrm{d})\omega = 0$ decomposes into the so-called descent equations

$$\mathrm{s}[\omega]_4 + \mathrm{d}[\omega]_3 = 0, \quad \mathrm{s}[\omega]_3 + \mathrm{d}[\omega]_2 = 0, \ldots, \mathrm{s}[\omega]_0 = 0,$$

which in particular shows that the 4-form-part $[\omega]_4$ of ω indeed solves (1.4).

(ii) respectively (6.11) follows simply from the fact that δ_+ increases the translation ghost number by one, δ_0 does not change this number and δ_- decreases it by one.

The first statement (6.13) of (iii) is a direct consequence of the first of eqs. (6.11) and of (6.5) (due to $G \geqslant 0 \Rightarrow \mathrm{g} \geqslant 4$ there is no contribution $\Theta R(\phi)$ to $f_{\underline{n}}$ since Θ has ghost number 3) *. The second statement (6.14) of (iii) follows from the second of eqs. (6.11) which requires $\delta_0 f_{\underline{n}}$ to be δ_--trivial. $\delta_0 f_{\underline{n}}$ is explicitly given by

$$\delta_0 f_{\underline{n}} = Z + bP + \bar{b}Q, \qquad Z = \delta_0 \delta_- Y_{\underline{n}+1} + \bar{b}P + bQ.$$

By means of (6.5) one concludes that Z is δ_--trivial since (a) Z is δ_--invariant due to $\delta_- P = \delta_- Q = 0$ and $\{\delta_-, b\} = \{\delta_-, \bar{b}\} = 0$ and (b) each contribution to Z depends both on ξ and $\bar{\xi}$ but none of these contributions is of the form $\Theta R(\phi)$ since Z has ghost number $(g+1) \geqslant 5$. The remaining contributions bP and $\bar{b}Q$ to $\delta_0 f_{\underline{n}}$ are also δ_--invariant but depend on the ghosts only via $\bar{\vartheta}$ and ξ (in the case of bP) respectively via ϑ and $\bar{\xi}$ (in the case of $\bar{b}Q$) and thus they are not δ_--trivial unless they vanish, cf. (6.7). This yields (6.14).

(iv) follows from (6.14) by means of (6.8). Namely for the various values of $\underline{n}$, P is explicitly given by

$$\begin{aligned} &\underline{n} = 0: \quad P = \omega^g(\xi, \phi), && \underline{n} = 1: \quad P = \bar{\vartheta}^{\dot\alpha} \omega_{\dot\alpha}^{g-2}(\xi, \phi), \\ &\underline{n} = 2: \quad P = \bar{\vartheta}_{\dot\alpha} \bar{\vartheta}^{\dot\alpha} \omega^{g-4}(\xi, \varphi), && \underline{n} = 3, 4: \quad P = 0, \end{aligned} \tag{6.19}$$

where the superscript of the ω's denotes their ghost number (degree in ξ). It is easy to see that the first condition (6.14) is equivalent to

$$\underline{n} = 0: \quad b\omega^g = 0, \qquad \underline{n} = 1: \quad b\omega_{\dot\alpha}^{g-2} = 0, \quad \underline{n} = 2: \quad b\omega^{g-4} = 0. \tag{6.20}$$

* ΘR contributes to $f_{\underline{n}}$ for $\underline{n} = 1$ if further symmetries are included into the investigation since then R can depend also on the ghosts referring to these symmetries. E.g. Fayet–Iliopoulos contributions [10] to lagrangians of super-Yang–Mills theories arise from $f_{\underline{n}} = \Theta C$ where C is an abelian ghost [6,8].

Since the ω's are $l_{\alpha\beta}$-invariant (due to the Lorentz invariance of ω_0^g) (6.8) can be applied to (6.20). Due to $g \geqslant 4$ there is only one case in which the respective ω does not depend on ξ, namely the case $g = 4$, $\underline{n} = 2$. In this case (6.8) states that P is of the form $\bar{\vartheta}_{\dot{\alpha}}\bar{\vartheta}^{\dot{\alpha}}p$ given in (6.15). In all other cases the respective ω is b-trivial according to (6.8). This implies that P is b-trivial as well (due to $b\bar{\vartheta}^{\dot{\alpha}} = 0$). Treating the second condition (6.14) analogously and inserting the results into (6.19) gives (6.15).

The first statement (6.16) of (v) is derived from (6.15) as follows: In the case $g \geqslant 5$, P is given by

$$P = bg_{\underline{n}}(\bar{\vartheta}, \xi, \phi) = \delta_0 g_{\underline{n}} - \bar{b}g_{\underline{n}}.$$

The part $\bar{b}g_{\underline{n}}$ is δ_--invariant (since $g_{\underline{n}}$ depends on the C^a only via $\bar{\vartheta}$), depends both on ξ and $\bar{\xi}$ and is not of the form $\Theta R(\phi)$ since it has ghost number not less than 5. Therefore $\bar{b}g_{\underline{n}}$ is δ_--trivial according to (6.5), i.e. P can be written in the form

$$P = \delta_0 g_{\underline{n}} + \delta_- \hat{g}_{\underline{n}+1}(C^A, \phi)$$

for some $\hat{g}_{\underline{n}+1}$. Analogously one concludes

$$Q = \delta_0 h_{\underline{n}} + \delta_- \hat{h}_{\underline{n}+1}.$$

Inserting this into (6.13) and the result into (6.12) shows that ω_0^g is of the form

$$\omega_0^g = \delta H + \mathrm{O}(\underline{n}+1), \qquad H = g_{\underline{n}} + h_{\underline{n}} + \hat{g}_{\underline{n}+1} + \hat{h}_{\underline{n}+1} + Y_{\underline{n}+1}. \tag{6.21}$$

where $\mathrm{O}(n)$ denotes contributions whose translation ghost number is not less than n (note that $\delta H = f_{\underline{n}} + \mathrm{O}(\underline{n}+1)$ due to $\delta_- g_{\underline{n}} = \delta_- h_{\underline{n}} = 0$). Now the function

$$\omega_0'^g := \omega_0^g - \delta H$$

is considered. It solves (6.9) as well (due to $\delta^2 = 0$) but starts at higher translation ghost number than ω_0^g. $\omega_0'^g$ is treated like ω_0^g and finally one concludes analogously to (6.21):

$$\omega_0'^g = \delta(H') + \mathrm{O}(\underline{n}+2) \quad \Leftrightarrow \quad \omega_0^g = \delta(H + H') + \mathrm{O}(\underline{n}+2). \tag{6.22}$$

One repeats the arguments until one has shown

$$\omega_0^g = \delta(H + H' + H'' + \dots) + \mathrm{O}(5) = \delta(H + H' + H'' + \dots),$$

which holds since there are no contributions O(5) with translation ghost number $\geqslant 5$ since the translation ghosts anticommute. This proves (6.16).

The case $g=4$, $\underline{n}<2$ can be traced back to the case $g=4$, $\underline{n}\geqslant 2$. Namely by the same arguments used in the case $g\geqslant 5$ to derive (6.21) and (6.22) one concludes

$$g=4,\ \underline{n}<2:\quad \omega_0^4=\delta(H+\dots)+\mathrm{O}(2).$$

Subtracting the trivial contribution $\delta(H+\dots)$ from ω_0^4 one gets a δ-invariant function $\omega_0'^4=\omega_0^4-\delta(H+\dots)$ which solves (6.9) but contains only contributions of translation ghost number not less than 2.

Now consider the case $g=4$, $\underline{n}=2$. According to (6.15) ω_0^4 in this case is of the form

$$g=4,\ \underline{n}=2:\quad \omega_0^4=\delta_-Y_3+P+Q+\mathrm{O}(3),\quad P=\bar{\vartheta}_{\dot{\alpha}}\bar{\vartheta}^{\dot{\alpha}}p(\phi),\quad Q=\vartheta^{\alpha}\vartheta_{\alpha}q(\phi),$$

where p and q are given in (6.15). Each P of this form can be completed to a solution $\hat{P}$ of (6.9) which is given in (6.17) and reads more explicitly

$$\hat{P}=\left\{\frac{i}{4}C^{\#}\bar{D}^2-\tfrac{1}{6}\epsilon_{abcd}C^aC^bC^c\xi^{\alpha}\sigma^d{}_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}}+\bar{\vartheta}_{\dot{\alpha}}\bar{\vartheta}^{\dot{\alpha}}\right\}\left[D^2p_1(\phi)+p_2(S)\right]. \quad (6.23)$$

Most easily one checks that $\hat{P}=\frac{1}{4}i\bar{D}'^2(C^{\#}p)$ solves $\delta\hat{P}=0$ using

$$\delta=\bar{\xi}^{\dot{\alpha}}\bar{D}'_{\dot{\alpha}}+b+\delta_+,\qquad \left\{\bar{D}'_{\dot{\alpha}},\,b+\delta_+\right\}=0,$$

where $\bar{D}'_{\dot{\alpha}}$ has been defined in (6.18). Analogously one completes Q to a solution $\hat{Q}$ of (6.9) (cf. eq. (6.17)). The proof of (6.17) is completed by treating the function

$$\tilde{\omega}_0^4=\omega_0^4-\hat{P}-\hat{Q}-\delta Y_3$$

like ω_0^g in the case $g\geqslant 5$ which finally leads to $\tilde{\omega}_0^4=\delta(Y_3+Y_4)$. Solutions $\hat{P}+\hat{Q}$ of (6.9) are trivial if and only if $(\bar{D}^2p-D^2q)$ is a total derivative.

Nontrivial $\hat{P}+\hat{Q}$ yield according to (6.10) solutions ω_4^0 of (1.4) given by

$$\omega_4^0=\mathrm{d}^4x\ \frac{i}{4}\left\{\bar{D}^2\left[D^2p_1(\phi)+p_2(S)\right]-D^2\left[\bar{D}^2q_1(\phi)+q_2(\bar{S})\right]\right\}.$$

The real part of ω_4^0 is the result stated in sect. 5 for QDS theories.

Remark (i). It is interesting to compare the b-cohomology (6.8) to the algebraic Poincaré lemma (2.5). The operators $b=\xi^{\alpha}D_{\alpha}$ and $d=\mathrm{d}x^a\,\partial_a$ have a similar form. However in the case of d the differentials anticommute and the derivatives ∂_a commute while in the case of b the ξ^{α} commute and the derivatives D_{α} anticommute. Now compare (6.8) and (2.5): The nontrivial cohomology classes of d are given by constants $\hat{\omega}_0(C^A)$ $(\partial_aC^A=0)$ and by those volume forms which have non-vanishing Euler derivative with respect to at least one φ^i. Volume forms contain the maximal number of the anticommuting differentials dx^a and are therefore always d-invariant. In the QDS case the nontrivial cohomology classes of b have a similar structure where the function $h(S)$ take the part of the constants ($D_\alpha S = 0$) and the remaining nontrivial cohomology classes can be obtained by applying the maximal number of the anticommuting derivatives to functions of ϕ.

Remark (ii). The δ-cohomology is now compared to the ordinary Lie algebra cohomology [11] of semisimple Lie algebras $\mathscr{G}$. In the BRS language the latter is the cohomology of the operator δ_{ym} given by

$$\delta_{ym} = \hat{\delta} - \left(C^I T_I \phi^r\right)\frac{\partial}{\partial \phi^r}, \qquad \hat{\delta} = -\tfrac{1}{2}C^J C^K f_{JK}{}^I \frac{\partial}{\partial C^I},$$

where the C^I denote anticommuting Yang–Mills ghosts and the T_I span a finite-dimensional representation of $\mathscr{G}$ according to which the ϕ^r transform. δ_{ym} of course is the BRS operator of ordinary Yang–Mills theories on the ghosts and on tensor fields ϕ^r. The cohomology of δ_{ym} is given essentially by that of $\hat{\delta}$, more precisely:

$$\delta_{ym} f(C^I, \phi) = 0 \quad \Leftrightarrow \quad f = P^i(\Theta_K)\mathscr{L}_i(\phi) + \delta_{ym} g(C^I, \phi),$$

where $\mathscr{L}_i$ are group scalars constructed of the ϕ and the P^i are polynomials in $\Theta_K = \Theta_K(C^I)$, $K = 1, \dots, \mathrm{rank}(\mathscr{G})$ which are certain polynomials of the ghosts and span the cohomology of $\hat{\delta}$ (they can be found e.g. in ref. [4]). The fact that nontrivial δ_{ym}-invariants depend on the ϕ only via group scalars $\mathscr{L}_i(\phi)$ expresses a result well known in mathematical literature, namely the vanishing of the cohomology groups $\mathrm{H}^g(\mathscr{G}, \mathrm{M})$ for all g unless the $\mathscr{G}$-module M is the trivial one (provided M is finite dimensional). In the SUSY case there are non-zero cohomology groups H^g for nontrivial (infinite dimensional) M. The representatives of these H^g are those solutions of (6.9) which depend on the ϕ and these are simultaneously those solutions of (6.9) which correspond to solutions of (1.4) (e.g. eq. (6.23) yields representatives of H^4). δ_- plays the same part for the algebra (1.3) as $\hat{\delta}$ does for an ordinary Lie algebra. Eq. (6.5) yields H^g for trivial M: those $P(\bar{\vartheta}, \xi)$, $Q(\vartheta, \bar{\xi})$, ΘR which do not depend on the ϕ and have ghost number g make up a complete list of representatives of H^g for trivial M.

7. Examples for the non-QDS case

To demonstrate that the results which are valid for QDS theories do not hold generally I discuss a simple example of a SUSY multiplet which has not QDS structure. In particular it is shown that in the presence of this multiplet there are nontrivial solutions of the consistency equation (1.4) with ghost number 1, i.e. candidates for pure SUSY anomalies.

TABLE 1

	χ_β	A	$V_{\beta\dot\beta}$	$\bar\psi_{\dot\beta}$	η_β	π	
D_α	$\epsilon_{\beta\alpha}A$	0	$-2i\partial_{\alpha\dot\beta}\chi_\beta-\epsilon_{\alpha\beta}\bar\psi_{\dot\beta}$	$2i\partial_{\alpha\dot\beta}A$	$2i\partial_{\alpha\dot\alpha}V_\beta{}^{\dot\alpha}+\epsilon_{\alpha\beta}\pi$	$2i\partial_{\alpha\dot\alpha}\bar\psi^{\dot\alpha}$	(7.1)
$\bar D_{\dot\alpha}$	$V_{\beta\dot\alpha}$	$-\bar\psi_{\dot\alpha}$	$\epsilon_{\dot\alpha\dot\beta}\eta_\beta$	$\epsilon_{\dot\alpha\dot\beta}\pi$	0	0	

The multiplet consists of the fields χ_α, A, $V_{\alpha\dot\alpha}$, ψ_α, η_α, π and the respective complex conjugate fields where χ, ψ, η are spin-$\frac{1}{2}$ fermions, V is a complex vector field, and A and π are complex scalar fields. On these fields D_α, $\bar D_{\dot\alpha}$ are defined according to table 1. One can check that the fields and their partial derivatives form a multiplet of (1.3) without leading to differential equations for the fields. It is obvious that this multiplet has not QDS structure since it contains an indecomposable D_α-multiplet of the (Z)-type represented by the diagram fig. 4. This (Z)-multiplet is not a (D)-multiplet but a (D′)-multiplet (cf. appendix A) since the arrow in fig. 4 points to the left.

An investigation of the complete D_α-multiplet structure (like performed in sect. 4 for the chiral multiplet) shows that this (D′)-multiplet in fact is the only D_α-multiplet arising from table 1 which is not a (Q) or (D) multiplet or a singlet $S^{(0)}$. Therefore this SUSY multiplet has QDSD′ structure (cf. appendix A).

Furthermore one immediately verifies that (6.8) is not valid in this case since there are nontrivial b- and $l_{\alpha\beta}$-invariant functions of the fields which depend on ξ given by

$$\xi^\alpha\chi_\alpha, \qquad \xi^\alpha\chi_\alpha\bar\psi'_{\dot\alpha}\bar\psi'^{\dot\alpha}, \tag{7.2}$$

where $\bar\psi'$ is the only D_α-singlet arising from (7.1),

$$\bar\psi'_{\dot\alpha}=\bar\psi_{\dot\alpha}-2i\partial_{\alpha\dot\alpha}\chi^\alpha.$$

The functions (7.2) are examples of contributions K to the b-cohomology (A.24) in QDSD′ theories ($NK=-K$, $n=1$ in this case). The existence of the nontrivial b-invariants (7.2) is responsible for the existence of nontrivial solutions of (1.4) with ghost number 1 given by the $\bar D^2$-transformations of these functions (times the volume element). A solution of (1.4) arising from $\xi^\alpha\chi_\alpha$ is given by

$$\omega_4^1=-\tfrac{1}{2}\mathrm{d}^4x\ \xi^\alpha\bar D^2\chi_\alpha=\mathrm{d}^4x\ \xi^\alpha\eta_\alpha. \tag{7.3}$$

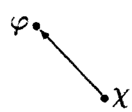


Fig. 4.

The solution (7.3) is obviously nontrivial since according to (7.1) there is no field whose D_α-transformation contains η_α. A solution of (1.4) arising from the second function (7.2) is given by

$$\omega_4^1 = \mathrm{d}^4x\ \xi^\alpha \overline{D}^2\left(\chi_\alpha \overline{\psi}'_{\dot\alpha} \overline{\psi}'^{\dot\alpha}\right). \tag{7.4}$$

One can check that (7.4) is nontrivial too. One also can construct solutions ω_4^1 which depend on the fields of the multiplet (7.1) and of further SUSY multiplets. E.g. from (7.1) and scalar chiral multiplets (4.1) one can construct the following solution of (1.4):

$$\omega_4^1 = \mathrm{d}^4x\ \xi^\alpha \overline{D}^2[\chi_\alpha f(\overline{\varphi})], \tag{7.5}$$

where $f(\overline{\varphi})$ is a function of the lowest component fields $\overline{\varphi}$ of the chiral multiplets which according to eq. (4.1) satisfy $D_\alpha \overline{\varphi} = 0$. The functions $\xi^\alpha \chi_\alpha f(\overline{\varphi})$ are further examples of contributions K to (A.24). These examples show that generally there exist solutions of (1.4) with ghost number 1 which are therefore called candidates for pure SUSY anomalies. They are not present in QDS theories (cf. sect. 5).

There is also a solution ω_4^0 of (1.4) containing the fields of the SUSY multiplet (7.1) which cannot be written in the form (5.2), namely

$$\omega_4^0 = \frac{a}{2} \mathrm{d}^4x\ \overline{D}^2 A + \text{c.c.} = \mathrm{d}^4x\ a\pi + \text{c.c.},$$

where a is a constant. This solution to (1.4) arises from the nontrivially b-invariant function given just by φ which represents an example of a contribution L to (A.24) (since $NA = -2A$, cf. (A.5)). I note that this contribution to a lagrangian is not of the form (5.2) (since A is not a D_α-singlet) but it is of the form of the integrand of (1.1) (since $D_\alpha A = 0$). To construct an example of a solution ω_4^0 which cannot be written in the form of the integrand of (1.1) I introduce a further SUSY multiplet consisting of the fields $\tilde\varphi$, $\tilde\chi_\alpha$, ρ_α, $\tilde V_{\alpha\dot\alpha}$, $\tilde\pi$, $\tilde\eta_\alpha$ and the respective complex conjugate fields. $\tilde\varphi$ and $\tilde\pi$ are complex scalar fields, $\tilde V$ is a complex vector field and $\tilde\chi$, ρ, $\tilde\eta$ are spin-$\frac{1}{2}$ fermions. If one chooses the SUSY transformations of these fields

TABLE 2

	$\tilde\varphi$	$\tilde\chi_\beta$	$\overline{\rho}_{\dot\beta}$	$\tilde V_{\beta\dot\beta}$	$\tilde\pi$	$\tilde\eta_\beta$
D_α	$\tilde\chi_\alpha$	0	$-2i\partial_{\alpha\dot\beta}\tilde\varphi - \tilde V_{\alpha\dot\beta}$	$-2i\partial_{\alpha\dot\beta}\tilde\chi_\beta$	$\tilde\eta_\alpha + 2i\partial_{\alpha\dot\alpha}\overline{\rho}^{\dot\alpha}$	$2i\partial_{\alpha\dot\alpha}\tilde V_\beta{}^{\dot\alpha}$
$\overline{D}_{\dot\alpha}$	$\overline{\rho}_{\dot\alpha}$	$\tilde V_{\beta\dot\alpha}$	$\epsilon_{\dot\alpha\dot\beta}\tilde\pi$	$\epsilon_{\dot\alpha\dot\beta}\tilde\eta_\beta$	0	0

according to table 2 one gets a SUSY multiplet which can be proved to have QDS structure.

From the multiplets given in tables 1 and 2 one can construct the following D_α-invariant function which is an example of a contribution K to (A.24):

$$f=\chi^\alpha\tilde{\chi}_\alpha-A\tilde{\varphi}. \tag{7.6}$$

f is D_α-invariant but cannot be written in the form $D^2g(\phi)+h(\phi_c)$ for any (polynomial) g and h. The corresponding solution

$$\omega_4^0=\mathrm{d}^4x\,\bar{D}^2\left(\chi^\alpha\tilde{\chi}_\alpha-A\tilde{\varphi}\right)$$

of (1.4) cannot be written in the form of the integrand of (1.1).

From the multiplet (7.1) one obtains a further SUSY multiplet which has not QDS structure by setting to zero the fields A, ψ, π (this can be done without imposing differential equations for one of the remaining fields or introducing further fields, see also remark at the end of this section). The resulting SUSY multiplet is an antichiral multiplet whose lowest component field is a D_α-singlet $S^{(1)}$ given by χ_α. This SUSY multiplet therefore has not QDS structure (recall that by definition the only D_α-singlets appearing in QDS theories are singlets $S^{(0)}$, i.e. singlets which carry only dotted but no undotted indices). I note that there are even more D_α-multiplets which are not (Q) or (D) multiplets or singlets $S^{(0)}$ in this case, namely (Z)-multiplets which have four component fields. This shows that the SUSY multiplet obtained from (7.1) by setting to zero A, ψ, π has not QDSD′ structure like the original multiplet (7.1) but has a more complicated D_α-multiplet structure. Such antichiral multiplets have been considered in ref. [9] where the solutions (7.5) have been found too (of course (7.5) solves (1.4) also in the case $A=\psi=\pi=0$). The antichiral multiplet arising from (7.1) for $A=\psi=\pi=0$ probably cannot be coupled to (minimal) supergravity unless one enlarges it again to the complete multiplet (7.1).

Remark. The multiplet (7.1) has been constructed starting from the (D′)-multiplet (χ_α, A) and completing it to a SUSY multiplet by introducing the $\bar{D}_{\dot\alpha}$- and $\bar{D}^2$-transformations of χ_α and A as further elementary fields. The D_α- and $\bar{D}_{\dot\alpha}$-transformations of all fields then are uniquely determined by requiring (respectively using) the SUSY algebra (1.3) which leads to (7.1). It can be shown that in the same way each D_α-multiplet given in fig. 1 of sect. 4 whose component fields are elementary fields can be completed to a SUSY multiplet without imposing differential equations for one of the fields. The SUSY multiplet in table 2 is constructed in this way starting from the (D)-multiplet ($\tilde{\varphi}$, $\tilde{\chi}_\alpha$) and the SUSY multiplet arising from (7.1) for $A=\psi=\pi=0$ also can be obtained in this way starting from the $S^{(1)}$-singlet χ_α.

8. Conclusion

It has been shown that the most general form of $N = 1$ globally supersymmetric actions and the answer to the question whether there are anomaly candidates in four-dimensional global SUSY depends on the structure of the SUSY multiplets, especially on the representations of the subalgebra $\{D_\alpha, D_\beta\} = 0$ present in a given theory. For a class of theories which contain only special representations of this subalgebra (QDS theories) all solutions of (1.4) have been classified: in QDS theories there are only solutions with ghost number 0 and these can always be written in the form of the integrand of (1.1) (more precisely: in the form (5.2)). In particular there are no pure SUSY anomalies in QDS theories. This result however is restricted to the QDS case and examples for the non-QDS case have been discussed for which there exist solutions of (1.4) with ghost number 1 and solutions with ghost number 0 which cannot be written in the form of the integrand of (1.1). Some of these solutions have been given explicitly (cf. sect. 7).

These results originate in the cohomology of the operator $b = \xi^\alpha D_\alpha$. In the QDS case this cohomology has been proved to be of the remarkable simple form (6.8) which implies the particular form (5.2) of the most general invariant action and the absence of pure SUSY anomalies. In the non-QDS case the b-cohomology has been shown to have more complicated structure in general. In particular generally there are nontrivial b-cohomology classes in the non-QDS case from which the candidates of pure SUSY anomalies arise (cf. sect. 7 and eq. (A.24) of appendix A).

The results presented in this paper have been used also within an investigation of the form of the most general invariant action and the anomaly-problem in globally supersymmetric Yang–Mills theories and supergravity theories. This investigation can be found in refs. [6,8].

I thank N. Dragon for many fruitful discussions on various questions which arose during the work on the subject of this paper.

Appendix A

b-COHOMOLOGY IN QDS- AND QDSD′-THEORIES

This appendix contains the complete proof of (6.8) and derives a more general result. Namely the investigation of the b-cohomology is extended to theories which may contain apart from (Q) and (D) multiplets and singlets $S^{(0)}$ also (D') multiplets (cf. fig. A1). The latter are doublets of the (Z)-type like (D)-multiplets

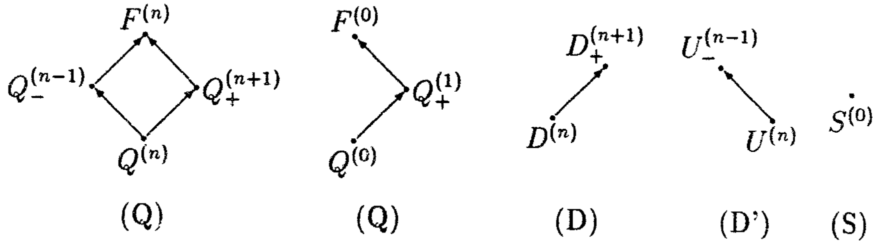


Fig. A.1.

but the arrow appearing in the diagram representing them points to the left instead of pointing to the right as in the case of (D)-multiplets. Analogously to the definition of a QDS theory I define:

Definition. A theory is called QDSD′ theory if its D_α-representation decomposes into a sum of (Q), (D) and (D′) multiplets and singlets $S^{(0)}$.

A basis whose elements make up these D_α-multiplets is denoted by $\hat{\mathscr{B}} = \{\hat{\phi}^r\}$ as in sect. 4. QDSD′ theories have a remarkable property which distinguishes them from all other theories: They allow to define operators t_β on $\hat{\mathscr{B}}$ whose anticommutators $O_{\beta\alpha}$ with the D_α are given by

$$O_{\beta\alpha} := \{t_\beta, D_\alpha\} = l_{\beta\alpha} + \tfrac{1}{2}\epsilon_{\beta\alpha} N \tag{A.1}$$

on all component fields of (Q), (D) and (D′) multiplets and on the singlets $S^{(0)}$. In (A.1) $l_{\alpha\beta}$ denote the generators of SL(2, ℂ) transformations of undotted spinor indices (cf. appendix C) and N is an operator which is diagonal on $\hat{\mathscr{B}}$, i.e. each $\hat{\phi}^r$ is an eigenfunction of N (see below). It can be shown that operators t_β yielding (A.1) exist only on (Q), (D) and (D′) multiplets and on singlets $S^{(0)}$ but not on other (Z) multiplets than (D) and (D′) and not on singlets $S^{(n)}$, $n > 0$. (A.1) turns out to be sufficient to compute the b-cohomology completely for QDS theories and almost completely for QDSD′ theories. In the following the component fields of (Q)-multiplets are denoted by $Q^{(n)}$, $Q_-^{(n-1)}$, $Q_+^{(n+1)}$ and $F^{(n)}$ like in sect. 4. D_α acts on them according to

$$D_\alpha Q^{(n)}{}_{\alpha_1\dots\alpha_n} = (-)^n \epsilon_{\alpha(\alpha_1} Q_-^{(n-1)}{}_{\alpha_2\dots\alpha_n)} + (n+1) Q_+^{(n+1)}{}_{\alpha\alpha_1\dots\alpha_n},$$

$$D_\alpha Q_-^{(n-1)}{}_{\alpha_2\dots\alpha_n} = n F^{(n)}{}_{\alpha\alpha_2\dots\alpha_n},$$

$$D_\alpha Q_+^{(n+1)}{}_{\alpha_0\alpha_1\dots\alpha_n} = (-)^{n+1} \epsilon_{\alpha(\alpha_0} F^{(n)}{}_{\alpha_1\dots\alpha_n)},$$

$$D_\alpha F^{(n)}{}_{\alpha_1\dots\alpha_n} = 0. \tag{A.2}$$

The component fields of the (D)-multiplets are denoted by $D^{(n)}$, $D_+^{(n+1)}$, those of the (D′)-multiplets by $U^{(n)}$, $U_-^{(n-1)}$ and the singlets by $S^{(0)}$. D_α acts on them according to

$$D_\alpha D^{(n)}{}_{\alpha_1\ldots\alpha_n} = (n+1) D_+^{(n+1)}{}_{\alpha\alpha_1\ldots\alpha_n}, \qquad D_\alpha D_+^{(n+1)}{}_{\alpha_0\ldots\alpha_n} = 0,$$

$$D_\alpha U^{(n)}{}_{\alpha_1\ldots\alpha_n} = (-)^n \epsilon_{\alpha(\alpha_1} U_-^{(n-1)}{}_{\alpha_2\ldots\alpha_n)}, \qquad D_\alpha U_-^{(n-1)}{}_{\alpha_2\ldots\alpha_n} = 0.$$

$$D_\alpha S^{(0)} = 0. \tag{A.3}$$

t_β is defined as follows:

$$t_\beta Q^{(n)}{}_{\alpha_1\ldots\alpha_n} = 0,$$

$$t_\beta Q_-^{(n-1)}{}_{\alpha_1\ldots\alpha_{n-1}} = (-)^n \frac{nk}{n+1} Q^{(n)}{}_{\beta\alpha_1\ldots\alpha_{n-1}},$$

$$t_\beta Q_+^{(n+1)}{}_{\alpha_0\ldots\alpha_n} = \frac{n+1-k}{n+1} \epsilon_{\beta(\alpha_0} Q^{(n)}{}_{\alpha_1\ldots\alpha_n)},$$

$$t_\beta F^{(n)}{}_{\alpha_1\ldots\alpha_n} = \frac{n+1-k}{n+1} \epsilon_{\beta(\alpha_1} Q_-^{(n-1)}{}_{\alpha_2\ldots\alpha_n)} + (-)^{n+1} k Q_+^{(n+1)}{}_{\beta\alpha_1\ldots\alpha_n},$$

$$t_\beta D^{(n)}{}_{\alpha_1\ldots\alpha_n} = 0,$$

$$t_\beta D_+^{(n+1)}{}_{\alpha_0\ldots\alpha_n} = \epsilon_{\beta(\alpha_0} D^{(n)}{}_{\alpha_1\ldots\alpha_n)},$$

$$t_\beta U^{(n)}{}_{\alpha_1\ldots\alpha_n} = 0,$$

$$t_\beta U_-^{(n-1)}{}_{\alpha_2\ldots\alpha_n} = (-)^n n U^{(n)}{}_{\beta\alpha_2\ldots\alpha_n},$$

$$t_\beta S^{(0)} = 0. \tag{A.4}$$

On functions of the fields t_β is defined as anti-derivation (fermionic first-order differential operator),

$$|t_\beta| = 1.$$

One can check explicitly by means of (A.2)–(A.4) that (A.1) holds on $\hat{\mathscr{B}}$. E.g. one calculates on $Q^{(n)}$:

$$\{t_\beta, D_\alpha\} Q^{(n)}{}_{\alpha_1\ldots\alpha_n} = \frac{nk}{n+1} \epsilon_{\alpha(\alpha_1} Q^{(n)}{}_{\alpha_2\ldots\alpha_n)\beta} + \frac{n(n+1-k)}{n+1} \epsilon_{\beta(\alpha_1} Q^{(n)}{}_{\alpha_2\ldots\alpha_n)\alpha}$$
$$+ \frac{n+1-k}{n+1} \epsilon_{\beta\alpha} Q^{(n)}{}_{\alpha_1\ldots\alpha_n}.$$

On the other hand one has

$$l_{\beta\alpha}Q^{(n)}{}_{\alpha_1\ldots\alpha_n}=\frac{n}{2}\epsilon_{\alpha(\alpha_1}Q^{(n)}{}_{\alpha_2\ldots\alpha_n)\beta}+\frac{n}{2}\epsilon_{\beta(\alpha_1}Q^{(n)}{}_{\alpha_2\ldots\alpha_n)\alpha}.$$

Subtracting this from the result obtained for $\{t_\beta, D_\alpha\}Q^{(n)}$ gives

$$\begin{aligned}&\left(\{t_\beta, D_\alpha\}-l_{\beta\alpha}\right)Q^{(n)}{}_{\alpha_1\ldots\alpha_n}\\&=\frac{2nk-n(n+1)}{2(n+1)}\left(\epsilon_{\alpha(\alpha_1}Q^{(n)}{}_{\alpha_2\ldots\alpha_n)\beta}-\alpha\leftrightarrow\beta\right)\\&+\frac{n+1-k}{n+1}\epsilon_{\beta\alpha}Q^{(n)}{}_{\alpha_1\ldots\alpha_n}=\tfrac{1}{2}(n-2k+2)\epsilon_{\beta\alpha}Q^{(n)}{}_{\alpha_1\ldots\alpha_n}\end{aligned}$$

which proves (A.1) on $Q^{(n)}$ and determines $NQ^{(n)}$: $NQ^{(n)}=(n-2k+2)Q^{(n)}$. Analogously one verifies (A.1) on all other fields and obtains N. The result for the (D) and (D′) multiplets and the singlets is

$$\begin{aligned}&ND^{(n)}=(n+2)D^{(n)}, \quad ND^{(n+1)}_{+}=(n+1)D^{(n+1)}_{+},\\&NU^{(n)}=-nU^{(n)}, \quad NU^{(n-1)}_{-}=-(n+1)U^{(n-1)}_{-},\\&NS^{(0)}=0,\end{aligned}\tag{A.5}$$

while on the (Q) multiplets it reads

$$\begin{aligned}&NQ^{(n)}=(n-2k+2)Q^{(n)}, \quad NQ^{(n-1)}_{-}=(n-2k+1)Q^{(n-1)}_{-},\\&NQ^{(n+1)}_{+}=(n-2k+1)Q^{(n+1)}_{+}, \quad NF^{(n)}=(n-2k)F^{(n)}.\end{aligned}\tag{A.6}$$

k can be chosen freely for each (Q) multiplet, i.e. there is a freedom in the definition of t_β on the (Q)-multiplets while on the (D), (D′) and (S) multiplets the definition (A.4) of t_β and the N-eigenvalues (A.5) are uniquely determined by the requirement (A.1) and the D_α-transformations (A.3). Eqs. (A.2)–(A.6) imply the following identities on $\hat{\mathscr{B}}$:

$$\begin{aligned}&\{t_\alpha, t_\beta\}=0, \quad [l_{\alpha\beta}, t_\gamma]=-\epsilon_{\gamma(\alpha}t_{\beta)}, \quad [N, t_\alpha]=t_\alpha, \quad [N, D_\alpha]=-D_\alpha,\\&[N, l_{\alpha\beta}]=0.\end{aligned}\tag{A.7}$$

Each function $f(\xi, \hat{\phi})$ can be decomposed into eigenfunctions of N (where N is defined trivially on ξ, $N\xi=0$)

$$f(\xi, \hat{\phi})=\sum_r f_r(\xi, \hat{\phi}), \quad Nf_r=rf_r.\tag{A.8}$$

The fourth of eqs. (A.7) implies $[N, b] = -b$. Thus bf_r has eigenvalue $(r-1)$ to N and therefore $bf = 0$ requires $bf_r = 0$ for all r,

$$bf = 0 \quad \Leftrightarrow \quad bf_r = 0 \quad \forall r. \tag{A.9}$$

One can therefore investigate $bf = 0$ without loss of generality for functions f with definite eigenvalue of N. Furthermore f can of course be assumed to have definite degree in ξ since b increases this degree by one. f therefore can be assumed to be given by

$$f = \xi^{\alpha_1} \dots \xi^{\alpha_n} \omega_{\alpha_1 \dots \alpha_n}(\hat{\phi}), \qquad N\omega_{\alpha_1 \dots \alpha_n} = r\omega_{\alpha_1 \dots \alpha_n}, \tag{A.10}$$

where ω is totally symmetric in its undotted indices,

$$\omega_{\alpha_1 \dots \alpha_n} = \omega_{(\alpha_1 \dots \alpha_n)}.$$

$bf = 0$ is equivalent to

$$D_{(\alpha}\omega_{\alpha_1 \dots \alpha_n)} = 0. \tag{A.11}$$

Application of $O^{\beta}{}_{\alpha}$, eq. (A.1), to $\omega_{\alpha_1 \dots \alpha_n}$ and symmetrization in all lower indices gives

$$O^{\beta}{}_{(\alpha}\omega_{\alpha_1 \dots \alpha_n)} = D_{(\alpha}t^{\beta}\omega_{\alpha_1 \dots \alpha_n)} = \frac{n+r}{2}\delta^{\beta}{}_{(\alpha}\omega_{\alpha_1 \dots \alpha_n)}. \tag{A.12}$$

The first equality in (A.12) follows from the definition of $O_{\beta\alpha}$ due to (A.11), the second equality in (A.12) follows from $O_{\beta\alpha} = l_{\beta\alpha} + \frac{1}{2}\epsilon_{\beta\alpha}N$ due to (A.10) and

$$l_{\beta(\alpha}\omega_{\alpha_1 \dots \alpha_n)} = \frac{n}{2}\epsilon_{\beta(\alpha}\omega_{\alpha_1 \dots \alpha_n)}. \tag{A.13}$$

(A.13) holds due to the assumption of $l_{\alpha\beta}$-invariance of f which implies that the index picture of $\omega_{\alpha_1 \dots \alpha_n}$ indicates its actual transformations property with respect to $l_{\alpha\beta}$-transformations of the $\hat{\phi}^r$. Contracting (A.12) with δ^{α}_{β} gives

$$-Dt\omega_{\alpha_1 \dots \alpha_n} + nD_{(\alpha_1}t^{\beta}\omega_{\alpha_2 \dots \alpha_n)\beta} = \frac{(n+r)(n+2)}{2}\omega_{\alpha_1 \dots \alpha_n}, \quad Dt := D^{\alpha}t_{\alpha}. \tag{A.14}$$

Applying D_{α} to this equation and symmetrizing in all indices yields due to (A.11)

$$D_{(\alpha}Dt\omega_{\alpha_1 \dots \alpha_n)} = 0, \tag{A.15}$$

i.e. $Dt\omega_{\alpha_1 \dots \alpha_n}$ satisfies (A.11) as well. Analogously to the derivation of (A.14) one concludes ($Dt\omega_{\alpha_1 \dots \alpha_n}$ has N-eigenvalues r like $\omega_{\alpha_1 \dots \alpha_n}$ due to (A.7)):

$$-(Dt)^2\omega_{\alpha_1 \dots \alpha_n} + nD_{(\alpha_1}t^{\beta}Dt\omega_{\alpha_2 \dots \alpha_n)\beta} = \frac{(n+r)(n+2)}{2}Dt\omega_{\alpha_1 \dots \alpha_n}. \tag{A.16}$$

The contribution $(Dt)^2\omega_{\alpha_1\dots\alpha_n}$ to (A.16) is treated by means of the following identities:

$$(Dt)^2 = Dt + D_\beta t^\gamma O^\beta{}_\gamma - \tfrac{1}{2}D^2t^2, \quad D^2 := D^\alpha D_\alpha, \quad t^2 := t^\alpha t_\alpha, \tag{A.17a}$$

$$D_\beta t^\gamma O^\beta{}_\gamma \omega_{\alpha_1\dots\alpha_n} = nD_{(\alpha_1}t^\beta \omega_{\alpha_2\dots\alpha_n)\beta} + \frac{n-r}{2}Dt\omega_{\alpha_1\dots\alpha_n}, \tag{A.17b}$$

$$n \neq 0: \quad D_{(\alpha_1}t^\beta D^2t^2\omega_{\alpha_2\dots\alpha_n)\beta} = \frac{n+r+2}{2}D^2t^2\omega_{\alpha_1\dots\alpha_n}. \tag{A.17c}$$

These identities can be proved using (A.1), (A.7) and (A.10). E.g. (A.17c) can be obtained using $t^\beta D^2t^2 = [t^\beta, D^2]t^2$ first (which holds since $t^\beta t^2 = 0$ due to the first of eqs. (A.7)), then working out the commutator $[t^\beta, D^2]$ using (A.1) and finally evaluating the SL(2, ℂ) generators and N-operators which arise.

Inserting (A.17a) into (A.16), using then (A.17b), (A.17c) and solving the resulting equation for $Dt\omega_{\alpha_1\dots\alpha_n}$ gives

$$\tfrac{1}{2}(n+1)(n+r+2)Dt\omega_{\alpha_1\dots\alpha_n} = D_{(\alpha_1}t^\beta\left\{\frac{1}{n+r+2}D^2t^2 + nDt - n\right\}\omega_{\alpha_2\dots\alpha_n)\beta}, \tag{A.18}$$

where $(n+r+2) \neq 0$ and $n \neq 0$ (the latter due to (A.17c)). Finally (A.18) is inserted into (A.14) which is then solved for $\omega_{\alpha_1\dots\alpha_n}$. The result is

$$\omega_{\alpha_1\dots\alpha_n} = -\frac{4}{(n+r+2)(n+r)(n+2)(n+1)}D_{(\alpha_1}t^\beta P^{(n,r)}\omega_{\alpha_2\dots\alpha_n)\beta}, \tag{A.19}$$

where the operator $P^{(n,r)}$ is given by

$$P^{(n,r)} = \frac{1}{n+r+2}D^2t^2 + nDt - \frac{n}{2}(n^2 + rn + 3n + 4).$$

This shows that $\omega_{\alpha_1\dots\alpha_n} = D_{(\alpha_1}\eta_{\alpha_2\dots\alpha_n)}$, i.e. f is trivial unless $n \in \{0, -r, -r-2\}$:

$$n \notin \{0, -r, -r-2\}: \quad f = bg, \quad g = \xi^{\alpha_1}\dots\xi^{\alpha_{n-1}}\eta_{\alpha_1\dots\alpha_{n-1}},$$

$$\eta_{\alpha_2\dots\alpha_n} = -\frac{4}{(n+r+2)(n+r)(n+2)(n+1)}t^\beta P^{(n,r)}\omega_{\beta\alpha_2\dots\alpha_n}. \tag{A.20}$$

I mentioned already that the constant k appearing in (A.4) and (A.6) can be arbitrarily chosen for each (Q) multiplet. Therefore the N-eigenvalue r of all parts of f which depend on component fields of (Q)-multiplets can be always chosen to be different from the values $(-n)$ and $(-n-2)$. One concludes that components of (Q) multiplets can contribute to nontrivial parts of solutions f of $bf=0$ only if $n=0$. This reflects a more general result holding for the dependence of b-invariant functions on component fields of (Q) multiplets (cf. remark at the end of this appendix).

The case $n=0$ needs a special treatment due to (A.17c). In this case one has

$$n=0: \quad f=\omega(\hat{\phi}), \quad bf=0 \Leftrightarrow D_\alpha \omega = 0. \tag{A.21}$$

Since ω is $l_{\alpha\beta}$-invariant one concludes from (A.1), (A.21) and $N\omega = r\omega$:

$$r \neq 0: \quad r\omega = \{t^\alpha, D_\alpha\}\omega = D_\alpha t^\alpha \omega = -Dt\omega \quad \Leftrightarrow \quad \omega = -\frac{1}{r} Dt\omega. \tag{A.22}$$

Making use of (A.22) (several times), (A.1), and $l_{\alpha\beta}\omega = 0$ one calculates for $n=0$:

$$\omega = -\frac{1}{r} Dt\omega = \frac{1}{r^2}(Dt)^2\omega = \frac{1}{r^2} D_\alpha (O^\alpha{}_\beta - D_\beta t^\alpha) t^\beta \omega$$

$$= -\frac{1}{2r^2} D^2 t^2 \omega - \frac{1}{r^2} D_\alpha (t^\alpha - t^\beta O^\alpha{}_\beta)\omega = -\frac{1}{2r^2} D^2 t^2 \omega + \frac{2-r}{2r^2} \overbrace{Dt\omega}^{-r\omega}$$

$$\Rightarrow \omega = -\frac{1}{r(r+2)} D^2 t^2 \omega \quad (r \neq 0, -2). \tag{A.23}$$

This shows that if $bf=0$ and $n=0$ then f is of the form $D^2 g(\phi)$ unless $r=0$ or $r=-2$. By the same arguments used below (A.20) one concludes that component fields of (Q) multiplets contribute only to solutions which can be written in the form $D^2 g(\phi)$. Altogether the result for QDSD′ theories can be stated in the following form:

Lemma 1. In QDSD′ theories the solutions $f(\xi, \hat{\phi})$ of $bf = l_{\alpha\beta} f = 0$ with ghost number n are of the form

$$n=0: \quad sf = D^2 g(\hat{\phi}) + L(\hat{\phi}) + K(\hat{\phi}),$$

$$n>0: \quad f = L(\xi, \hat{\phi}) + K(\xi, \hat{\phi}) + bY(\xi, \hat{\phi}), \tag{A.24}$$

where the N-eigenvalues of K and L equal $(-n)$ respectively $(-n-2)$

$$(N+n)K = 0, \qquad (N+n+2)L = 0,$$

and K and L do not depend on the component fields of (Q) multiplets:

$$L = L\left(\xi,\, D^{(n)},\, D_{+}^{(n+1)},\, U^{(n)},\, U_{-}^{(n-1)},\, S^{(0)}\right),$$

$$K = K\left(\xi,\, D^{(n)},\, D_{+}^{(n+1)},\, U^{(n)},\, U_{-}^{(n-1)},\, S^{(0)}\right).$$

This of course does not mean that each function K or L with the properties mentioned in the lemma is b-invariant. It only states that if there are any nontrivial b-invariants in a QDSD′ theory which are not of the form $D^2 g(\phi)$ then they contain necessarily a contribution $K+L$ (examples of nontrivial b-invariants K and L can be found in sect. 7). Therefore this lemma is not a complete determination of the b-cohomology in QDSD′ theories.

However lemma 1 yields a complete result for the b-cohomology in the QDS case, i.e. in the case where there are no (D′)-multiplets. Namely in this case there are no elements $\hat{\phi}^r$ which have negative N-eigenvalues due to (A.5), (A.6) since the eigenvalues of the (Q)-components can be chosen positive by choice of k. This shows that there are no contributions L to (A.24) (since these have negative N-eigenvalues for all n) and the only contributions K which can appear are those with $n = NK = 0$. In the QDS case such K can depend only on the singlets since they are the only fields which have vanishing eigenvalue to N while the eigenvalues of all remaining $\hat{\phi}^r$ are positive (by choice of k). Therefore the only contributions K which can contribute to (A.24) in the QDS case are functions $h(S)$ which obviously are b- and $l_{\alpha\beta}$-invariant. This proves (6.8).

Remark. One can prove the following statement which holds independently of the multiplet structure of a theory and is not restricted to $l_{\alpha\beta}$-invariant functions:

Lemma 2. Nontrivial contributions to b-invariant functions $f(\xi, \hat{\phi})$ with ghost number $n > 0$ do not depend on component fields of (Q)-multiplets and those with ghost number 0 depend on component fields of (Q)-multiplets only via functions of the form $D^2 g(\hat{\phi})$:

$$\begin{aligned} n = 0:&\quad f = D^2 g(\hat{\phi}) + M\left(Z_l^{(n)},\, Z_u^{(n)},\, S^{(n)}\right),\\ n > 0:&\quad f = M\left(\xi,\, Z_l^{(n)},\, Z_u^{(n)},\, S^{(n)}\right) + bY(\xi, \hat{\phi}). \end{aligned}$$

Here $Z_l^{(n)}$, $Z_u^{(n)}$, $S^{(n)}$ denote the component fields of D_α-multiplets of the (Z) and (S) type (cf. fig. 1 of sect. 4). The lemma is proved analogously to the proof of lemma 1 by means of operators r_β whose anticommutators with D_α on all component fields of the (Q), (Z) and (S) multiplets are given by

$$\{r_\beta, D_\alpha\} = \tfrac{1}{2}\epsilon_{\beta\alpha} N_Q,$$

where N_Q is the counting operator of the component fields of the (Q) multiplets. r_β is defined according to

$$r_\beta Q^{(n)}{}_{\alpha_1\ldots\alpha_n} = 0,$$

$$r_\beta Q_-^{(n-1)}{}_{\alpha_1\ldots\alpha_{n-1}} = (-)^{n+1}\frac{n}{2(n+1)}Q^{(n)}{}_{\beta\alpha_1\ldots\alpha_{n-1}},$$

$$r_\beta Q_+^{(n+1)}{}_{\alpha_0\ldots\alpha_n} = \frac{1}{2(n+1)}\epsilon_{\beta(\alpha_0}Q^{(n)}{}_{\alpha_1\ldots\alpha_n)},$$

$$r_\beta F^{(n)}{}_{\alpha_1\ldots\alpha_n} = \frac{1}{2(n+1)}\epsilon_{\beta(\alpha_1}Q_-^{(n-1)}{}_{\alpha_2\ldots\alpha_n)} + (-)^n\tfrac{1}{2}Q_+^{(n+1)}{}_{\beta\alpha_1\ldots\alpha_n},$$

$$r_\beta Z_l^{(n)}{}_{\alpha_1\ldots\alpha_n} = r_\beta Z_u^{(n)}{}_{\alpha_1\ldots\alpha_n} = r_\beta S^{(n)}{}_{\alpha_1\ldots\alpha_n} = 0.$$

Appendix B

REPRESENTATION THEORY OF $\{D_\alpha, D_\beta\} = 0$

Like in sect. 4 $A^{(n)}$ denotes an irreducible SL(2, ℂ) tensor field, i.e. $A^{(n)}$ is totally symmetric in its undotted and dotted indices respectively. The superscript (n) indicates the number of undotted indices of $A^{(n)}$, dotted indices are omitted. The D_α-transformation of $A^{(n)}$ generally is given by

$$D_\alpha A^{(n)}{}_{\alpha_1\ldots\alpha_n} = (-)^n\epsilon_{\alpha(\alpha_1}B^{(n-1)}{}_{\alpha_2\ldots\alpha_n)} + (n+1)B^{(n+1)}{}_{\alpha\alpha_1\ldots\alpha_n}.$$

where $B^{(n-1)}$ and $B^{(n+1)}$ are irreducible SL(2, ℂ) tensors carrying $(n-1)$ respectively $(n+1)$ indices ($B^{(n-1)}$, $B^{(n+1)}$ or both may be zero). The action of D_α is represented by the diagrams introduced in sect. 4, see fig. 2.

The possible finite-dimensional indecomposable D_α-multiplets are easily obtained from the following statements:

(i) Diagrams representing D_α-multiplets do not contain one of the segments of fig. B.1

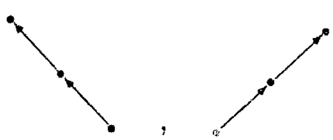
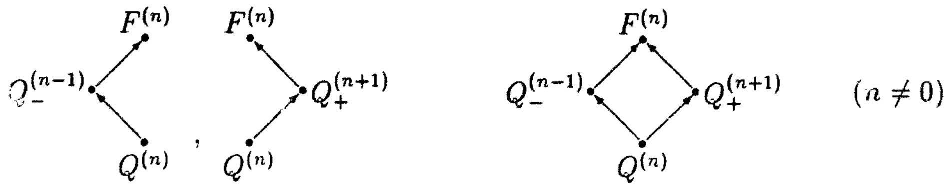


Fig. B.1.



(a) (b)

Fig. B.2.

(ii) If a diagram representing a D_α-multiplet contains one of the segments of fig. B.2a then it contains also fig. B.2b unless $n=0$ (Q_- vanishes if $n=0$).

(i) holds due to the algebra $\{D_\alpha, D_\beta\}=0 \Rightarrow D_\alpha D_\beta = \frac{1}{2}\epsilon_{\alpha\beta}D^2$ and due to the symmetry of irreducible SL(2, $\mathbb{C}$) tensors which imply the following identities whose graphical translation is (i):

$$D^{\alpha_1}D^{\alpha_2}A^{(n)}{}_{\alpha_1\ldots\alpha_n} = \tfrac{1}{2}D^2\epsilon^{\alpha_2\alpha_1}A^{(n)}{}_{\alpha_1\ldots\alpha_n} = 0,$$

$$D_{(\alpha}D_\beta A^{(n)}{}_{\alpha_1\ldots\alpha_n)} = \tfrac{1}{2}D^2\epsilon_{(\alpha\beta}A^{(n)}{}_{\alpha_1\ldots\alpha_n)} = 0.$$

(ii) states that $D^2Q^{(n)} \neq 0$ requires $Q_+^{(n+1)} \neq 0$ and $Q_-^{(n-1)} \neq 0$ unless $n=0$ ($D^2Q^{(0)} \neq 0$ requires $Q_+^{(1)} \neq 0$). E.g. $Q_+^{(n+1)}=0$ implies $D^\alpha Q_+^{(n+1)}{}_{\alpha\alpha_1\ldots\alpha_n}=0$ which contradicts $D^2Q^{(n)} \neq 0$ since

$$(n+1)D^\alpha Q_+^{(n+1)}{}_{\alpha\alpha_1\ldots\alpha_n} = D^\alpha D_{(\alpha}Q^{(n)}{}_{\alpha_1\ldots\alpha_n)} = \frac{n+2}{2(n+1)}D^2Q^{(n)}{}_{\alpha_1\ldots\alpha_n}.$$

From (i) and (ii) one easily concludes that fig. 1 of sect. 4 gives a complete list of diagrams representing indecomposable D_α-diagrams, i.e. there exists a basis for the irreducible SL(2, $\mathbb{C}$) tensor fields such that the representation of $\{D_\alpha, D_\beta\}=0$ on the tensor fields decomposes into a sum of D_α-multiplets represented by the diagrams given in fig. 1 of sect. 4.

Appendix C. Conventions and notation

C.1. GRADING

The grading $|\varphi| \in \{0, 1\}$ of a variable φ (a field or one of its partial derivatives or a differential or a coordinate) is defined modulo 2 by the sum of its ghost number (gh), its form degree (deg) and the number of its spinor indices (dotted and undotted):

$$\left|\varphi_{\alpha_1\ldots\alpha_n}^{\dot\alpha_1\ldots\dot\alpha_m}\right| = \deg(\varphi) + \mathrm{gh}(\varphi) + m + n \quad (\text{modulo } 2).$$

The grading of the variables determines their statistics,

$$\varphi^1\varphi^2 = (-)^{|\varphi^1||\varphi^2|}\varphi^2\varphi^1.$$

An object X (variable or operator) is called fermionic if $|X| = 1$. The grading is additiv modulo 2, i.e.

$$|XY| = |X| + |Y| \quad (\text{modulo } 2).$$

Derivatives with respect to a variable act from the left according to

$$\frac{\partial\varphi^s}{\partial\varphi^r} = \delta_r^s$$

and therefore have the same grading as the variable itself,

$$\left|\frac{\partial}{\partial\varphi}\right| = |\varphi|.$$

The grading of operators which can be represented in the form

$$\mathscr{O}_{(n)} = \sum_{k=0}^{n} \sum_{r_1\ldots r_k} f^{r_1\ldots r_k}(\varphi)\frac{\partial}{\partial\varphi^{r_1}}\cdots\frac{\partial}{\partial\varphi^{r_k}}$$

follows from the grading of the variables. First-order differential operators $\mathscr{O}_{(1)}$ satisfy the product rule

$$\mathscr{O}_{(1)}(\varphi^1\varphi^2) = (\mathscr{O}_{(1)}\varphi^1)\varphi^2 + (-)^{|\mathscr{O}_{(1)}||\varphi^1|}\varphi^1(\mathscr{O}_{(1)}\varphi^2).$$

C.2. COMPLEX CONJUGATION

Complex conjugation of an object X (a variable or an operator) is denoted by X^* or by $\bar{X}$. The complex conjugate of a product of two objects X, Y (variables or operators) is defined by

$$(XY)^* = (-)^{|X||Y|}X^*Y^*.$$

In particular the complex conjugate $\mathscr{O}^*$ of an operator $\mathscr{O}$ is defined by

$$\mathscr{O}^*\varphi = (-)^{|\mathscr{O}||\varphi|}(\mathscr{O}\varphi^*)^*.$$

Note that $(\partial/\partial\varphi)^*$ and $\partial/\partial\varphi^*$ generally are not identical but related by

$$\left(\frac{\partial}{\partial\varphi}\right)^* = (-)^{|\varphi|}\frac{\partial}{\partial\varphi^*}.$$

In particular the derivative with respect to a real fermionic variable is a purely imaginary operator. The operators D_A have the following reality properties:

$$(\partial_a)^* = \partial_a, \qquad (D_\alpha)^* = \bar{D}_{\dot\alpha}.$$

C.3. SYMMETRIZATION, ANTISYMMETRIZATION

Symmetrization respectively antisymmetrization of indices are denoted by brackets ( ) respectively [ ], e.g.

$$T_{(a_1\dots a_n)} = \frac{1}{n!}\sum_\pi T_{a_{\pi(1)}\dots a_{\pi(n)}}, \qquad T_{[a_1\dots a_n]} = \frac{1}{n!}\sum_\pi (-)^{\mathrm{sign}(\pi)} T_{a_\pi(1)\dots a_\alpha(n)},$$

where Σ_π runs over all permutations in the symmetric group, $\pi \in S_n$.

C.4. LORENTZ (SL(2, ℂ)) ALGEBRA

Minkowski metric, ϵ^{abcd}:

$$\eta_{ab} = \mathrm{diag}(1, -1, -1, -1), \quad \epsilon^{abcd} = \epsilon^{[abcd]}, \quad \epsilon^{0123} = 1.$$

Invariant SL(2, ℂ) tensors

$$\epsilon^{\alpha\beta} = -\epsilon^{\beta\alpha}, \quad \epsilon^{\dot\alpha\dot\beta} = -\epsilon^{\dot\beta\dot\alpha} = \left(\epsilon^{\alpha\beta}\right)^*, \quad \epsilon^{12} = \epsilon^{\dot1\dot2} = 1,$$

$$\epsilon_{\alpha\gamma}\epsilon^{\gamma\beta} = \delta_\alpha^\beta = \mathrm{diag}(1, 1), \quad \epsilon_{\dot\alpha\dot\gamma}\epsilon^{\dot\gamma\dot\beta} = \delta_{\dot\alpha}^{\dot\beta} = \mathrm{diag}(1, 1).$$

σ-matrices: $\sigma^a{}_{\alpha\dot\beta}$, α: row index, $\dot\beta$: column index

$$\sigma^0 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \quad \sigma^1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad \sigma^2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \quad \sigma^3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}.$$

$\bar\sigma$-matrices:

$$\bar\sigma^{a\dot\alpha\alpha} = \epsilon^{\dot\alpha\dot\gamma}\epsilon^{\alpha\gamma}\sigma^a{}_{\gamma\dot\gamma}.$$

Raising and lowering of spinor indices:

$$\psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta, \quad \psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta, \quad \bar\psi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\bar\psi^{\dot\beta}, \quad \bar\psi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\bar\psi_{\dot\beta}$$

σ^{ab}, $\bar\sigma^{ab}$-matrices:

$$\sigma^{ab}{}_\alpha{}^\beta = \tfrac{1}{4}(\sigma^a\bar\sigma^b - \sigma^b\bar\sigma^a)_\alpha{}^\beta, \quad \bar\sigma^{ab\dot\alpha}{}_{\dot\beta} = \tfrac{1}{4}(\bar\sigma^a\sigma^b - \bar\sigma^b\sigma^a)^{\dot\alpha}{}_{\dot\beta}$$

Contraction of spinor indices:

$$\psi\chi := \psi^\alpha\chi_\alpha, \qquad \bar\psi\bar\chi := \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha}.$$

Vector indices in spinor notation:

$$V_{\alpha\dot\alpha} = \sigma^a{}_{\alpha\dot\alpha}V_a.$$

Lorentz (SL(2, $\mathbb{C}$)) generators:

$$l_{ab} = \sigma_{ab}{}^{\alpha\beta}l_{\alpha\beta} - \bar\sigma_{ab}{}^{\dot\alpha\dot\beta}\bar l_{\dot\alpha\dot\beta}, \quad \bar l_{\dot\alpha\dot\beta} = (l_{\alpha\beta})^*,$$

$$[l_{ab}, l_{cd}] = \eta_{bc}l_{ad} + \eta_{ad}l_{bc} - \eta_{bd}l_{ac} - \eta_{ac}l_{bd}.$$

Lorentz (SL(2, $\mathbb{C}$)) transformations of spinor- and vector-indices:

$$l_{\alpha\beta}\psi_\gamma = -\epsilon_{\gamma(\alpha}\psi_{\beta)}, \quad \bar l_{\dot\alpha\dot\beta}\psi_\gamma = 0, \quad \bar l_{\dot\alpha\dot\beta}\bar\psi_{\dot\gamma} = -\epsilon_{\dot\gamma(\dot\alpha}\bar\psi_{\dot\beta)}, \quad l_{\alpha\beta}\bar\psi_{\dot\gamma} = 0,$$

$$l_{ab}V_c = \eta_{bc}V_a - \eta_{ac}V_b, \quad l_{ab}\psi_\alpha = -\sigma_{ab\alpha}{}^\beta\psi_\beta, \quad l_{ab}\bar\psi^{\dot\alpha} = -\bar\sigma_{ab}{}^{\dot\alpha}{}_{\dot\beta}\bar\psi^{\dot\beta}$$

References

[1] L. Baulieu, Phys. Rep. 129 (1985) 1

[2] C. Becchi, A. Rouet and R. Stora, Ann. Phys. (N.Y.) 98 (1976) 287

[3] F. Brandt, N. Dragon and M. Kreuzer, Nucl. Phys. B332 (1990) 224

[4] F. Brandt, N. Dragon and M. Kreuzer, Nucl. Phys. B332 (1990) 250

[5] F. Brandt, N. Dragon and M. Kreuzer, Nucl. Phys. B340 (1990) 187

[6] F. Brandt, Lagrangedichten und Anomalien in vierdimensionalen supersymmetrischen Theorien, PhD thesis, Technische Informationsbibliothek der Universität Hannover (1991)

[7] F. Brandt, Dependence of BRS invariant local functionals of tensor- gauge- and ghost-fields, preprint ITP-UH 11/91

[8] F. Brandt, Lagrangians and anomaly candidates of $D = 4$, $N = 1$ super-Yang–Mills and supergravity theories, preprint ITP-UH 4/92

[9] J.A. Dixon, Phys. Rev. Lett. 67 (1991) 797; Does Supersymmetry Break Itself?, preprint UTTG-01-91

[10] P. Fayet and J. Iliopoulos, Phys. Lett. B51 (1974) 461

[11] G. Hochschild and J.-P. Serre, Ann. Math. 57 (1953) 591

[12] P.J. Olver, Application of Lie groups to differential equations (Springer, New York, 1986)

[13] O. Piguet, M. Schweda and K. Sibold, Nucl. Phys. B174 (1980) 183; O. Piguet and K. Sibold, Nucl. Phys. B247 (1984) 484

[14] O. Piguet, M. Schweda and K. Sibold, preprint UGVA-DPT 1992/5-763