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Anomalous dimension of the twist four gluon operator and pomeron cuts in deep inelastic scattering

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The anomalous dimension of the twist four gluonic operator (γ_4) in deep inelastic scattering is calculated in the double log approximation (DLA) of perturbative QCD. It turns out that at $N \to 1$ $\gamma_4(N-1)$ is close to $2\gamma_2((N-1)/2)$ where γ_2 is the anomalous dimension of the leading twist operator. It means that at $N \to 1$ the contribution of the twist four operator to the deep inelastic structure function becomes very important and gives rise to the screening correction to the deep inelastic structure function as was taken into account in the nonlinear GLR evolution equation.

1. Introduction

It is well known that the deep inelastic structure function rapidly increases in the low-x region. It is the reason why we have to take into account the absorption (shadowing) correction (see ref. [1] for details). In refs. [1,2] it was shown that the main contribution to the absorption corrections comes from the "fan" diagrams (see fig. 1a) where two gluons annihilate in one. In the t-channel these diagrams are those where two gluons transit into four (see fig. 1a).

On the other hand there is the widespread opinion that operators with different dimensions cannot mix in the renormalization group equation (see ref. [3] where this point of view has been advocated in the most direct and explicit way). The above transition is nothing more than the specific example of such mixing. Thus the question arises how to explain this conflict.

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Our answer to this question is given in sect. 2, where we consider as an example the "fan" diagrams. It is shown in sect. 2 that for these Feynman diagrams the anomalous dimension of the twist four gluonic operator (γ_4) becomes equal to

$$\gamma_4(\omega) = 2\gamma_2(\omega/2) \tag{1}$$

where γ_2 is the anomalous dimension of the leading twist operator while ω is equal to $N-1$ (N is energy–moment index). It means that γ_4 increases at small ω ($x \to 0$) more rapidly than γ_2 and at $\omega = \omega_{\rm cr}(q^2)$ the full (canonical plus anomalous) dimensions of the leading twist operator and the twist four operator turn out to be equal. Thus we cannot neglect the contribution of the next twist operator to the deep inelastic structure function at $\omega \leqslant \omega_{\rm cr}$.

In sects. 3 and 4 the anomalous dimensions of all twist four gluonic operator are calculated in the double log approximation (DLA) of perturbative QCD. In other words it means that only the terms of the order of $(\alpha_2 \ln(1/x) \ln q^2)^n$ are taken into account in the perturbation expansion. In the general case not only two pairs of gluons in t-channel can interact but also all gluons. The first case is related to two pomeron-like exchange in t-channel in the old-fashioned reggeon language while the second one is nothing more than the so-called many particle Regge poles or the new bounded states in t-channel [4]. It is worthwhile mentioning that we try to use here the reggeon language because we firmly believe that namely the reggeon approach is very close technically to the usual energy–moments representation, has a clear space-time interpretation and is more adequate to Feynmann diagrams which is our perturbative QCD than the formal approach based on Wilson's operator expansion and the renormalization group. We see our goal in establishing a more transparent relation between the both languages.

It turns out that the anomalous dimension of the operator which corresponds to two Pomeron exchange (see fig. 1c) is very close to the position of the two pomeron cut (see fig. 1b). This result confirms the assumption that was made in refs. [1,2,5] and allows us to neglect as a first step the contribution of all diagrams except the diagrams of fig. 1b in the region of large q^2 and small x. Namely this approach has led to the nonlinear GLR-evolution equation [1], and this paper can be considered as a first step in the improvement of it.

In sect. 5 the emission of the soft gluons is considered and it is shown that such an emission does not lead to infrared singularities.

We would like also to mention from the beginning that the correct evolution equation for the anomalous dimensions of higher twist operators have been written in ref. [6]. Our goal was to solve these equations in the region of small x or $\omega \to 0$. However on the way we got the equations for the anomalous dimension of the twist four operator in the simpler DLA form than the one used in ref. [6].

We would also like to note that we do not use the technique developed in ref. [7] for DLA preferring the direct summation of the Feyman diagrams since it looks

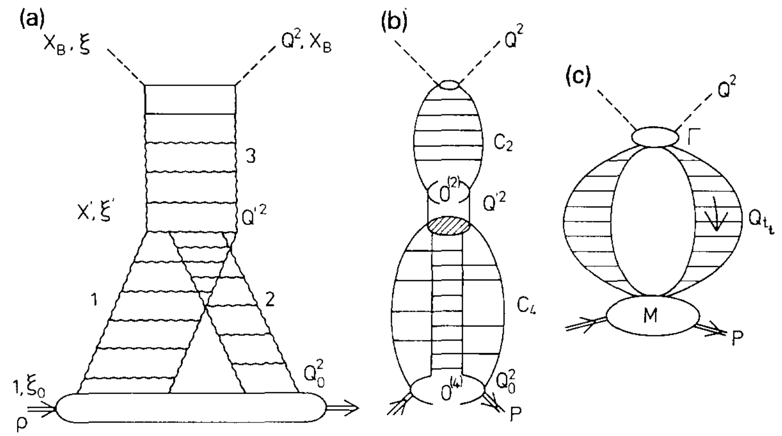

Fig. 1. (a) The simplest "fan" diagram. (b) The mixing of the operators in the simplest "fan" diagram. (c) The anomalous dimension of the twist four operator in the two-ladder approximation.

more transparent from our point of view. However we checked that all our results could be obtained in a such technique too.

2. The mixing of operators of different twists

In this section we would like to clarify what is meant by mixing of operators of different twists. Let us start from an attempt to rewrite the 'fan' diagram of fig. 1 in terms of operators of different twists in the DLA of QCD. The easiest way to do this is to go to the moment representation. In this representation the contribution of the diagram of fig. 1 to the ω moment of the deep inelastic structure function looks as follows:

$$F(\omega) = \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{\omega Q'^2} C_2(Q^2, \mu^2) \langle \text{gluon}(Q'^2) | O^{(2)} | \text{gluon}(Q'^2) \rangle$$

$$\times C_4(Q'^2, \mu^2) \langle N | O^{(4)} | N \rangle, \tag{2}$$

where Q'^2 is the virtuality of the intermediate gluonic state (see fig. 1b) and μ^2 is an arbitrary scale of QCD. $1/\omega Q'^2$ is the propagator of two gluons in the t-channel in DLA.

$F(\omega)$ as a physical quantity does not depend on the scale μ. We can choose this scale $\mu = Q'$. In this case the matrix element $\langle \text{gluon}(Q'^2) | O^{(2)} | \text{gluon}(Q'^2) \rangle$ is very simple and is equal to 1 in our normalization. The coefficient function $C_2(Q^2, Q'^2)$ is also well known, namely

$$C_2(Q^2, Q'^2) = \left(\frac{Q^2}{Q'^2} \right)^{-1+\gamma_2(\omega)},$$

where γ_2 is the anomalous dimension of the leading twist operator while 1 is the canonical dimension of it.

The dependence of the matrix element of $O^{(4)}$ on the large virtuality Q'^2 is

$$\langle N | O^{(4)}(Q'^2) | N \rangle = \left(\frac{Q'^2}{Q_0^2} \right)^{-2+\gamma_4(\omega)} \langle N | O^{(4)}(Q_0^2) | N \rangle,$$

where Q_0^2 is the typical virtuality inside of the nucleon (N) and 2 and γ_4 are the canonical and anomalous dimension of the twist four operator.

Thus we are able to calculate the integral (2) integrating over Q' in an explicit way. Finally we have

$$\begin{aligned} F(\omega) &= \int_{Q_0^2}^{Q^2} \frac{\mathrm{d}Q'^2}{\omega Q'^2} \left(\frac{Q^2}{Q'^2} \right)^{-1+\gamma_2(\omega)} \left(\frac{Q'^2}{Q_0^2} \right)^{-2+\gamma_4(\omega)} \langle N | O^{(4)}(Q_0^2) | N \rangle \\ &= \frac{1}{\omega} \cdot \frac{1}{1 - \gamma_4(\omega) + \gamma_2(\omega)} \left\{ \left(\frac{Q^2}{Q_0^2} \right)^{-1+\gamma_2(\omega)} - \left(\frac{Q^2}{Q_0^2} \right)^{-2+\gamma_4(\omega)} \right\} \\ &\quad \times \langle N | O^{(4)}(Q_0^2) | N \rangle. \end{aligned} \tag{3}$$

Looking directly at the integral (2) we can conclude that the first term in eq. (3) corresponds to the value of Q' of the order of Q_0. It means that in this case we cannot calculate in an explicit way the matrix element of the operator $O^{(2)}$, so it is better to rewrite the first term in the form

$$\frac{1}{\omega} \cdot \frac{1}{1 - \gamma_4 + \gamma_2} \left(\frac{Q^2}{Q_0^2} \right)^{1+\gamma_2(\omega)} \langle N | O^{(2)}(Q_0^2) | N \rangle,$$

absorbing all the bottom part of the diagram in fig. 1b as some renormalization of the matrix element $\langle N | O^{(2)}(Q_0^2) | N \rangle$.

In the second term of eq. (3) the typical value of Q' is of the order of Q, so for it the form in eq. (3) is correct. Finally eq. (3) can be reduced to

$$\begin{aligned} F(\omega) &= \frac{1}{\omega} \cdot \frac{1}{1 - \gamma_4(\omega) + \gamma_2(\omega)} \\ &\times \left\{ \left(\frac{Q^2}{Q_0^2} \right)^{-1+\gamma_2(\omega)} \langle N | O^{(2)}(Q_0^2) | N \rangle - \left(\frac{Q^2}{Q_0^2} \right)^{-2+\gamma_4(\omega)} \langle N | O^{(4)}(Q_0^2) | N \rangle \right\}. \end{aligned} \tag{4}$$

From eq. (4) we can easily see that for the small values of the anomalous dimensions γ_2 and γ_4 which are of the order of α_s * the attempt to take into account the mixing of operators of different twists leads us only to a renormalization of their matrix elements. This statement is in an agreement with the general theorems [3,8].

However the situation changes crucially if $\gamma_4 - \gamma_2$ tends to 1. This is just the case when the total dimensions of the leading twist operator and the next twist one become equal and they both give contributions of the same order to the deep inelastic structure function. The integral (3) gives an extra $\log(Q^2/Q_0^2)$, and the final answer for $F(\omega)$ can be written in the form

$$F(\omega) = \alpha \log\left(\frac{Q^2}{Q_0^2}\right) \cdot \left(\frac{Q^2}{Q_0^2}\right)^{-1+\gamma_2(\omega)} \langle N \,|\, O^{(4)}(Q_0^2) \,|\, N \rangle \tag{5}$$

in the region $\omega \to \omega_{\mathrm{cr}}$ where ω_{cr} can be found from the relation

$$1 + \gamma_2(\omega_{\mathrm{cr}}) - \gamma_4(\omega_{\mathrm{cr}}) = 0. \tag{6}$$

Eq. (4) can be interpreted as a mixing of two operators since the typical virtuality in the diagram of fig. 1b was sufficiently large, namely $Q'^2 \sim QQ_0$. However within the same accuracy we can consider the above result as the contribution to the anomalous dimension of the twist four operator in mixing with the leading twist operator because the leading and the next twist operators give the same contribution at $\omega = \omega_{\mathrm{cr}}$.

The above statement does not contradict any general theorems [8]. However it should be stressed that we introduce the new mass scale (Q_0^2) $(\langle N \,|\, O^{(4)} \,|\, N \rangle)$ which we call the typical virtuality of the gluon inside the nucleon. Even more we would like to note that Q_0^2 was introduced in such a way that $\langle N \,|\, O^{(4)} \,|\, N \rangle \to 0$ at $4Q_0^2 \to 0$. This scale is irrelevant to the initial virtuality of the photon from which we start to solve the evolution equation (see ref. [7] for detail discussion of this point). The physical origin of this new scale is, of course, confinement that breaks the conformal symmetry of QCD. This is the reason why we can apply to the operator expansion only general theorems of a massive theory but not for massless ones **. The last remark is the real origin of some misunderstanding between the standard approach advocated in ref. [3] and the one which we are discussing now.

To estimate the value of the anomalous dimension of the twist four operator let us consider the diagram of fig. 1c in DLA of perturbative QCD. In DLA the

* For simplicity we neglect here the running of α_s.

** We are very grateful to J. Bartels and J. Blumlein for discussion on this subject.

solution of GLAP evolution equation [9,10] for the deep inelastic structure function of gluons in the leading twist approximation looks as follows:

$$xG(x, Q^2) \propto \exp\left(\sqrt{\frac{4N_c\alpha_s}{\pi}\ln\frac{Q^2}{Q_0^2}\ln\frac{1}{x}}\right). \tag{7}$$

In the moment representation the anomalous dimension related to the above solution is equal to

$$\gamma_2(\omega) = \frac{N_c\alpha_s}{\pi\omega} + O(\alpha_s), \tag{8}$$

where the correction of the order of α_s does not increase in the region of small ω.

The diagram of fig. 1c can be written in the form

$$C_4(Q'^2, Q_0^2, \omega)\langle N | O^{(4)}(Q_0^2, \omega) | N\rangle$$
$$= \int \frac{d\omega'}{2\pi i} d^2Q_t M(Q_t, Q_0)\Gamma \exp\{\gamma_2(\omega-\omega')r' + \gamma_2(\omega')r' - 2r'\}, \tag{9}$$

where $r' = \ln(Q'^2/Q_0^2)$. In eq. (9) we took into account that the anomalous dimension of the leading twist operator did not depend on the squared momentum transfer Q_t^2 in DLA (see ref. [1] for the detail discussion of this point). The amplitude $M(Q_t, Q_0)$ describes the emission of four gluons from the nucleon. Thus it is obvious that we can interpret the integral over Q_t as follows:

$$\int d^2Q_t M(Q_t, Q_0) = \langle N | O^{(4)}(Q_0^2) | N\rangle.$$

We would like to draw attention to the fact that the above integral introduces the new scale Q_0^2 which has the physical meaning of the correlation radius between two gluons inside the nucleon (see also ref. [7]). It is also very essential that we assume that the above integral is convergent with respect to the integration over Q_t.

For large r' we can use the saddle-point approximation to calculate the integral over ω' which gives the answer

$$C_4(Q'^2, Q_0^2, \omega)\langle N | O^{(4)}(Q_0^2, \omega) | N\rangle$$
$$\propto \exp\left\{\gamma_2\left(\frac{\omega}{2}\right)r' + \gamma_2\left(\frac{\omega}{2}\right)r' - 2r'\right\}\langle N | O^{(4)}(Q_0^2) | N\rangle. \tag{10}$$

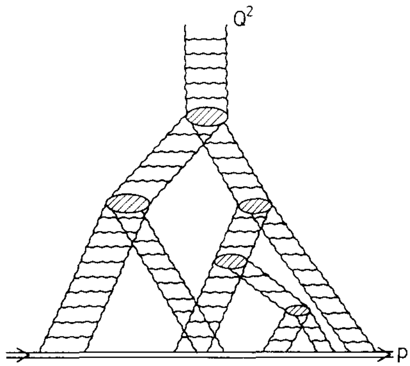

Fig. 2. The "fan" diagram.

Thus one can see directly from eq. (10) that the anomalous dimension of the twist four operator for the diagrams of fig. 1c is equal to

$$\gamma_4(\omega) = 2\gamma_2\left(\frac{\omega}{2}\right) = \frac{4N_c\alpha_s}{\pi\omega}. \tag{11}$$

We can find the value of ω_{cr} from eq. (6) which leads to

$$\omega_{cr} = \frac{3N_c\alpha_s}{\pi}.$$

The detail analysis of all 'fan' diagrams (see fig. 2) shows that

$$\omega_{cr} = \frac{2N_c\alpha_s}{\pi}, \tag{12}$$

(see refs. [1,11,12]). It means that the contributions of the leading twist operator and the next one become of the same order at $x = x_{cr}$ where

$$\ln\frac{1}{x_{cr}} = \frac{b}{114}\ln^2\frac{Q^2}{Q_0^2},$$

if we take $N_c = 3$ and $\alpha_s = 4\pi/b \ln(Q^2/\Lambda^2)$.

However the principal dynamical assumption in ref. [1] was that the main contribution to the anomalous dimension of the twist four gluon operator comes from the diagrams of fig. 1c type. In the next three sections we would like to prove this assumption and we are going to calculate the anomalous dimension of the twist four gluon operator in DLA in a consistent way taking into account all essential diagrams.

3. Anomalous dimension of the twist four operator in DLA (general approach)

3.1. SELECTION OF DIAGRAMS

We restrict ourselves to the calculation of the anomalous dimension of the twist four gluon operator in DLA of perturbative QCD since we are interested in the kinematical region where the anomalous dimensions are large enough $\gamma \approx 1$. This is just the region of small x, where the smallness of the QCD coupling constant α_s is compensated by the large value of the $\ln(1/x)$ ($\alpha_s \ln(1/x) \geqslant 1$). In DLA the kernel of the GLAP [9,10] evolution equation ($P_G^G(\omega)$) looks as follows:

$$P_G^G(\omega) = \frac{N_c \alpha_s}{\pi \omega} \tag{13}$$

and describes the exchange of a gluon in the s-channel between two t-channel gluons as shown in fig. 3a. It is worthwhile mentioning that the kernel of the FKL equation [13] has the same form despite the fact that FKL equation was the result of the summation of contributions of the order of $(\alpha_s \ln(1/x))^n$ in each order of the perturbation expansion but not $(\alpha_s \ln Q^2)^n$ as it was done in the GLAP equation.

Let us denote by arrows the direction of the longitudinal momenta or the fraction of the hadron momentum x in the Breit frame of the t-channel (vertical) gluons in fig. 3b. It was shown in the early seventies [4] that the only interaction between lines with different direction of the arrows gives rise to the large $\ln(1/x)$. It means that only pairs of gluons with the colours indices "a" and "b", "c" and "b", "a" and "d" or "c" and "d" but not "a" and "c" or "b" and "d" could interact in fig. 3b. Thus there are only four type of s-channel gluons in fig. 3b that are able to interact in arbitrary order. Each of them is 'hard' in the sense that it carries the dominant fraction of the longitudinal component of the momentum x transferred in the t-channel ($x_{k_1} \approx x_{q_1} \gg x_{q_2}$ in fig. 3b).

Using the direction of lines in the diagrams we can classify them in the following way:

[2,2]: the state of four gluons in the t-channel in which two of them have the same direction of the energy fraction x_i (the same arrows) as shown in fig. 3b.

[3,1]: the state in which three gluons have the same direction of the arrows (see fig. 3d).

These are two states which we have to discuss separately in the framework of DLA. They cannot by mixed since otherwise two gluons with the same direction of the arrows would have to interact in order to change the direction of one of the arrows. However such an interaction does not give a $\log(1/x)$ contribution as it has been mentioned before [4].

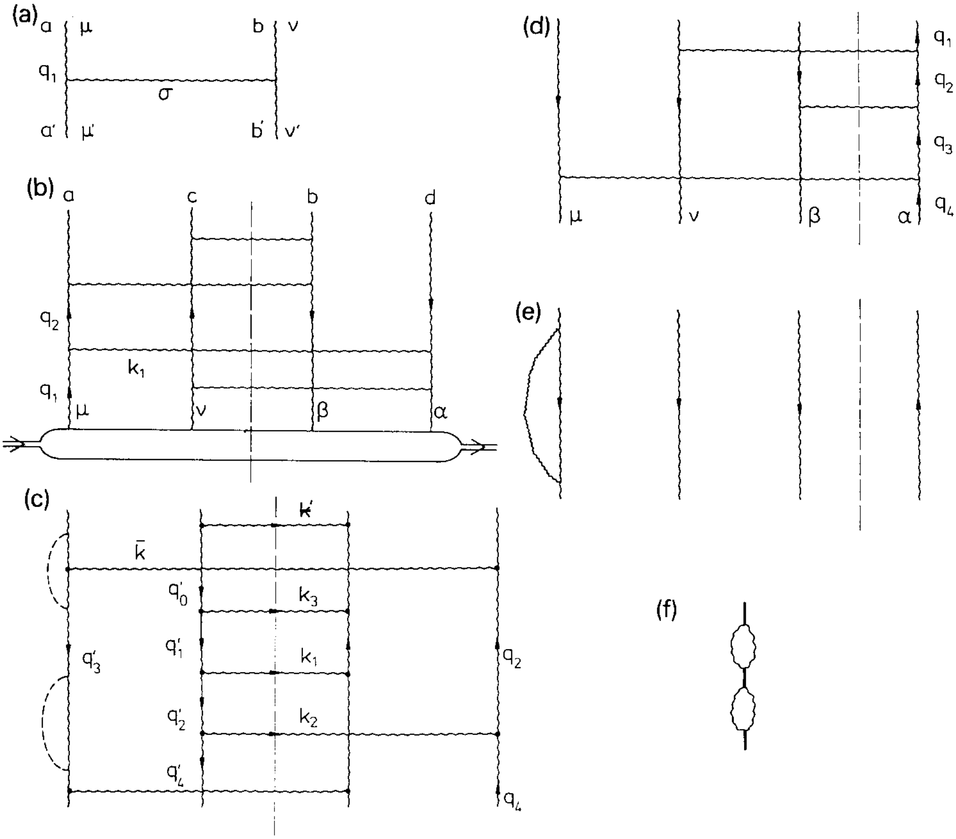

Fig. 3. (a) The interaction of gluons in the Born approximation. (b) The interaction of gluons in the [2,2] state. (c) The diagram for the [2,2] state. (d) The diagram for the [3,1] state. (e) Self-energy correction.

It turns out that the most important state is [2,2] which we are going to discuss in a separate section (sect. 4). The contribution of [3,1] will be considered in subsect. 3.5.

Generally speaking the double log contributions could also come from the diagrams with emission of "soft" gluons for which $x_{k_1} \gg x_{q_1} \approx x_{q_2}$ in fig. 3b. However such gluons do not contribute to the anomalous dimension since four gluons in the t-channel create the colourless state. So the double log contributions from "soft" gluon emission are cancelled in the sum of the diagrams, including both real and virtual "soft" gluons. The last ones are shown as dashed lines in fig. 3c. This property has been discussed many times (see for example ref. [17]) and has very transparent physical meaning. Indeed, a "soft" longwave gluon could be only emitted coherently by the state of four gluons as a whole. However such an emission is very small (vanishes in DLA) due to the zero global charge of our four gluon operator. In sect. 5 we illustrate this point in detail.

There is another type of "soft" gluons (with small transverse momenta) that corresponds to the interaction between the t-channel lines. All these gluons together with the self-energy (reggeization) corrections of t-channel gluons should be summed up separately. However it is shown that this emission also does not contribute to the value of anomalous dimension (see sect. 5).

3.2. COLOUR STRUCTURE

Before performing the calculation let us discuss the colour of the four gluon state in t-channel. Four colour indices of SU(3) can be contracted in the following way:

$$\begin{aligned}&1.\,\delta_{ab}\delta_{cd}; \quad && 2.\ \delta_{ac}\delta_{bd}; \quad && 3.\ \delta_{ad}\delta_{bc};\\ &4.\,d_{abe}d_{cde}; && 5.\ d_{ace}d_{bde}; && 6.\ d_{ade}d_{bce};\\ &7.\,f_{abe}f_{cde}; && 8.\ f_{ace}f_{bde}; && 9.\ f_{ade}f_{bce}.\end{aligned} \tag{14}$$

However in SU(3) we have four relations between these tensors, namely

$$f_{abe}f_{cde}+f_{ace}f_{dbe}+f_{ade}f_{bce}=0, \tag{15}$$

which exists in any SU(N) group.

$$d_{abe}d_{cde}+d_{ace}d_{dbe}+d_{ade}d_{bce}=\tfrac{1}{3}(\delta_{ab}\delta_{cd}+\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc}), \tag{16}$$

$$f_{abe}f_{cde}+f_{cbe}f_{ade}+\delta_{ab}\delta_{cd}+\delta_{ad}\delta_{bc}-\delta_{ac}\delta_{bd}=3d_{ace}d_{bde}, \tag{17}$$

$$d_{abe}d_{cde}-d_{ade}d_{bce}-f_{ace}f_{bde}=\tfrac{2}{3}(\delta_{ad}\delta_{bc}-\delta_{ab}\delta_{cd}). \tag{18}$$

The last three relations are valid only in SU(3) *. Thus in SU(3) there are only three independent colour tensors. It is convenient to choose them as the projectors on the SU(3) representation for the pair of gluons. For the pair (a, b) they look as follows:

$$\begin{aligned}P_0&=\tfrac{1}{8}\delta_{ab}\delta_{cd},\\ P_8&=\frac{i^2}{3}f_{abe}f_{cde},\\ P_{\bar{8}}&=\tfrac{3}{5}d_{abe}d_{cde},\\ P_{27}&=\tfrac{1}{2}\{\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc}-\tfrac{1}{4}\delta_{ab}\delta_{cd}-\tfrac{6}{5}d_{abe}d_{cde}\},\\ P_{10}+P_{\overline{10}}&=\tfrac{1}{2}\left\{\delta_{ad}\delta_{bc}-\delta_{ac}\delta_{bd}-\frac{2i^2}{3}f_{abe}f_{cde}\right\}.\end{aligned} \tag{19}$$

* We are grateful to A. Bukhvostov who point out these relations.

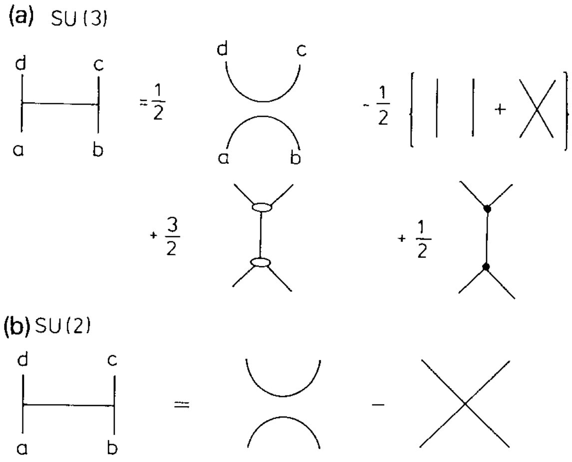

Fig. 4. The Firtz relation for the gluon interaction.

From eqs. (17) and (18) we can derive a new identity which looks like Firtz' one and turns out to be very convenient in all practical calculations ⋆.

$$i^2 f_{ade} f_{cbe} = \tfrac{1}{2}\{\delta_{ab}\delta_{cd} - \delta_{ad}\delta_{bc} - \delta_{ac}\delta_{bd}\} + \tfrac{3}{2} d_{abe} d_{cde} + \frac{i^2}{2} f_{abe} f_{cde}$$

$$= 3P_0 + \tfrac{3}{2}P_8 + \tfrac{3}{2}P_{\bar{8}} - P_{27}, \tag{20}$$

which is pictured in fig. 4a using a circle for d-tensor and a point for if_{abc}. It is interesting to note that 10 and $\overline{10}$ do not contribute to eq. (20) so the transition amplitudes to these states due to the exchange of one s-channel gluon are equal to zero. The situation for SU(2) is much simpler since there is no d-tensor in this group and only the first three colour structures from eq. (14) are enough to describe the colour states of four gluons. The Firtz-like identity for SU(2) is shown in fig. 4b.

3.3. POLARIZATION STRUCTURE

Let us discuss now the polarization structure of the twist four operator. It is well known (see refs. [1,9,19]) that only longitudinal (nonsense) polarization of t-channel gluons with $\lambda = 0$ gives the main leading $\log(1/x)$ contribution. In this sense the polarization structure of the amplitudes shown in figs. 3b and 3d is very simple,

⋆ We are grateful to J. Bartels for discussions of this point with us. We would like also to mention that he is using eq. (20) in his attempt to get the equation for anomalous dimension of high twist operator [6] from his reggeon technique [15].

namely A_{0000}. However we have to work in the specific axial gauge where we have for the gluonic field A_μ the following relation:

$$Q'_\mu A_\mu = 0, \qquad Q'_\mu = Q_\mu - x_B p_\mu, \qquad Q'^2 = 0,$$

since only in such a gauge [16] we were able to reduce the whole set of Feynman diagrams to the ladder ones in DLA. In the axial gauge the polarization structure of the gluon propagator looks as follows:

$$d_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu Q'_\nu + Q'_\mu q_\mu}{(qQ')} \tag{21}$$

and transforms the vector p_μ into the vector $-q_{t\mu}/x_q$ *. It could be seen directly from eq. (21) since

$$p_\mu d_{\mu\nu}(q) = -\frac{q_{t\nu}}{x_q} - \frac{2\alpha_q}{x_q} \cdot Q'_\nu$$

and the second term does not contribute in the leading $\log(1/x)$ approximation (see ref. [1] for details).

Thus the four gluon amplitude turns out to be proportional to the product of four transverse momenta $q_{ta\mu} q_{tc\nu} q_{tb\beta} q_{td\alpha}$ **, which could be contracted into a scalar in two ways:

$$e_1 = (q_{ta} q_{td}) \cdot (q_{tb} q_{tc}) \quad \text{and} \quad e_2 = (q_{ta} q_{tb}) \cdot (q_{tc} q_{td}). \tag{22}$$

We cannot add an additional power of the transverse momenta since it violates the log integration over virtuality dq_i^2 or the condition to get $\ln Q^2$ contribution.

It is easily to see that the amplitude of the "hard" gluon exchange in the s-channel (see fig. 3a) has the form $\delta^{\perp}_{\mu\alpha}$ in DLA. Indeed, in DLA the product of the triple gluon vertex $\Gamma_{\mu\mu'\sigma}$ with the polarization $-q_{t\mu} Q'_{\mu'}/x_q(pQ')$ is equal to

$$-q'_{1t\mu'} Q'_\mu \frac{\Gamma_{\mu\mu'\sigma}}{x_q(pQ')} = -2q_{1t\sigma}$$

in axial gauge (see eq. (21)) and the amplitude of the interaction works as $\delta^{\perp}_{\mu\alpha}$ since

$$A \propto 2q_{1t\mu} d_{\mu\alpha} 2q_{3t\alpha}.$$

* We use here the Sudakov variables, namely $q_\mu = x_q p_\mu + \alpha_q Q'_\mu + q_{t\mu}$.

** The fact that the amplitude is proportional to q_{ti} reflects the gauge invariance of QCD. In the limit $q_i^2 \to 0$ ($q_i^2 \approx q_{ti}^2$ in our case) the longitudinal polarization coincides with the scalar one ($e^{(0)}_\mu = q_\mu$) and an emission of such gluons should be negligible.

TABLE 1
The transition matrix for e_1 and e_2

Initial\Final	e_1	e_2
e_1	1	$\frac{1}{2}$
e_2	$\frac{1}{2}$	1

Strictly speaking it is possible to write one more scalar, namely $(q_{ta}q_{tc})(q_{tb}q_{tb})$. However this scalar cannot enter the game in DLA for the state [2,2] (see fig. 3b) since the exchange of gluons between lines with the same direction of the arrows ((a, c) or (b, d) in fig. 3b) cannot lead to a $\log(1/x)$ contribution. We have to take into account this structure for the state [3,1] which is shown in fig. 3d.

Now let us observe how the Lorentz structure $\delta^{\perp}_{\mu\alpha}$ acts if we add one additional interaction, say between lines c and b. It is obvious that the structure e_1 turns out to be the same after the interaction if the previous one (k_0 in fig. 3c) gave us the structure e_1 (see eq. (22)) as the result of the interaction between lines (c, b) or (a, d). The product of transverse momenta $q'_{1t} \approx q_{1t}$ * reduced one gluon propagator and provided the log integration over q^2_{1t} (factor $\mathrm{d}q^2_{1t}/q^2_{1t}$ in the integral). As a result the product $(q'_{3t}q_{2t})(q'_{1t}q_{1t})$ goes to $(q'_{3t}q_{2t})(q'_{2t}q_{3t})$ which also has the form of e_1. The situation changes for the exchange of a gluon (k_2 in fig. 3c) between the lines c and d. In this case we get the new structure of e_2 type from $e_1 = (q'_{3t}q_{2t})(q'_{2t}q_{3t})$ since

$$\int (q'_{3t}q_{2t})(q'_{2t}q_{3t})(q'_{4t}q_{3t})\frac{\mathrm{d}\phi}{2\pi} = \tfrac{1}{2}q^2_{2t}(q'_{4t}q_{4t})(q'_{3t}q_{3t})$$

after trivial integration over the azimuthal angle ϕ of the momentum $q'_{2t} \approx q_{2t}$.

Finally, the interaction between lines (a, d) or (b, c) transfers the structures e_1 and e_2 to e_1 with additional factor $\frac{1}{2}$ in front of the transition $e_2 \to e_1$. The interaction between (a, b) and (c, d) works in the same way. So we get the transition matrix of table 1.

3.4. THE FULL TRANSITION MATRIX

In this subsection we are going to discuss the full transition matrix including both the transition between different momentum tensors and different colour multiplets of SU(3). The transition matrix depends on what state [2,2] or [3,1] we are discussing. Here we would like to continue mostly the discussion of the [2,2]

* In DLA it is always possible to numerate the s-channel gluons in such a way that their transverse momenta are in the order of increase, namely $k_{0t} \gg k_{1t} \gg k_{2t} \gg \dots$ So $q_{0t} \approx q'_{0t} \approx k_0$; $q'_{1t} \approx q_{1t} \approx k_1 \ll q_{0t}$; $q_{2t} \approx q'_{2t} \approx k_2 \ll q_{1t}$ and so on (see fig. 3b for notation).

state but we are preparing all elements to calculate the anomalous dimension for the [3,1] state, too, postponing the detailed consideration to subsect. 3.5.

The first remark is very simple. The point is that the transitions between e_1 and e_1 or e_2 and e_2 are also diagonal for different multiplets of colour SU(3). It means that their transition amplitude could be written in the form

$$M^{(a)}_{\sigma i,\rho j} = \delta_{\sigma\rho}\delta_{ij}\lambda_i N_c P_i^{(\sigma)}, \tag{23}$$

where we label the colour SU(3) multiplets 0,8, $\bar{8}$, $10+\overline{10}$ and 27 by j, i, respectively; σ and ρ denote the momentum structure: $\sigma = 1$ or 2 means e_1 or e_2. $P_i^{(1)}$ is the same as in eq. (19) while $P_i^{(2)}$ are the projectors on the colour state of gluon pair (b, c) which can be obtained from $P_i^{(1)}$ by the substitution $a \leftrightarrow c$ in eq. (19). Using eq. (20) it is easy to calculate the values of λ_i in eq. (23) which are equal to

$$\lambda_0 = 1, \qquad \lambda_8 = \lambda_{\bar{8}} = \tfrac{1}{2}, \qquad \lambda_{10} = \lambda_{\overline{10}} = 0, \qquad \lambda_{27} = -\tfrac{1}{3}. \tag{24}$$

Nondiagonal transitions are described by a somewhat more complicated formula, namely

$$M^{(b)}_{\sigma i,\rho j} = \tfrac{1}{2}(1-\delta_{\sigma\rho})A_{ij}P_i^{(\sigma)}P_j^{(\lambda)}. \tag{25}$$

The factor $\frac{1}{2}$ in eq. (24) is correlated with the transition matrix for the momentum structures (see table 1) and $A_{i,j}$ are given in the table 2.

At this moment it seems that the problem has practically been solved since at the first sight we are able to write the simple evolution equation for the coefficient function $C_4^{[2,2]}$, namely

$$\frac{\mathrm{d}C_4^{[2,2]}(\rho, j)}{\mathrm{d}\ln Q^2} = \frac{\alpha_s}{\pi\omega}\cdot 2\cdot\left\{M^{(a)}_{\rho j,\sigma i} + M^{(b)}_{\rho j,\sigma i}\right\}C_4^{[2,2]}(\sigma, i). \tag{26}$$

So the only problem is to find the eigenvalues of eq. (26). Let us note that the

TABLE 2
The transition matrix for the multiplets of colour SU(3) ($A_{i,j}$)

i/j	0	8	$\bar{8}$	$10+\overline{10}$	27
0	$\frac{3}{8}$	$3/4\sqrt{2}$	$3/4\sqrt{2}$	0	$-3\sqrt{3}/8$
8	$3/2\sqrt{2}$	$\frac{3}{4}$	$\frac{3}{4}$	0	$\sqrt{3}/2\sqrt{2}$
$\bar{8}$	$3/2\sqrt{2}$	$\frac{3}{4}$	$-\frac{9}{20}$	0	$-3\sqrt{3}/10\sqrt{2}$
$10+\overline{10}$	$3\sqrt{5}/4$	0	$-3\sqrt{2}/2\sqrt{5}$	0	$\sqrt{3}/4\sqrt{5}$
27	$9\sqrt{3}/8$	$-3\sqrt{3}/4\sqrt{2}$	$9\sqrt{3}/20\sqrt{2}$	0	$-\frac{7}{40}$

factor 2 in front reflects the fact that the gluon exchange between lines (a, b) and (c, d) gives the same contribution as the exchange between lines (a, d) and (b, c).

However the situation in [2,2] turns out much more complicated and we will discuss it in sect. 4. Such a simple equation as eq. (26) we can obtain only for the state [3,1] but also with some modifications which we are going to consider in subsect. 3.5.

3.5. THE ANOMALOUS DIMENSION OF THE STATE [3,1]

In the case of the state [3,1] all gluons are emitted from one vertical line (a in fig. 3d). This is the reason why we can integrate over gluons in DLA in the kinematical region where gluons have strong ordering both for transverse momenta q_{ti} and for the fraction of energies x_i (see fig. 3d for notations):

$$Q^2 \gg q_{1t} \gg q_{2t} \gg q_{3t} \gg q_{4t} \gg \ldots \gg Q_0;$$

$$x_b \ll x_1 \ll x_2 \ll x_3 \ll x_4 \ll \ldots \ll 1. \tag{27}$$

The integration over x in the kinematical region eq. (27) leads to ω in the dominator of the kernel in the ω-representation as was written in eq. (26). Therefore we can use for the [3,1] state an equation of the type of eq. (26) with three important new ingredients:

(1) We should take into account the new momentum structure e_3 which is

$$e_3 = (q_{ta} q_{tc})(q_{tb} q_{td}). \tag{28}$$

(2) The three gluons (μ, ν, and β in fig. 3d) are identical so we have to symmetrize the coefficient function C_4 with respect to all permutation of these three gluons. It means that

$$C_4^{[3,1]}(\rho, j) = e_1 C_4^{[3,1]}(1, j) + e_2 C_4^{[3,1]}(2, j) + e_3 C_4^{[3,1]}(3, j). \tag{29}$$

As a result of this symmetrization the matrix elements between the states with different symmetry in the colour SU(3), namely symmetric ($0, \bar{8}, 27$) and antisymmetric ($8, 10, \overline{10}$) are vanishing. So the new matrix $\hat{M}^{(b)}$ becomes quasi-diagonal. It contains only two subblocks $j, i = 1, 3, 5$ and $j, i = 2, 4$ of the matrix of table 2. Moreover the diagonal elements of $\hat{M}^{(b)}$ do not contain the factor 2 in front since only one s-channel gluon contributes to each component in eq. (29).

(3) By now we have discussed only the emission of the emission of the hard gluons with the strong ordering in the transverse momenta. As was mentioned before there are other sources of the double logs, the self-energy (reggeization) diagrams (see fig. 3e) which give the double logs in the axial gauge and the "soft"

gluon emission with $k_{1t} \ll q_{1t} \approx q_{2t}$. In the whole sum of the reggeizations and "soft" gluon emissions the double log contribution is cancelled as shown in sect. 5.

It means that we have to include the additional matrix $M^{(c)}_{\rho j,\sigma i}$ into the r.h.s. of the evolution equation:

$$\frac{\mathrm{d}C_4^{[3,1]}(\rho, j)}{\mathrm{d}\ln Q^2} = \frac{\alpha_s}{\pi\omega}\left\{2\hat{M}^{(b)}_{\rho j,\sigma_i} + M^{(a)}_{\rho j,\sigma i}\right\}C_4^{[3,1]}(\sigma, i), \tag{30}$$

where $C^{[3,1]}$ was defined in eq. (26).

The eigenvalues of this equation give the anomalous dimensions of the [3,1] state. We computed these values and the answer for anomalous dimensions look as follows:

$$\gamma_i([3,1]) = l_i \frac{\alpha_s N_c}{\pi\omega} \tag{31}$$

where the l_i are equal:

$$\text{in SU(3)}: \quad l_i = 2.12; \quad 1\tfrac{1}{4}; \quad 0.80; \quad 0; \quad -0.665; \tag{32}$$

$$\text{in SU(2)}: \quad l_i = 2\tfrac{1}{4}; \quad 1\tfrac{1}{4}; \quad -1. \tag{33}$$

Thus the anomalous dimensions of this state are smaller than the contribution of two pomeron cut (see fig. 1b) which led to formula (1) for the value of the anomalous dimension, or, in the other words, to $l_i = 4$.

4. Anomalous dimensions of the twist four gluon operator for the channel [2,2]

4.1. PROBLEMS AND STRATEGY

The main problem that does not allow to use the ordinary evolution equation in the form of eq. (31) is the following one. The momenta of gluons in the diagram of fig. 3c (for example k', k_0, k_1 and $\bar{k}$) have no such strong ordering as it was in fig. 3d (see eq. (27)) in that part of it where only diagonal transitions occur. In these parts of the diagram the state could be considered as an exchange of two independent ladders. Inside of each of them we see the strong ordering in transverse momenta

$$k'_t \gg k_{0t} \gg k_{1t}, \qquad x' \ll x_0 \ll x_1.$$

An analogous ordering of the momenta $\bar{k}$ holds in the other ladder, but there is no correlation between the gluon momenta from different ladders. The only correlation exists in the point of branching (gluon with momentum k_2 in fig. 3c) where both ladders have common boundary, namely

$$k_{1t} \gg k_{2t}, \quad x_1 \ll x_2, \quad \bar{k}_t \gg k_{2t}, \quad \bar{x}_1 \ll x_2.$$

To sum these two-ladder-reduced part of the diagram we cannot use only one total moment $N = 1 + \omega$ but have to introduce two different N_1 and N_2 (ω_1 and ω_2) for the description of each separate ladder. Of course $\omega = \omega_1 + \omega_2$. Fortunately we know the amplitude of each ladder in DLA and are able to sum all double logs in a direct way for the two-ladder reduced part of the diagram in fig. 3c.

Our strategy in calculating the anomalous dimension for the [2,2] channel looks as follows:

(1) First of all we sum the diagrams which cannot be reduced to the exchange of two ladders. For this purpose we solve eq. (26) with $M^{(a)} = 0$ and find the anomalous dimensions (γ_i) of such substates. The explicit solution gives us

$$\gamma_i = \frac{\alpha_s N_c}{\pi \omega} \lambda_i \tag{34}$$

where the λ_i for SU(3) are equal to

$$\lambda_i = 0.597; \quad 0; \quad -0.316, \tag{35}$$

and for SU(2):

$$\lambda_i = 0.677. \tag{36}$$

(2) The next step is to calculate the amplitude for the exchange of two ladders at the same value of ω and γ. Of course each ladder can belong to different representations of colour SU(3) (0, 8, $\bar{8}$, 10, $\overline{10}$, 27), and the two-ladder amplitude will be different for each of them.

Finally we mix the two-ladder amplitude and the irreducible state that we have calculated. We find the new diagonal matrix $\hat{M}^{(a)}$, built from two-ladder amplitudes, and solve the complete equation of the type of eq. (26). This procedure is shown in fig. 3f where fat lines denote the irreducible states and wavy ones are used for ladder amplitudes.

(3) As we explained below we do not need to take into account the emission of the "soft" gluons since this emission is cancelled in the whole sum.

4.2. TWO-LADDER AMPLITUDE

To discuss the property of the two-ladder amplitude it is more convenient to introduce the general (γ, ω) representation than to use the anomalous dimension as we have done so far. In this representation the coefficient function $C_4(Q^2, \omega)$ looks as follows:

$$C_4(Q^2, \omega) = \int \frac{d\gamma}{2\pi i} C_4(\gamma, \omega) \exp(\gamma \ln(Q^2/Q_0^2)), \tag{37}$$

where the integration contour is situated to the right of the singularities in γ.

In the (γ, ω) representation the exchange of two gluons or the Born approximation for the ladder amplitude has the form

$$A_1^{(\mathrm{Born})}(\omega, \gamma) = \frac{1}{\omega\gamma},$$

and the full answer for the one ladder amplitude in DLA can be written as

$$A_1^{(\mathrm{DLA})}(\omega, \gamma) = \frac{1}{\omega\gamma(1 - (\alpha_s N_c/\pi\omega\gamma)\lambda_i)}, \tag{38}$$

where the λ_i are the same as in eq. (24). It is easy to check that for colourless state in the t-channel we immediately get the anomalous dimension $\gamma_1(\omega) = \alpha_s N_c/\pi\omega$ substituting $\lambda_0 = 1$ in eq. (39) and closing the contour on the pole in $\gamma(\gamma = \gamma_1(\omega))$ *.

It should be stressed that the values of λ_i in eq. (40) is different from eq. (40), since we took into account the soft gluon contribution to the ladder kernel.

In a direct way or using the rules of Reggeon Diagram Technique [14] we can calculate the amplitude for two ladder exchange in the form

$$A_2^{(\mathrm{DLA})}(\omega, \gamma) = \int \frac{d\omega_1\, d\omega_2 \delta(\omega_1 + \omega_2 - \omega)}{(2\pi i)^2} \frac{d\gamma_1\, d\gamma_2}{\omega_1\gamma_1\omega_2\gamma_2} \frac{\delta(\gamma - \gamma_1 - \gamma_2)}{\left(1 - \frac{\alpha_s N_c}{\pi\omega_1\gamma_1} \cdot \lambda_i\right)} \frac{1}{\left(1 - \frac{\alpha_s N_c}{\pi\omega_2\gamma_2} \cdot \lambda_i\right)}. \tag{39}$$

Using δ-functions for the integration over ω_2 and γ_2, and closing the integration contour avoiding the pole of $\gamma_1\omega_1\gamma_1 - \lambda_i\alpha_s N_c/\pi = 0$ we reduce eq. (39) to the form

$$A_2^{(\mathrm{DLA})} = \int \frac{d\omega_1}{2\pi i} \frac{1}{\omega_1(\omega - \omega_1)\gamma - \lambda_i\alpha_s N_c/\pi}.$$

* For the running coupling constant $\alpha_s = (4\pi/b)\ln(q^2/\Lambda^2)$ we should go to the variable $\xi = \ln\ln(q^2/\lambda^2)$ in which the evolution of operators looks as $O(\xi) = O(\xi_0)\exp(\nu(\xi - \xi_0))$. So in this case all our formulas are valid if we replace the variables in the following way: $\gamma \to \nu$ and $\alpha_s/\pi \to 4/b$.

The remaining integral could be done closing the contour avoiding the pole

$$\omega_1 = \frac{\omega}{2}\left\{1 - \sqrt{1 - \frac{4\alpha_s N_c \lambda_i}{\pi\gamma}}\right\}.$$

As a result the amplitude for the exchange of two ladders belonging to the ith multiplet of colour SU(3) is equal to *

$$A_2^{(i)}(\omega, \gamma) = \frac{1}{\omega\gamma\sqrt{1 - 4(\alpha_s N_c/\pi\omega\gamma)\lambda_i}}. \tag{40}$$

4.3. EQUATION FOR THE ANOMALOUS DIMENSION

As in the case of the [3,1] state we should take into account the identity of gluons "a" and "c" in fig. 3b and introduce the symmetric coefficient function

$$C_4^{[2,2]}(\rho, j) = e_1 C_4^{[2,2]}(1, j) + e_2 C_4^{[2,2]}(2, j). \tag{41}$$

For this coefficient function we can write the evolution equation in the form

$$\frac{\mathrm{d}C_4^{[2,2]}(\rho, j; \omega, \gamma)}{\mathrm{d}\ln Q^2}$$

$$= \gamma C_4^{[2,2]}(\rho, j; \omega, \gamma) = \frac{\alpha_s N_c}{\pi\omega} \cdot 2 \cdot \left\{\hat{M}^{(a)}_{\rho j,\sigma i} \cdot M^{(b)}_{\rho j,\sigma i}\right\} C_4^{[2,2]}(\sigma, i; \omega, \gamma), \tag{42}$$

where

$$\hat{M}^{(a)}_{\sigma i,\rho j} = \delta_{\sigma\rho}\delta_{ij} P_i^{(\sigma)} \frac{1}{\sqrt{1 - 4\lambda_i \alpha_s N_c/\pi\omega\gamma}}. \tag{43}$$

Eq. (44) is nonlinear in respect to γ but we solved it numerically.
The first eigenvalue of eq. (44) that we found is the pole at **

$$\lambda = 4.0053.$$

* We are grateful to J. Bartels who pointed out that AGK cutting rules [20] which we used in our previous publication [18] does not work in this case. So the additional signature factor (η_i) does not appear in eq. (42) contrary to the same equation in ref. [18].

** We would like to mention that J. Bartels (private communication) was the first who got the value of anomalous dimension larger than one corresponded to two pomeron cut ($\lambda > 4$) using his reggeon technique [15].

The next ones are

$$\lambda = -0.032 \pm i\ 0.062, \qquad \lambda = -0.28 \pm i\ 0.027,$$

for SU(3) and

$$\lambda = 4.029, \qquad \lambda = -0.074 \pm i\ 0.14,$$

for SU(2).

It turns out that the rightmost singularity is very close to the position of the two pomeron cut while the others are sufficiently to the left. This fact allows us to illustrate the result of numerical calculations using the simplified model.

Let us consider the values of γ which are very close to

$$\gamma = \frac{4\lambda_0 \alpha_s N_c}{\pi\omega}.$$

In this case we can reduce eq. (44) to the form

$$\frac{\mathrm{d}C_4^{[2,2]}(\rho, 0; \omega, \gamma)}{\mathrm{d}\ln Q^2} = \gamma C_4^{[2,2]}(\rho, 0; \omega, \gamma)$$

$$= \frac{\alpha_s N_c}{\pi\omega} \cdot \left(\frac{1}{8}\right) \cdot \frac{1}{\sqrt{1 - 4\lambda_0 \alpha_s N_c/\pi\omega\gamma}} \cdot C_4^{[2,2]}(\rho, 0; \omega, \gamma). \quad (44)$$

The solution of this equation can be written in the form

$$C_4^{[2,2]}(\rho, 0; \omega, \gamma) = \frac{C(\omega)}{\gamma - \dfrac{\alpha_s N_c}{\pi\omega} \cdot \left(\dfrac{1}{8}\right) \cdot \dfrac{1}{\sqrt{1 - 4\lambda_0 \alpha_s N_c/\pi\omega\gamma}}}. \quad (45)$$

Here the function $C(\omega)$ should be found from initial condition, but it does not affect the value of the anomalous dimension. The value of the anomalous dimension is determined by the rightmost singularity in γ in eq. (45). It is easy to see that the singularities in eq. (45) are originated either by the zero of the dominator or by the square root singularity

$$\gamma = \frac{4\lambda_0 \alpha_s N_c}{\pi\omega} = 2\gamma_2\left(\frac{\omega}{2}\right).$$

It is easy to see that we have the zero of the dominator which is very close to $\gamma = 2\gamma_2(\omega/2)$ but corresponds to the larger value of λ in eq. (35), namely

$$\lambda \simeq 4 + \tfrac{1}{256}. \quad (46)$$

This is just that eigenvalue which we found from numerical calculation.

5. Soft gluon emission

In this section we are going to discuss the cancellation of the infrared divergencies due to the "soft" gluon emission in the anomalous dimension of the twist four operator. The physical meaning of such a cancellation has been discussed in subsect. 3.1 so here we concentrate on the proof how it occurs in the calculation of the value of the anomalous dimension of the twist four operator.

5.1. THE LEADING TWIST OPERATOR

We start with the same cancellation for the leading twist, in order to illustrate the physical origin of it. We prefer to calculate the moments of the deep inelastic structure function in the so-called leading $\ln(1/x)$ approximation which leads to the FKL equation [13] for the function ϕ which is equal

$$F_2(\omega, Q^2) = \frac{1}{4\sqrt{2}\,\pi^3} \int_{Q_0^2}^{Q^2} \phi(\omega, q^2)\, \mathrm{d}q^2, \tag{47}$$

where $F_2(\omega, Q^2)$ is the moment of the deep inelastic structure function (see eq. (5)).

The FKL equation looks as follows:

$$\omega\phi(\omega, q_1, q_1 - q_2) = \frac{\alpha_s N_c}{\pi^2} \int_{Q_0^2}^{Q^2} K(q_1 - q_2, q_1, q_2, q')\phi(\omega, q', q_1 - q_2)\, \mathrm{d}^2 q', \tag{48}$$

where the kernel K has the form

$$K(q_1 - q_2, q_1, q_2, q')\phi(\omega, q', q_1 - q_2)\, \mathrm{d}^2 q' = \frac{1}{2(q_1 - q')_t^2}$$
$$\times \left\{ \left(-\frac{(q_1 - q_2)_t^2 (q_1 - q')_t^2}{(q_1 - q_2 - q')_t^2 q_{1t}^2} + 1 + \frac{q_{2t}^2 q_t'^2}{(q_1 - q_2 - q')_t^2 q_{1t}^2} \right) \cdot \phi(\omega, q', q_1 - q_2) \right.$$
$$\left. - \left(\frac{q_{1t}^2}{q_t'^2 + (q_1 - q')_t^2} + \frac{q_{2t}^2}{q_t'^2 + (q_2 - q')_t^2} \right) \cdot \phi(\omega, q_1, q_1 - q_2) \right\}, \tag{49}$$

where the first term in the kernel is responsible for the emission of the gluon in the ladder diagrams of fig. 5a while the second one describes the reggeization of the gluon (see refs. [1,13] for details).

From the explicit form of the kernel one can see that there are two kinematical regions for log contributions:

(1) $|q_1 - q_2| \ll q' \ll q_1 \sim q_2$, which corresponds to hard gluon emission from the ladder.

(2) $q' \to q_1$. Namely in this kinematical region we have the log contribution from soft gluon emission, but we can see that the log contribution from the emission in the ladder diagram (the first term in the kernel) reduces due to contribution of the gluon reggeization for any values of q_1 and q_2. Practically this cancellation reflects the same colour factor for gluon emission and gluon reggeization, namely N_c for the first and $N_c/2$ for the reggeization of each gluon in the t-channel (two in our case).

The resulting answer for the scattering amplitude ($A(x_B, q_1 - q_2, q_1^2, q_2^2, q_1'^2, q_2'^1)$) is very simple, namely A is pure imaginary and equal to

$$A\left(x_B, q_1 - q_2, q_1^2, q_2^2, q_1'^2, q_2'^1\right) = \frac{i}{x_B}\phi\left(x_B, q_1 - q_2, q_1^2, q_2^2, q_1'^2, q_2'^1\right). \tag{50}$$

In the colour state for two gluons in the t-channel ($i = 8, \bar{8}, 10 + \overline{10}, 27$) the situation looks quite different, since there is no cancellation between the gluon reggeization and emission. In this case we can rewrite the equation in the following way:

$$\begin{aligned} &\omega\phi(\omega, q_1, q_1 - q_2) \\ &\quad = \frac{1}{q_1^2}\delta^{(2)}(q_1 - Q_0) + \frac{\alpha_s C_i}{\pi^2}\int_{Q_0^2}^{q_1^2} K(q_1 - q_2, q_1, q_2, q')\phi(\omega, q', q_1 - q_2)\, d^2q' \\ &\qquad + \{C_i - N_c\} \cdot \left(\alpha(q_1^2) + \alpha(q_2^2)\right)\phi(\omega, q_1, q_1 - q_2), \end{aligned} \tag{51}$$

where we use the notation $(\alpha(q_1^2) + \alpha(q_2^2))$ for the reggeization part of the kernel K, including all α_s and all numerical factors except N_c. C_i corresponds to the colour coefficient of the diagram of fig. 5b. We included also the inhomogeneous term (Born approximation) in eq. (51) which has the principal meaning for our discussion.

The solution of eq. (51) looks as follows:

$$\phi(\omega, q_1, q_1 - q_2) = \frac{\hat{\phi}(\omega, q_1, q_1 - q_2)}{\omega - \{C_i - N_c\} \cdot \left(\alpha(q_1^2) + \alpha(q_2^2)\right)}, \tag{52}$$

where $\hat{\phi}$ is the solution of eq. (48) with the same inhomogeneous term as in eq. (51).

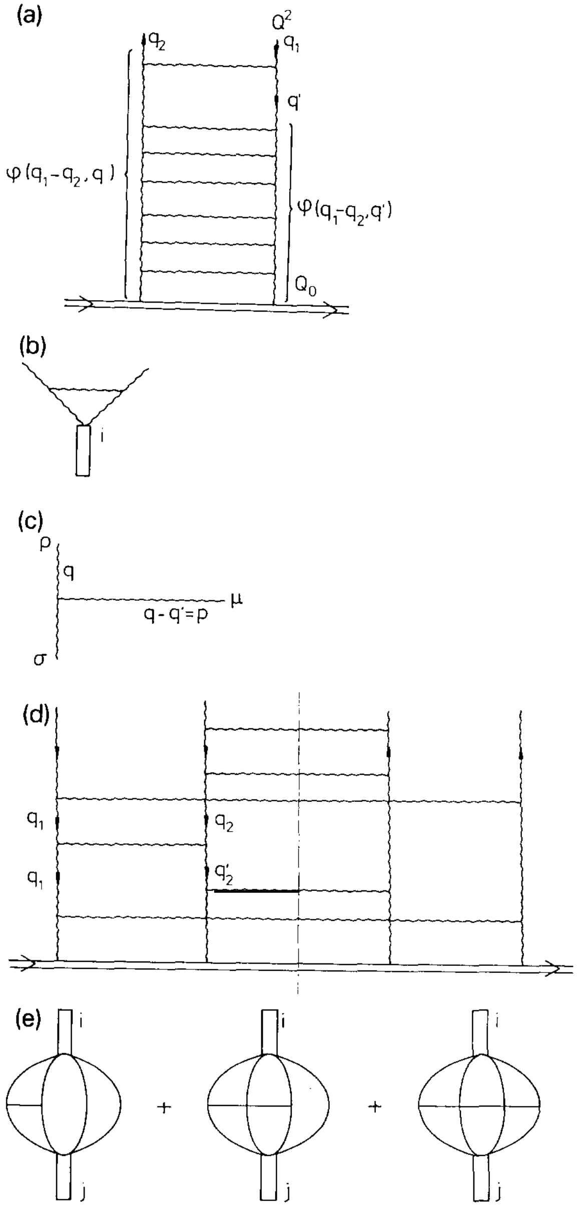

Fig. 5. (a) The ladder diagram for the anomalous dimension of the leading twist operator. (b) Colour coefficient C_i in eq. (53). (c) Vertex for gluon emission. (d) Interaction between gluon lines with the same directions of the arrows. (e) Colour coefficient C_i^j in eq. (56). (f) Colour states of eq. (19). (g) Colour coefficient for a gluon interaction in state 2. (h) Colour coefficient C_i^j for the transition $4 \to 4$ (1), $7 \to 7$ (2) and $4 \to 7$ (3).

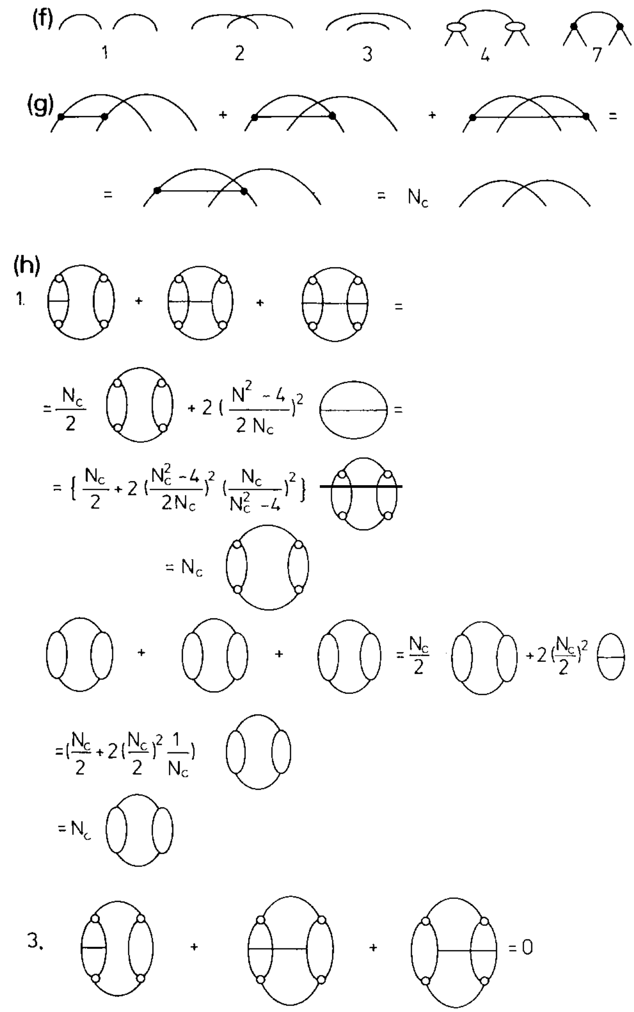

Fig. 5. (continued).

The main message from eq. (52) is that the scattering amplitude has a real part even in the lowest order of α_s. Indeed, even the Born term in $\hat{\phi}$ generates the amplitude which is equal to

$$A^{\mathrm{Born}}\left(x_B,\, q_1 - q_2,\, q_1^2,\, q_2^2,\, q_1'^2,\, q_2'^1\right) = \frac{1}{x_B} \cdot \ln\frac{1}{x_B} \cdot \{C_i - N_c\} \cdot \left(\alpha\left(q_1^2\right) + \alpha\left(q_2^2\right)\right). \tag{53}$$

5.2. POLARIZATION STRUCTURE

Here we are starting to consider the soft gluon contribution to the anomalous dimension of the twist four operator. The first remark is that the polarization structure of the soft emission is very simple, namely the vertex for such an emission is equal to

$$-\frac{q_{t\rho}q'_{t\sigma}}{\alpha_q x_{q'}(pQ')}\Gamma_{\mu\rho\sigma}=\frac{2}{p_t^2}\left(p_t^2 q_\mu - q^2 p_{t\nu}\right), \tag{54}$$

where all notations are clear from fig. 5c (see also ref. [1] for details).

From the above expression it is obvious that the soft gluon emission does not change the structure of the ladder, since q^2 in the numerator of eq. (52) reduces the gluon propagator. Thus we have the ladder without the emitted gluon and the extra factor

$$C_i(\text{colour})\alpha_s\int^{x_q}\frac{dx_p}{x_p}\int^{q^2}\frac{d^2p_t}{p_t^2}.$$

Our main problem is to calculate the colour coefficients to see that they are the same as for the gluon reggeization.

5.3. INTEGRATION OVER LONGITUDINAL MOMENTA

However first of all we have to revise the integration over longitudinal momenta. Namely this integration led us to the rule that the only interaction between lines with different direction of the arrows gives rise to the large $\log(1/x)$ contribution in DLA.

Our statement is that this is not the case for soft gluon emission, and in this case we should take into account the interaction with all gluons in the t-channel (with four of them in our problem).

The point is that zero of the dominator $(q-q')_t^2$ in eq. (49) comes from the emission of the gluon in the initial state (not from t-channel gluons with the define direction of arrows [13]).

Two statements follow from these simple observation:

(1) We can neglect the interaction between lines with different directions of the arrows for hard gluon emission as we did in our previous consideration.

(2) The interactions between all lines should be taken into account for soft gluon emission. It should be stressed that the resulting formula for this contribution looks very simply, namely

$$C_4^{[2,2]}(q_1^2, q_2^2, q_3^2, q_4^2; \rho, i)=\frac{\alpha_s N_c}{\pi\omega}\cdot\{\ln q_1^2+\ln q_2^2+\ln q_3^2+\ln q_4^2\}$$
$$\times M_{\rho i,\rho j}\cdot C_4^{[2,2]}(q_1^2, q_2^2, q_3^2, q_4^2; \rho, j). \tag{55}$$

5.4. COLOUR STRUCTURE

The structure of the colour matrix $M_{\rho i,\rho j}$ is very simple and it is equal to

$$M_{\rho i,\rho j} = C_i^j - N_c \delta_{ij}, \tag{56}$$

where C_i^j is the colour coefficient for the transition from the colour state i to j due to interaction between one definite line and all others, as shown in fig. 5e.

We use the same complete set of colour states (see eq. (19)) as before. As one can see from the explicit formula for our complete set of colour states of eq. (19) we need to find the expression for the transition between the colour states 1, 2, 3, 4, 7 from eq. (14) to calculate the coefficient C_i^j (see also fig. 5f, where points and circles denote $if_{\alpha\beta\gamma}$ and $d_{\alpha\beta\gamma}$, respectively).

There are several observations that help us to calculate C_i^j:

(1) The interaction between all lines in the states 1, 2, 3 in fig. 5f is diagonal and gives a factor N_c due to the fact that in these states one pair of gluons has the opposite colour charge. It follows directly from the antisymmetry of $f_{\alpha\beta\gamma}$ in the colour indices. Fig. 5g illustrates this statement.

(2) It means that we need only to calculate the transition between the states 4, 7 as shown in fig. 5h. It is easy to see that the interaction with two gluons "a" and "b" is equal to zero for the states "i" and "j" with the different symmetry with respect to permutation of the indices "a" and "b". Thus the transition $4 \to 7$ vanishes.

(3) The explicit calculation of C_i^i for the states 4 and 7 shows (see fig. 5h) that they are equal to N_c.

Therefore the colour coefficients C_i^j for the complete set of the colour state of eq. (19) are equal to

$$C_i^j = N_c \delta_{ij}. \tag{57}$$

Substituting the above values of C_j^i in eq. (53) one can see that the soft emission gives no contribution to the anomalous dimension of the twist four operator in DLA.

6. Conclusions

The main results of the paper are the following:

(1) Operators of the leading twist and of twist four can mix at $\omega = \omega_{cr}$ where the full dimensions of these two operators become equal, namely

$$-1 + \gamma_2(\omega_{cr}) = -2 + \gamma_4(\omega_{cr}).$$

It happens due to the fact that the anomalous dimension of the twist four operator increases more rapidly at small ω than the leading twist one. However this statement does not contradict any general theorem since namely at $\omega = \omega_{cr}$ this mixing could be interpreted as the contribution of the anomalous dimension of the leading twist operator to the anomalous dimension of the twist four operator. Such a mixing illustrates only that all general theorems are not well defined in the case when the anomalous dimension of the next twist operator becomes of the same order as the leading twist.

(2) In DLA we found the value of the anomalous dimension and wrote the corresponding evolution equation. It turns out that the anomalous dimension of the twist four operator is equal to

$$\gamma_4(\omega) = 2\gamma_2\left(\frac{\omega}{2}\right)(1+\delta) \tag{58}$$

where $\delta \sim 10^{-3}$ is very small. The smallness of δ has very simple origin. As was shown the main contribution comes from the pomeron–pomeron interaction near the threshold $\omega\gamma = 4\alpha_s N_c/\pi$. The pomeron–pomeron vertex is non-planar one and is suppressed by the colour factor $1/(N_c^2-1)$. The solution of eq. (45) gives $\delta \propto 1/N_c^4$.

The above result confirms the hypothesis made in ref. [1] that the rightmost singularity comes from the exchange of many pomerons (ladders) in t-channel.

Strictly speaking the pomeron–pomeron interaction has not been taken into account in GLR evolution equation. Now we can improve this equation using eq. (58). However the corrections ($\sim O(\delta)$) are so small that they give the noticeable contribution only at astronomically high energies (small x_B) of the order of $\ln(1/x_B) > 100$.

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