Resonances in K_{l4} decays

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The axial form factors for the K_{l4} decay are calculated using models with hidden local symmetry to describe $J^P=1^-$ resonances. The relation between this approach and low-energy expansion is clarified. It is shown that the explicit breaking of chiral symmetry has no significant effect on the predictions. It turns out that in order to achieve a satisfying description an additional S-wave contribution is needed, which can be described by a $J^P=0^+$ resonance.

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I. INTRODUCTION

K_{l4} decays are an interesting subject to study with phenomenological meson theories, being simple enough to be calculable, but having enough structure to give interesting physical information. The construction of such phenomenological models can be based on the observation that QCD possesses an approximate chiral $SU(3)_L\otimes SU(3)_R$ flavor symmetry, which is both spontaneously broken with the $J^P=0^-$ mesons of lowest mass appearing as Goldstone bosons and directly by the quark masses. In leading order $O(P^2)$ of an expansion in small momenta, this leads to a unique chiral Lagrangian $\mathcal{L}^{(2)}$ for pseudoscalar mesons, the nonlinear σ model based on the manifold G/H with $G=SU(3)_L\otimes SU(3)_R$ and $H=SU(3)_V$. It is now known [1] that any nonlinear σ model based on G/H is gauge equivalent to a linear model with a $G_{\text{global}}\otimes H_{\text{local}}$ symmetry. If G is the chiral symmetry of QCD, the vector mesons ($J^P=1^-$) appear as dynamical gauge bosons of the hidden local symmetry H_{local}.

In this paper I will calculate K_{l4} decays using chiral Lagrangians, including vector mesons in this way as gauge bosons of $SU(3)_{\text{local}}$. Special attention will be addressed to the effect of the direct breaking of chiral symmetry by the masses. The main question shall be whether or not the mass splitting has *dynamical* effects on K_{l4} decays, i.e., effects which result from symmetry breaking in the couplings, as opposed to the obvious *kinematical* effects such as the shift of the poles of the propagators and the change in the phase space factor. These dynamical effects have not been included in papers such as [2,3], which discuss resonance factor enhancements improving the $O(P^2)$ predictions for K_{l4} decays.

Another aim of this paper is to clarify the relation between the resonance approach to K_{l4} decays and low-energy expansion, as developed by Gasser and Leutwyler in [4] and applied to K_{l4} decays in [5,6].

The paper is organized as follows. In Sec. II, I define the form factors and review the experimental results. In Sec. III, I define the chiral models with hidden local symmetry which have been used to perform the calculations in this paper. In Sec. IV, K_{l4} decays are calculated with the Lagrangians defined before. In Sec. V, a scalar resonance channel is discussed, and in Sec. VI, I draw the conclusions.

II. EXPERIMENTAL RESULTS ON K_{l4} DECAYS

In this paper I will consider the K_{l4} decay mode

$$K^+(k)\to\pi^+(p_1)\pi^-(p_2)e^+(p_3)\nu_e(p_4)\ . \tag{1}$$

The transition amplitude for this decay can be written as a current-current coupling:

$$A=\frac{G_F\sin\theta_C}{\sqrt{2}}\langle\pi^+(p_1)\pi^-(p_2)|J^\mu_{\text{hadronic}}|K^+(k)\rangle \times\langle e^+(p_3)\nu_e(p_4)|J_{\mu\ \text{leptonic}}|0\rangle\ , \tag{2}$$

where the matrix element H^μ of the hadronic current can only be calculated using phenomenological models. On general grounds of covariance, it can be parametrized by four form factors F, G, R, and H:

$$H^\mu=\langle\pi^+(p_1)\pi^-(p_2)|J^\mu_{\text{hadronic}}|K^+(k)\rangle =\frac{1}{m_K}\left\{F(p_1+p_2)^\mu+G(p_1-p_2)^\mu+R\,(k-p_1-p_2)^\mu +\frac{1}{m_K^2}H\epsilon^{\mu\nu\alpha\beta}k_\nu(p_1+p_2)_\alpha(p_1-p_2)_\beta\right\}\ . \tag{3}$$

The Lorentz-invariant form factors can only depend on invariant products of the momenta ($p_1\cdot p_2$, $k\cdot p_1$, and $k\cdot p_2$). The contribution of the form factor R to the decay probability is suppressed by a factor $m_e^2/4m_\pi^2$, and so R cannot be measured.

The Watson-Fermi final-state theorem tells us that the phases of the two pions in K_{l4} decays carry direct information on low-energy $\pi\pi$ scattering. This fact was one of the original motivations why the K_{l4} experiments were performed. The partial-wave expansion of the measur-

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able form factors is given by [7]

$$F=f_s e^{i\delta_0^0}+f_p e^{i\delta_1^1}\cos\theta_\pi+D \text{ wave },$$
$$G=e^{i\delta_1^1}+D \text{ wave },\tag{4}$$
$$H=he^{i\delta_1^1}+D \text{ wave },$$

where δ_0^0 and δ_1^1 are phases for s wave $I=0$ and p wave $I=1$ scattering, respectively, and θ_π is the angle between the line of flight of the positive pion and the direction opposite to the kaon line of flight, measured in the dipion rest frame.

In the experiment described in [8], 30 318 K_{l4} decays were analyzed to extract the form factors F, G, and H. As far as the dependence of the form factors on the kinematical variables is concerned, only a little variation with the invariant mass $\sqrt{s_{\pi\pi}}$ of the dipion system

$$s_{\pi\pi}=(p_1+p_2)^2 \tag{5}$$

was found; the dependence of the other invariants was negligible. This dependence was expressed in a variable:

$$x=\frac{s_{\pi\pi}}{4m_\pi^2}-1 . \tag{6}$$

The results for the form factors at threshold ($x=0$) are (here I use $\sin\theta_C=0.220$ to transcribe the results)

$$F(x=0)=5.59\pm0.14 ,$$
$$G(x=0)=4.77\pm0.27 ,\tag{7}$$
$$H(x=0)=-2.68\pm0.68 .$$

The energy dependence of $|F(x)|$ was parametrized as

$$|F(x)|=|F(0)|(1+\lambda x) ,\tag{8}$$

with the result

$$\lambda=0.08\pm0.02 .\tag{9}$$

Under the assumptions that have been made in [8] in the analysis of the data, $|G|$ and $|H|$ must have the same energy dependence as $|F|$. If one neglects the electron mass m_e, the invariant mass $\sqrt{s_{\pi\pi}}$ of the dipion system can in principle take any value in the interval

$$2m_\pi=280 \text{ MeV}\le\sqrt{s_{\pi\pi}}\le 494 \text{ MeV}=m_K ,\tag{10}$$

which corresponds to a maximum increase of the form factors of 17%. Actually, however, 80% of all events in [8] lie in the interval

$$280 \text{ MeV}\le\sqrt{s_{\pi\pi}}\le 347 \text{ MeV} ,\tag{11}$$

in which interval the energy dependence is below 4%. So the energy dependence of the form factors is a small effect, and therefore one often only computes the form factors at threshold. In this paper I will also take this attitude at first, but come back to the energy dependence in Sec. V.

III. CHIRAL MODELS WITH HIDDEN LOCAL SYMMETRY

The matrix element of the hadronic current will be calculated with the help of effective chiral Lagrangians, including vector mesons as gauge bosons of a hidden local symmetry. The direct breaking of chiral symmetry by mass splitting will be taken into account in both the pseudoscalar and vector-meson sectors. To this aim I consider the following four Lagrangians, which define the models (A)–(D):

$$\begin{aligned}&(\text{A})\ \mathcal{L}_m^{(2)}=\mathcal{L}_A+\mathcal{L}_m ,\\&(\text{B})\ \mathcal{L}^{(2)}=\mathcal{L}_A+\mathcal{L}_M ,\\&(\text{C})\ \mathcal{L}_{hg}=\mathcal{L}_A+a\mathcal{L}_V+\mathcal{L}_M ,\\&(\text{D})\ \mathcal{L}_{hg}(c_A,c_V)=\mathcal{L}_A(c_A)+a\mathcal{L}_V(c_V)+\mathcal{L}_m ,\end{aligned}\tag{12}$$

where

$$\begin{aligned}\mathcal{L}_A&=\frac{f_\pi^2}{4}\text{tr}(D_\mu U D^\mu U^\dagger)\\&=-\frac{f_\pi^2}{4}\text{tr}(D_\mu\xi_L\xi_L^\dagger-D_\mu\xi_R\xi_R^\dagger)^2 ,\end{aligned}\tag{13}$$

with

$$\xi_R=\xi_L^\dagger=\exp\left[\frac{i}{\sqrt{2}f_\pi}\Phi\right]\tag{14}$$

(this assumes unitary gauge in the sector of the hidden local symmetry; see below) and

$$U=\xi_R^2 .\tag{15}$$

f_π is the pion decay constant, $f_\pi\approx93$ MeV. Φ is the pseudoscalar matrix,

$$\Phi=\begin{bmatrix}\pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+\\ \pi^- & -\pi^0/\sqrt{2}+\eta/\sqrt{6} & K^0\\ K^- & \bar{K}^0 & -2\eta/\sqrt{6}\end{bmatrix} .\tag{16}$$

The covariant derivative of U is given by

$$D_\mu=\partial_\mu U-i\mathcal{L}_\mu U+iU\mathcal{R}_\mu ,\tag{17}$$

with external left- and right-handed gauge fields $\mathcal{L}_\mu$ and $\mathcal{R}_\mu$. For the K_{l4} decays (weak charged currents), one can put

$$\begin{aligned}\mathcal{L}_\mu&=\frac{g_w}{\sqrt{2}}\begin{bmatrix}0 & W_\mu^+\cos\theta_C & W_\mu^+\sin\theta_C\\ W_\mu^-\cos\theta_C & 0 & 0\\ W_\mu^-\sin\theta_C & 0 & 0\end{bmatrix} ,\\ \mathcal{R}_\mu&=0 .\end{aligned}\tag{18}$$

So $\mathcal{L}_A$ is the most general chiral-symmetric Lagrangian of order $O(P^2)$ which describes exactly massless pseudoscalar mesons. Of course, the actual pions, kaons, and the η do have small masses m_π, m_K, and m_η. One could try to take this into account by using exactly chiral-symmetric couplings derived from $\mathcal{L}_A$, but then considering the obvious *kinematical* implications of the mass splitting, e.g., the shift of the poles of the propagators $[1/(p^2-m^2)$ instead of $1/p^2]$ and the change of the phase-space factors. In terms of the Lagrangian, this amounts to adding the mass terms $\mathcal{L}_m$ to the Lagrangian, with

$$\mathcal{L}_m = -m_\pi^2\pi^+\pi^- - m_K^2 K^+K^- - \cdots \ . \tag{19}$$

Note, however, that this approach is not consistent with chiral symmetry. Rather, a term which transforms under chiral symmetry such as the quark-mass matrix

$$M = \mathrm{diag}(m_u, m_d, m_s) \tag{20}$$

should be considered. This is achieved by

$$\begin{aligned}\mathcal{L}_M &= \frac{f_\pi^2\mu}{2}\mathrm{tr}(MU+U^\dagger M)\\ &= \frac{f_\pi^2\mu}{2}\mathrm{tr}(\xi_R M\xi_L^\dagger + \xi_L M \xi_R^\dagger)\ ,\end{aligned} \tag{21}$$

which therefore should be added to $\mathcal{L}_A$ instead of $\mathcal{L}_m$. (The parameter combination μM is fixed by the physical masses m_π and m_K.) $\mathcal{L}_M$ includes $\mathcal{L}_m$, but also gives contributions to interaction vertices and therefore includes *dynamical* effects of symmetry breaking.

The K_{l4} form-factor predictions resulting from the Lagrangians $\mathcal{L}_m^{(2)}$ and $\mathcal{L}_M^{(2)}$ are well known, of course. I consider them here in order to trace what happens to the dynamical effects of symmetry breaking. It will turn out that $\mathcal{L}_m$ and $\mathcal{L}_M$ both give the same answer in the case of K_{l4} decays, and so it does not matter which term adds to the following hidden-gauge Lagrangians.

The Lagrangian $\mathcal{L}_A$ is based on the field U, which belongs to the differentiable manifold G/H and has a chiral symmetry G, where

$$G = \mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R \tag{22}$$

and

$$H = \mathrm{SU}(3)_V = \{(U,U)\} \subset G\ . \tag{23}$$

It is now known that such a nonlinear model is gauge equivalent to a model with a linear symmetry $G_{\mathrm{global}} \otimes H_{\mathrm{local}}$, based on the manifold G, parametrized by $(\xi_L, \xi_R) \in G$ [1]. This model is defined by the Lagrangian

$$\mathcal{L}_A + a\mathcal{L}_V\ , \tag{24}$$

with

$$\mathcal{L}_V = -\frac{f_\pi^2}{4}\mathrm{tr}(D_\mu\xi_L\xi_L^\dagger + D_\mu\xi_R\xi_R^\dagger)^2\ . \tag{25}$$

Here a is a free parameter. $a=2$ gives complete vector-meson dominance of the electromagnetic form factors, but experimental data on the ρ-meson parameters favor a slightly higher value of $a_{\mathrm{expt}} \approx 2.2$. The covariant derivative D_μ is covariant both with respect to H_{local}, the associated composite gauge bosons V_μ being the vector mesons, and with respect to the external gauge group $I \subset G_{\mathrm{global}}$, the associated gauge fields $\mathcal{L}_\mu$ and $\mathcal{R}$ describing the photon γ, and the bosons $W^\pm$ and Z^0:

$$D_\mu\xi_{L/R} = \mathcal{D}_\mu\xi_{L/R} - igV_\mu\xi_{L/R}\ , \tag{26}$$

where $\mathcal{D}_\mu$ is only covariant with respect to I:

$$\begin{aligned}\mathcal{D}_\mu\xi_L &= \partial_\mu\xi_L + i\xi_L\mathcal{L}_\mu\ ,\\ \mathcal{D}_\mu\xi_R &= \partial_\mu\xi_R + i\xi_R\mathcal{R}_\mu\ .\end{aligned} \tag{27}$$

The vector-meson field V_μ is given by

$$V = \frac{1}{\sqrt{2}}\left\{\begin{pmatrix} 0 & \rho^+ & K^{*+} \\ \rho^- & 0 & K^* \\ K^{*-} & \bar{K}^* & 0\end{pmatrix} + \rho^0\mathrm{diag}\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right) + \cdots\right\} \tag{28}$$

(suppressing the index μ for simplicity). As the Lagrangian stands, the fields V_μ are redundant variables, but in the usual manner I assume that the kinetic term for the V_μ is generated dynamically (see [1]).

The Lagrangian $\mathcal{L}_A + a\mathcal{L}_V$ includes SU(3)-symmetric mass terms for the vector mesons, given by

$$m_\rho^2 = m_{K^*}^2 = \cdots = m_V^2 = af_\pi^2 g^2\ . \tag{29}$$

For the special parameter choice $a=2$, the last equation becomes the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation [9] $m_V^2 = 2f_\pi^2 g^2$.

Model (D) also includes the symmetry breaking $m_\rho^2 \neq m_{K^*}^2$ in the vector-meson sector. This is described by changing $\mathcal{L}_A$ and $\mathcal{L}_V$ to [10]

$$\begin{aligned}\mathcal{L}_A(c_A) &= -\frac{f_\pi^2}{4}\mathrm{tr}\{(D_\mu\xi_L\xi_L^\dagger + D_\mu\xi_L\epsilon_A\xi_R^\dagger)\\ &\qquad -(D_\mu\xi_R\xi_R^\dagger + D_\mu\xi_R\epsilon_A\xi_L^\dagger)\}^2\ ,\\ \mathcal{L}_V(c_V) &= -\frac{f_\pi^2}{4}\mathrm{tr}\{(D_\mu\xi_L\xi_L^\dagger + D_\mu\xi_L\epsilon_V\xi_R^\dagger)\\ &\qquad +(D_\mu\xi_R\xi_R^\dagger + D_\mu\xi_R\epsilon_V\xi_L^\dagger)\}^2\ ,\end{aligned} \tag{30}$$

where

$$\epsilon_{A/V} = \mathrm{diag}(0,0,c_{A/V})\ . \tag{31}$$

The pseudoscalar fields must be renormalized according to

$$\Phi \to (\sqrt{1+\epsilon_A})^{-1}\Phi(\sqrt{1+\epsilon_A})^{-1}\ . \tag{32}$$

The Lagrangian $\mathcal{L}_A(c_A) + a\mathcal{L}_V(c_V)$ then gives

$$m_\rho^2 = m_\omega^2 = ag^2f_\pi^2 = \frac{m_{K^*}^2}{1+c_V} \tag{33}$$

and a symmetry breaking in the decay constants

$$\frac{f_K}{f_\pi} = \sqrt{1+c_A}\ . \tag{34}$$

IV. RESULTS FOR THE K_{l4} FORM FACTORS

A. Models (A) and (B)

In models (A) and (B), there are two Feynman diagrams contributing to the K_{l4} decay [see Figs. 1(a) and 1(b)]. The decay is then described by the form factors

$$F=G=\frac{m_K}{\sqrt{2}f_\pi}\ , \quad R=\frac{m_K}{3\sqrt{2}f_\pi}\,\frac{q^2-4q\cdot p_1-2q\cdot p_2-2p_1\cdot p_2+2m_K^2-m_\pi^2-\xi(m_K^2+m_\pi^2)}{q^2-m_K^2}\ , \tag{35}$$

and

$$H=0\ ,$$

where

$$\xi=\begin{cases}0 & \text{model (A)},\\ 1 & \text{model (B)}\ .\end{cases} \tag{36}$$

In the hadronic matrix element, the form factor H gets multiplied by $\epsilon^{\mu\nu\alpha\beta}$. Therefore it can only get a contribution from a suitable anomalous Lagrangian $\mathcal{L}^{\text{anomalous}}$, which should be added to the Lagrangians of models (A)–(D). Using Witten's reformulation [11] of the Wess-Zumino effective Lagrangian [12], one can easily calculate [13] H at threshold:

$$H=-\frac{1}{4\sqrt{2}\pi^2}\,\frac{m_K^3}{f_\pi^3}=-2.66\ , \tag{37}$$

in very good agreement with the experimental value. So the form factor H is understood quite well and I am not going to consider it any more.

At threshold ($x=0$) the other form factors are then given by

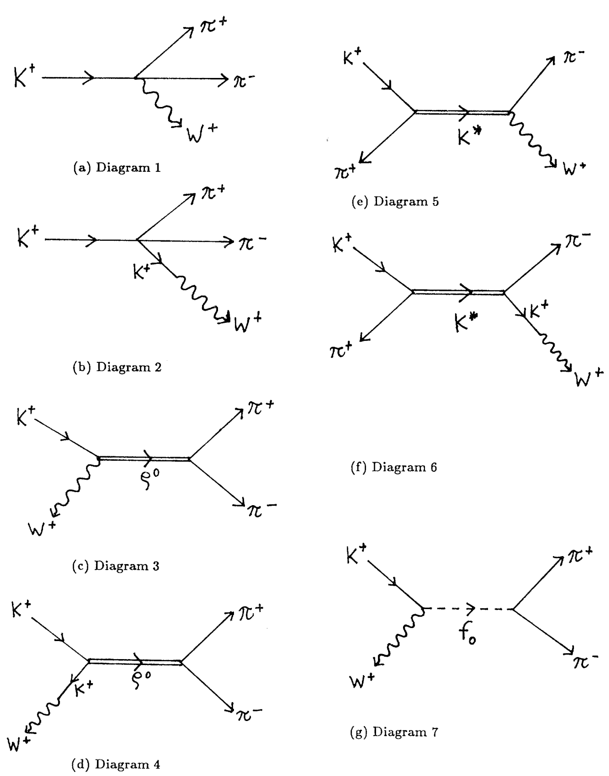

FIG. 1. Feynman diagrams for $K^+\to\pi^+\pi^-W^+$.

$$F=G=\frac{m_K}{\sqrt{2}f_\pi}\ ,$$
$$R=\frac{m_K\{11m_\pi^2-6m_Km_\pi-m_K^2+\xi(m_K^2+m_\pi^2)\}}{12\sqrt{2}m_\pi(m_\pi-m_K)}\ , \tag{38}$$

which in model (A) gives the numerical values

$$F=3.74,\quad G=3.74,\quad R=2.79\ , \tag{39}$$

and, in model (B),

$$F=3.74,\quad G=3.74,\quad R=1.13\ . \tag{40}$$

So the results for models (A) and (B) only differ in the form factor R, which is unmeasurable in electronic K_{l4} decays. One can understand easily how this comes about. The chiral-symmetric term

$$\frac{f_\pi^{p2}}{4}\text{tr}(D_\mu U D^\mu U^\dagger)$$

contains strong interaction vertices

$$\sim P^2\pi^n,\quad n=4,6,8,\ldots$$

and weak interaction values

$$\sim PW^\pm\pi^n,\quad n=1,2,3,\ldots$$

(here P generically denotes momenta of the pseudoscalars π). The symmetry-breaking term

$$\frac{f_\pi^2\mu}{2}\text{tr}(MU+U^\dagger M)\ ,$$

however, contains strong vertices

$$M\pi^n,\quad n=4,6,8,\ldots$$

only. So symmetry breaking does not affect diagram 1 [Fig. 1(a)], which only contains a $W^+\pi^3$ vertex, but only diagram 2 [Fig. 1(b)] via the strong π^4 vertex. But it is obvious that diagram 2 is proportional to the lepton mass, because in the limit of vanishing m_e the V-A weak decay of the virtual K^+ into $e^+\nu_e$ is forbidden by angular momentum conservation.

It should be stressed again that the suppression of the dynamical effects of symmetry breaking relies on the smallness of the lepton mass, and so in muonic K_{l4} decays (or even more so in strange semihadronic τ decays) dynamical effects are feasible.

B. Model (C)

In model (C) there are four additional diagrams which include vector mesons [see Figs. 1(c)–1(f)]. The diagrams with a kaon pole (diagrams 4 and 6), however, are again proportional to m_e and can be neglected. In order to compare my results with certain models in the literature (see below), I take

$$\frac{-i(g^{\alpha\beta}-\zeta p^\alpha p^\beta/m_V^2)}{p^2-m_V^2} \tag{41}$$

as the vector-meson propagator. Obviously, $\zeta=1$ gives the correct propagator for a massive spin-1 field. $\zeta=0$ neglects the factor for the correct spin projection, which will only induce a small error if the momentum transfer P^2 is small compared with m_V^2. I call the model with this approximation model (Ca):

$$\zeta=\begin{cases}1 & \text{model (C)}\ ,\\ 0 & \text{model (Ca)}\ .\end{cases} \tag{42}$$

The result for the measurable axial form factors is then given by

$$\frac{F}{F_0}=1+\frac{3}{4}a\left[\left[1-\zeta\frac{m_K^2-m_\pi^2}{3m_V^2}\right]F_V(t_{K\pi})-1\right] \tag{43}$$

and

$$\frac{G}{G_0}=1+\frac{3}{4}a\left[\left[1+\zeta\frac{m_K^2-m_\pi^2}{m_V^2}\right]\frac{F_V(t_{K\pi})}{3}+\frac{2}{3}F_V(s_{\pi\pi})-1\right],$$

where F_0 and G_0 denote the order $O(P^2)$ results for F and G as in models (A) and (B). The resonance factors are defined by

$$F_X(s)=\frac{m_X^2}{m_X^2-s} \tag{44}$$

and

$$s_{\pi\pi}=(p_1+p_2)^2,\quad t_{K\pi}=(k-p_1)^2\ . \tag{45}$$

Note that the spin projection factor (the one multiplied by ζ) does only contribute if there is pseudoscalar mass splitting $m_K^2\neq m_\pi^2$.

In order to take the vector-meson mass splitting into account, one could use the following approach. The resonance factor $F_V(t_{K\pi})$ results from K^* exchange [Fig. 1(e)] and $F_V(s_{\pi\pi})$ from ρ exchange [Fig. 1(c)]. Correspondingly, one could write

$$\frac{F}{F_0}=1+\frac{3}{4}a\left[\left[1-\zeta\frac{m_K^2-m_\pi^2}{3m_{K^*}^2}\right]F_{K^*}(t_{K\pi})-1\right] \tag{46}$$

and

$$\frac{G}{G_0}=1+\frac{3}{4}a\left[\left[1+\zeta\frac{m_K^2-m_\pi^2}{m_{K^*}^2}\right]\frac{F_{K^*}(t_{K\pi})}{3}+\frac{2}{3}F_\rho(s_{\pi\pi})-1\right], \tag{47}$$

taking for m_ρ and m_{K^*} their different physical values. This defines the models with tildes ($\tilde{\text{C}}$) and ($\tilde{\text{C}}$a), which, however, have not been derived from a Lagrangian without additional assumptions.

Diagrams 1 and 2 [Figs. 1(a) and 1(b)] are proportional to $(1-3/4a)$. So, for the special parameter choice $a=\frac{4}{3}$, the form factors F and G do not get any contribution from the contact diagrams 1 and 2, but only from diagrams 3–6 with vector mesons. So $a=\frac{4}{3}$ gives a complete vector-meson dominance of the weak form factors. The experimental data for a ($a_{\text{expt}}\approx 2.2$) and the KSRF relation, however, do not support such a complete vector-meson dominance.

How do the models considered in this paper compare with approaches in the literature, which start from the chiral current and enhance it by resonance factors? Neglecting the vector part and the part proportional to q_μ (assuming $m_e^2\approx 0$), the result for the hadronic matrix element H^μ from $\mathcal{L}^{(2)}$ (the chiral current) is given by

$$H^\mu=\frac{i\sqrt{2}}{3f_\pi}(2p_1-p_2+k)_\mu\ . \tag{48}$$

If one considers this as a threshold theorem for the pole-enhanced diagrams with ρ and K^* resonances, one finds

$$H^\mu=\frac{i\sqrt{2}}{3f_\pi}[(p_1-p_2)^\mu F_\rho(p_1+p_2)+(k+p_1)F_{K^*}(k-p_1)]\ . \tag{49}$$

This was the ansatz in [2] and is equivalent to my model ($\tilde{\text{C}}$a) with $a=\frac{4}{3}$. In [3] it is discussed that this ansatz is in disagreement with KSRF and the ρ meson parameters, and so in this paper the more general ansatz

$$H^\mu=\frac{i\sqrt{2}}{3f_\pi}[(1-\tfrac{3}{4}a)(2p_1-p_2+k)^\mu+\tfrac{3}{4}a(p_1-p_2)^\mu F_\rho(p_1+p_2)+\tfrac{3}{4}a(k+p_1)^\mu F_{K^*}(k-p_1)]$$

is made, in which the parameter a describes the ratio of contact and resonance diagrams. This is equivalent to my model ($\tilde{\text{C}}a$) with arbitrary parameter a.

Now for the numerical results. In Table I the predictions of the different models for the form factors at

TABLE I. Form factors of model (C) and its variations.

	F	G
Experiment [8]	5.59±0.14	4.77±0.27
Model		
(C)	4.29	5.54
($\tilde{\text{C}}$)	4.20	5.43
(Ca)	5.11	4.72
($\tilde{\text{C}}$a)	4.89	4.75
($\tilde{\text{C}}$a), a=$\frac{4}{3}$	4.44	4.36
[3]	4.91	4.77
[2]	5.30	4.67

threshold are compared. Obviously, in models (C) and (Ca) it is not quite clear which value one should take for the vector-meson mass m_V. But as the ρ and the K^* contribute,

$$m_V=\tfrac{1}{2}(m_\rho+m_{K^*})=833 \text{ MeV} \qquad (50)$$

seems to be a plausible choice, which was used in Table I. The parameter a was always taken as

$$a=2.2 \;, \qquad (51)$$

except for the line where $a=\frac{4}{3}$ (complete vector-meson dominance) has been indicated.

It was said before that model ($\tilde{\text{C}}$a) is equivalent to [2]. The better agreement with the experimental data of the numerical results in [2] as compared with my results can be traced to the approximation $(m_K-m_\pi)^2\approx m_K^2$, which has been made in [2] and is therefore not significant.

It is striking that the "a models," which are characterized by the approximation $\zeta=0$ in the propagator (and therefore also [3]), give results which are in better agreement with the experimental data and predict correctly $F>G$, whereas models (C) and ($\tilde{\text{C}}$) incorrectly predict $F<G$. But as $\zeta=0$ is only an approximation and $\zeta=1$ is the correct value giving the complete propagator with the correct spin projector, this must be considered as accidental.

C. Model (D)

Calculating the diagrams with the Lagrangian of model (D), one gets

$$\frac{F}{F_0}=\frac{1}{\sqrt{1+c_A}}\left\{\frac{a}{4}\left[\left[3+c_V+\zeta\frac{2c_V(q\cdot p_1+p_1\cdot p_2)+(1+c_V)(m_\pi^2-m_K^2)}{m_{K^*}^2}\right]F_{K^*}(t_{K\pi})-3-c_V\right]+\frac{1}{2}c_A+1\right\},$$

$$\frac{G}{G_0}=\frac{1}{\sqrt{1+c_A}}\left\{\frac{a}{4}\left[\left[1-c_V-\zeta\frac{2c_V(q\cdot p_1+p_1\cdot p_2)+(1+c_V)(m_\pi^2-m_K^2)}{m_{K^*}^2}\right]F_{K^*}(t_{K\pi})+2\left[1+\frac{c_A}{a}\right]F_\rho(s_{\pi\pi})-3+c_V\right]+1\right\}. \qquad (52)$$

The parameters c_A and c_V can be determined from the ratios of the vector-meson masses and decay constants:

$$\frac{m_{K^*}}{m_\rho}=\sqrt{1+c_V}\Longrightarrow c_V=0.354 \qquad (53)$$

and

$$\frac{f_K}{f_\pi}=\sqrt{1+c_A}\Longrightarrow c_A=0.488 \;. \qquad (54)$$

In Table II the numerical predictions for the form factors at threshold are displayed. I also include model (C), which corresponds to

$$c_V=c_A=0 \;,$$

and model ($\tilde{\text{C}}$a), which only considers the kinematical effects of the vector-meson mass splitting. One finds that the predictions are changed only a little by the mass splitting (F is changed into the wrong direction, G in the right direction). The additional inclusion of the *dynamical* effects of symmetry breaking in model (D) as compared with ($\tilde{\text{C}}$) changes F not at all and G only by a tiny (0.1) amount.

TABLE II. Models with and without symmetry breaking in the vector masses.

	F	G
Experiment [8]	5.59±0.14	4.77±0.27
Model		
(C)	4.29	5.54
($\tilde{\text{C}}$)	4.20	5.43
(D)	4.20	5.32

D. Comparison with the low-energy expansion

Lately, Bijnens [5] and Riggenbach *et al.* [6] independently calculated K_{l4} decay with the same model, viz., at order $O(P^4)$ in the expansion in terms of low momenta and small quark masses. In this approach one considers the most general chiral Lagrangian $\mathcal{L}^{(4)}$ of order $O(P^4)$:

$$\mathcal{L}^{(4)}=\sum_{j=1}^{10}L_jP_j \;, \qquad (55)$$

where $L_1,\ldots,L_{10}$ are ten additional free constants, to be fitted to experimental data, and $P_1,\ldots,P_{10}$ list all different possible terms in U and M which have all the necessary symmetries [4]. The tree diagrams from $\mathcal{L}^{(2)}$ are then corrected by tree diagrams with one vertex from $\mathcal{L}^{(4)}$ and by diagrams with one loop and vertices from $\mathcal{L}^{(2)}$, which are also of the order $O(P^4)$. The low-energy constants $L_1,\ldots,L_{10}$ have been determined by comparison with experimental data in [4], and so one gets a prediction for the K_{l4} form factors if one uses these values for the low-energy constants. On the other hand, one can make a fine-tuning of the constants to get a best fit to the experimental values for the K_{l4} form factors in order to get a better estimate for the constants $L_1, \ldots, L_{10}$.

In Table III Bijnens' results are compared with those of models (C) and (D). Both Bijnens' prediction using the parameters of [4] and his best fit are displayed. It is obvious that it is easy to get very good agreement between theory and experiment with ten free parameters, and so in comparing his results with those of models (C) and (D) one should consider his prediction and not his best fit. For this reason the main difference between the models considered in this paper and Bijnens' results must be seen in the fact that he gets a form factor F which is about 1 bigger (about 5.2 rather than 4.2, which the hidden-symmetry Lagrangian yields) and therefore in much better agreement with experiment. So where does the significantly better agreement of the low-energy expansion with experiment come from? As explained in detail in [6], the consideration of chiral loops (especially those which describe a $\pi\pi$ final-state interaction) enhances F by a sufficient amount, whereas G is left almost unchanged. So the difference between my results and those of [5,6] stems from the inclusion of loop diagrams. One might, however, ask the question whether or not the simultaneous consideration of resonances and loops amounts to a double counting, as it is not clear whether loops describe resonances or a nonresonant contribution. As explained in [6], the loop diagrams mainly give a $J^P=0^+$ contribution. The better agreement of the "a models" with experiment, which are defined by the neglect of the spin projection for a spin-1 field, also hints at the importance of an additional s-wave contribution. In the next section I will discuss if this s-wave contribution could be described by a $J^P=0^+$ resonance channel rather than by chiral loops.

Before I proceed to this, I would like to comment on another point. In Refs. [14,15] the role of resonances in chiral perturbation theory has also been discussed. The authors integrate over the vector-meson degrees of freedom and determine the contribution of the hidden-gauge Lagrangian $\mathcal{L}_A+a\mathcal{L}_V$ to the $O(P^4)$ low-energy constants $L_1, \ldots, L_{10}$. These values might then be inserted into the results for the K_{l4} form factors in [5,6]. Several comments, however, are in order.

(i) In this paper I am not working in a fixed order in P^2. Breit-Wigner resonances sum up all orders in P^2:

$$F_X(s)=\frac{m_X^2}{m_X^2-s}=\sum_{n=0}^{\infty}\left[\frac{s}{m_x^2}\right]^n . \quad (56)$$

The above-described approach amounts to approximating this series by

$$F_X(s)\approx\sum_{n=0}^{1}\left[\frac{s}{m_x^2}\right]^n . \quad (57)$$

(ii) In [14,15] an SU(3)-symmetric octet of vector mesons is assumed. The low-energy couplings derived from the hidden-gauge Lagrangian are proportional to the inverse vector-meson mass squared:

$$L_i(m_V)\sim\frac{1}{m_V^2} . \quad (58)$$

For simplicity, $m_V=m_\rho$ is assumed in [14], but I could equally well take $m_V=m_{K^*}$, which induces a considerable uncertainty on the couplings L_i,

$$L_i(m_\rho)=1.35\times L_i(m_{K^*}) \quad (59)$$

and a corresponding uncertainty on the predictions for F and G.

In the approach taken in this paper, retaining the vector-meson fields as dynamical degrees of freedom in the Lagrangian, I do not have these ambiguities. I can determine from the Feynman diagrams the mesons contributing to the resonance factors and then use the appropriate physical masses. This is what I did in model ($\tilde{\mathrm{C}}$). Or I can start from an improved Lagrangian which includes the vector-meson mass splitting (and dynamical effects thereof) from the very beginning. This is done in model (D).

(iii) If I insert the L_i derived from the completely SU(3)-symmetric hidden-gauge Lagrangian into the results of [5,6], I get $F=4.13-4.27$ and $G=5.04-5.50$, but in any case $F<G$ in contradiction with experiment. The vector-meson dominance models [2,3], on the other hand, give results with $F>G$, which are in good agreement with experiment. But if I expect these two approaches to give similar results, this is an apparent contradiction. The source of this contradiction can only be found if one does not integrate out the vector mesons. As shown above, the sign of F-G depends on the detailed form of the vector-meson propagator and the explicit chiral-symmetry breaking. When using vector-meson dominance, usually the S-wave projection is not subtracted. This is only justified by assuming complete chiral symmetry, in which case there is no S-wave part in the chiral current anyway.

TABLE III. Comparison with low-energy expansion.

	F	G
Experiment [8]	5.59±0.14	4.77±0.27
Model		
(C)	4.29	5.54
(D)	4.20	5.32
Bijnens [5]		
Prediction	5.22	5.42
Best fit	5.60	4.76

V. SCALAR RESONANCE CHANNEL

In this section I will discuss the effect of a scalar resonance S in the $\pi\pi$ channel [see Fig. 1(g)]. The S particle could be the $f_0(975)$, which decays predominantly into $\pi\pi$. The partial-wave expansion of the form factors shows that such a scalar resonance can only contribute to f_S, i.e., only to the form factor F, and so I consider the S-wave resonance by starting from model ($\tilde{\mathrm{C}}$) and changing F in the following way [7,16]:

$$\frac{F}{F_0} \to 1 + \frac{3}{4} a \left[\left(1 - \frac{m_K^2 - m_\pi^2}{3 m_{K^*}^2} \right) F_{K^*}(t_{K\pi}) - 1 + \epsilon [F_S(s_{\pi\pi}) - 1] \right] , \quad (60)$$

where

$$F_S(s_{\pi\pi}) = \frac{m_S^2}{m_S^2 - s_{\pi\pi}} . \quad (61)$$

I call this ansatz model ($\tilde{C}_S$). ϵ is an unknown free parameter, describing the strength of the S-wave resonance coupling as compared to the vector-meson resonances.

In order to get a handle on the parameter ϵ, I will now also consider the slope parameters λ_F and λ_G of the form factors, which have been neglected until now. They are given by

$$\lambda_F = \frac{1}{x} \left[\frac{F(x)}{F(0)} - 1 \right] , \qquad \lambda_G = \frac{1}{x} \left[\frac{G(x)}{G(0)} - 1 \right] , \quad (62)$$

where x has been defined in Eq. (6).

In the extraction of λ_F from the data in [8], a linear dependence $F(x)$ on x was assumed. If this was reproduced exactly by the theory, I could choose any $s_{\pi\pi}$ above threshold, calculate the corresponding $F(x)$, and get the slope by use of the above formula. The linear dependence, however, is not reproduced exactly by model ($\tilde{C}_S$). Therefore, I will calculate the slope parameters considering $F(x)$ at $\sqrt{s_{\pi\pi}} = 347$ MeV, so that the brackets $0, \ldots, x$ cover 80% of all events [8]. (The precise value used for $s_{\pi\pi}$ here has only a very small effect on the final results.) Fixing a value for $s_{\pi\pi}$ does not determine the value of $t_{K\pi}$, and so one should integrate over all possible values. But if the experimental finding that the form factors only depend significantly on $s_{\pi\pi}$ is reproduced by the model, one can use a simpler approach [5]. I express $t_{K\pi}$ in terms of $s_{\pi\pi}$, $\cos\theta_\pi$, and s_l, where $s_l = (p_3 + p_4)^2$ is the invariant mass of the leptonic system. Then I take average values for $\cos\theta_\pi = 0.1$ and $\sqrt{s_l} = 100$ MeV, giving $t_{K\pi} = 0.085$ GeV^2. Varying $\cos\theta_\pi$ from 0.0 and 0.9 or $\sqrt{s_l}$ from 0 to 147 MeV leaves $t_{K\pi}$ in the interval 0.072 to 0.092 GeV^2. So I use the value

$$t_{K\pi} = 0.082 \pm 0.010 \text{ GeV}^2 \quad \text{at } \sqrt{s_{\pi\pi}} = 0.347 \text{ GeV} . \quad (63)$$

I have found that the theoretical uncertainty in $t_{K\pi}$ induces errors in λ_F and λ_G which are always below 0.01 and therefore may be neglected.

This way λ_G for models ($\tilde{C}$) and ($\tilde{C}_S$) is found to be

$$\lambda_G = 0.04 . \quad (64)$$

The values for F and λ_F in model ($\tilde{C}_S$) are displayed in

TABLE IV. Predictions of model ($\tilde{C}_S$) in variation with ϵ.

ϵ	F	λ_F
0	4.20	−0.18
1	4.76	−0.02
1.2	4.87	0.00
1.4	4.98	0.02
1.6	5.10	0.05
1.8	5.21	0.07
2.0	5.32	0.09
2.2	5.43	0.11
2.4	5.54	0.13
2.6	5.67	0.15
2.8	5.77	0.17
3.0	5.88	0.19

Table IV in variation with ϵ. The value $\epsilon = 0$ corresponds to model ($\tilde{C}$), and it is seen that in this model not only is F too small, but also λ_F is in disagreement with the experimental finding. Allowing for one standard deviation, one can use the experimental value for F to get $\epsilon = 2.2 - 2.7$ or the experimental value for λ_F to get $\epsilon = 1.7 - 2.1$. The two results for ϵ are compatible and can be summarized by

$$\epsilon = 2.2 \pm 0.5 . \quad (65)$$

In Table V the results for model ($\tilde{C}_S$) with $\epsilon = 2.2$ are summarized and compared with the results of the low-energy expansion results given by Riggenbach *et al.* [6] (Bijnens does not give results for the slope parameters) and with experiment. One can see that this model with $J^P = 1^-$ and 0^+ resonances (and without loops) gives results which are of a quality comparable with those of the low-energy expansion, but with only one unknown parameter (viz., ϵ), the others (a and f_π) being fixed by the parameters of the ρ and π mesons.

VI. CONCLUSIONS

The axial form factors and slope parameters for K_{l4} decays have been calculated including virtual $J^P = 1^-$ and 0^+ resonances. It has been shown that the inclusion of $J^P = 1^-$ resonances only is not fully sufficient. The results from the Lagrangian with hidden local symmetries are in significantly worse agreement with experiment compared with the results of the low-energy expansion [5,6]. This can be explained by the importance of an additional S-wave contribution, which in the $O(P^4)$ calculation stems from $\pi\pi$ loops. It can alternatively be described by a scalar resonance in the $\pi\pi$ channel.

As far as the mass splitting is concerned, it has been

TABLE V. Model ($\tilde{C}_S$) (with $\epsilon = 2.2$) compared with the literature.

	F	G	λ_F	λ_G
Experiment [8]	5.59±0.14	4.77±0.27	0.08±0.02	0.08±0.02
Model ($\tilde{C}_S$)	5.43	5.43	0.11	0.04
Prediction in [6]	5.03	5.14	0.06	0.12

shown that the direct breaking of chiral symmetry has no significant effects on K_{l4} decays except for the importance of the spin projection factor in the vector-meson propagator (and the obvious changes in the phase space). The dynamical effect of the mass splitting in the pseudoscalar sector is suppressed by the smallness of the lepton mass, and the effect of the splitting in the vector-meson sector is very small because it is suppressed by the product of two small factors, viz.,

$$\left[\frac{m_{K^*}^2}{m_\rho^2}-1\right]\left[\frac{\text{four-momentum transfers}}{\text{vector-meson masses}}\right]^2 .$$

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