Physics Letters B 289 (1992) 449–454
North-Holland

PHYSICS LETTERS B

Constituent quarks and total cross sections at LHC/SSC

Jon Pumplin

Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA [1]
and Deutsches Elektronen-Synchrotron DESY, Notkestrasse 85, W-2000 Hamburg 52, FRG

Received 10 June 1992; revised manuscript received 10 July 1992

A constituent quark model is shown to describe the phenomenology of $\sigma_{\rm tot}$ and the elastic $d\sigma/dt$ at high energy in a natural way. Multiple scattering is found to be important. The model is used to predict $\sigma_{\rm tot}$ and the forward elastic slope B at energies of the LHC and SSC colliders. The predicted $\sigma_{\rm tot}$ is smaller than in most other models. Cross sections for single-diffractive scattering are also predicted.

1. Constituent quark model

Phenomenological descriptions of elastic scattering at high energy have previously been formulated in terms of continuous smooth "matter distributions" for the beam and target particles [1–3]. The overlap of b-space projected matter distributions determines the eikonal function in these models, which in turn determines the elastic amplitude in a manner consistent with s-channel unitarity.

It has lately become apparent again that, to the contrary, non-perturbative QCD may prefer a picture in which *constituent* quarks, together perhaps with pionic degrees of freedom, describe the typical configurations of the proton that are important for low Q^2 physics [4,5]. The constituent quarks would be expected to interact strongly in a high energy collision, but only at short range [6]. In this paper, we examine the implications of the constituent quark point of view for elastic and total cross sections, as an alternative to the eikonal picture.

We therefore assume an elastic amplitude for pp or $\bar{\rm p}$p scattering as a function of impact parameter in the form

$$\tilde{M}(b) = \int d\mathcal{P}(\boldsymbol{b}_1,\boldsymbol{b}_2,\boldsymbol{b}_3)\, d\mathcal{P}(\boldsymbol{b}'_1,\boldsymbol{b}'_2,\boldsymbol{b}'_3) \times\left(1-\prod_{j=1}^{3}\prod_{k=1}^{3}[1-T_{\rm qq}(|\boldsymbol{b}_j-\boldsymbol{b}'_k-\boldsymbol{b}|)]\right). \qquad (1)$$

The amplitude as a function of four-momentum transfer $t=-\Delta^2$ is given by the Hankel transform

$$M(t) = 2{\rm i}\int d^2\boldsymbol{b}\, \exp({\rm i}\boldsymbol{b}\cdot\boldsymbol{\Delta})\,\tilde{M}(|\boldsymbol{b}|) = 4\pi{\rm i}\int_0^\infty b\, db\, J_0(b\Delta)\tilde{M}(b), \qquad (2)$$

with normalization $\sigma_{\rm tot} = {\rm Im}M(0)$. We neglect the small real part of the amplitude: $\tilde{M}(b)$ is the absorptive part and $M(t)$ is pure imaginary.

$T_{\rm qq}(|\boldsymbol{b}|)$ is the interaction probability for a pair of quarks at impact parameter $\boldsymbol{b}$. Hence $1-T_{\rm qq}$ is the probability for a pair of quarks *not* to interact. In eq. (1), $\prod\prod(1-T_{\rm qq})$ is the probability for *no* pairs to interact, and therefore the factor $[1-\prod\prod(1-T_{\rm qq})]$ is simply the probability for *one or more* pairs of quarks to interact. To clarify the assumption, consider the simpler situation of a one-constituent object scattering on a two-constituent object, with no dependence on impact parameter. In that case the absorptive part reduces to $\tilde{M} = T_1 + T_2 - T_1T_2$. This formula is especially appealing in the limit of total absorption: if $T_1 \to 1$ then $\tilde{M} \to 1$ independently of T_2, as it should.

[1] E-mail address: PUMPLIN@MSUPA

It is a clear improvement over the single-scattering approximation $\tilde{M} = T_1 + T_2$, which would allow the absorption to be blacker than black. Eq. (1) is equivalent to the multiple scattering theory of Glauber [7]. For a related discussion see ref. [8].

Two parametrizations of T_{qq} will be considered here: a simple "black disk"

$$T_{qq}(b) = 1 \quad \text{if } b < D, \\ = 0 \quad \text{otherwise}, \tag{3}$$

which assumes the interaction strong and short range in the most extreme possible way, and a gaussian form

$$T_{qq}(b) = \eta \exp(-b^2/R^2). \tag{4}$$

The probability distribution for the constituent quarks in the proton as a function of their impact parameters, which appears in eq. (1), is assumed to be

$$\mathrm{d}\mathcal{P}(\boldsymbol{b}_1,\boldsymbol{b}_2,\boldsymbol{b}_3) = \prod_{j=1}^{3}[\mathrm{d}^2\boldsymbol{b}_j f(\boldsymbol{b}_j)]\delta^{(2)}\left(\sum_{k=1}^{3}\boldsymbol{b}_k\right). \tag{5}$$

This contains no correlations except for a δ-function that represents angular momentum conservation in the approximation that the quarks have equal longitudinal momentum. As a specific form, I use

$$f(\boldsymbol{b}) = c\int \mathrm{d}^2\boldsymbol{p}\,\exp(\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{b})\,(\boldsymbol{p}^2+\mu^2)^{-\nu-1} \\ = c'\,b^{\nu}K_{\nu}(b\mu). \tag{6}$$

This form is monotonic and smooth in both $|\boldsymbol{b}|$ and $|\boldsymbol{p}|$, and includes two parameters to govern the range and shape of the probability distribution. It is otherwise ad hoc. The normalization constant c (or c') is fixed by $\int \mathrm{d}\mathcal{P} = 1$.

The constituent quark probability distribution in the proton can be determined in principle from the electromagnetic form factor

$$G_{\mathrm{EM}}(t) = G_{\mathrm{w}}(t)\,G_{\mathrm{q}}(t), \tag{7}$$

where $G_{\mathrm{w}}(t)$ comes from the wave function

$$G_{\mathrm{w}}(-\Delta^2) = \int \mathrm{d}\mathcal{P}(\boldsymbol{b}_1,\boldsymbol{b}_2,\boldsymbol{b}_3)\,\exp(\mathrm{i}\boldsymbol{b}_1\cdot\boldsymbol{\Delta}) \\ \propto \int \mathrm{d}^2\boldsymbol{p}\,(\boldsymbol{p}^2+\mu^2)^{-2\nu-2}[(\boldsymbol{p}+\boldsymbol{\Delta})^2+\mu^2]^{-\nu-1}. \tag{8}$$

This can be derived from eqs. (1), (2) by replacing one of the scattering protons with a structureless electron. One of the probability distributions, say $\mathrm{d}\mathcal{P}(\boldsymbol{b}'_1,\boldsymbol{b}'_2,\boldsymbol{b}'_3)$, then becomes point-like and the multiple scattering terms go away because the interaction probability is small. Finally, each of the three quarks can be assumed to have the same distribution, so integrals with $\exp(\mathrm{i}\boldsymbol{b}_j\cdot\boldsymbol{\Delta})$ are independent of j. The second half of the equation is obtained using our parametrizations eqs. (5), (6).

$G_{\mathrm{q}}(t)$ is the intrinsic form factor of the constituent quark. It can be assumed to be something like $G_{\mathrm{q}}(t) = (1-t/m^2)^{-1}$, where $m^2 \approx 10\,\mathrm{GeV}^2$ can be expected on the basis of the small size that will be found below for the quarks. That small size is also consistent with the absence of quark excitation effects in hadron spectroscopy. The precise $G_{\mathrm{q}}(t)$ plays no major role, since the important region of the matter distribution is determined by small $-t$.

Fitting the measured $G_{\mathrm{EM}}(t)$ [9] leads to $\nu = 0.20$ and $\mu = 0.395\,\mathrm{GeV}$. This fit has $4G'_{\mathrm{w}}(0) = \langle b_1^2\rangle = 11.6\,\mathrm{GeV}^{-2}$, where $\langle b_1^2\rangle = \int \mathrm{d}\mathcal{P}(\boldsymbol{b}_1,\boldsymbol{b}_2,\boldsymbol{b}_3)\,\boldsymbol{b}_1^2 = 8(\nu+1)(3\nu+2)/3\mu^2(3\nu+4)$. It agrees with the observed derivative $G'_{\mathrm{EM}}(0) = 3.0\,\mathrm{GeV}^{-2}$, and hence reproduces the proton charge radius [10] exactly. It continues to agree well with $G_{\mathrm{EM}}(t)$ out to $t \simeq -2\,\mathrm{GeV}^2$ – the fit is much better, for example, than the traditional dipole fit. In the case of the black disk interaction [eq. (3)], this parametrization does *not*, however, lead to adequate phenomenology for $\bar{\mathrm{p}}\mathrm{p}$ scattering: the forward slope comes out a bit too small. For that case, I use instead $\nu = -0.25$ and $\mu = 0.257\,\mathrm{GeV}$, which gives the same $G'_{\mathrm{EM}}(0)$ and hence continues to reproduce the proton charge radius. This parametrization still fits the measured $G_{\mathrm{EM}}(t)$ better than the dipole fit for $-t < 0.10\,\mathrm{GeV}^2$, but rises $\sim 40\%$ above it by $t = -1.0\,\mathrm{GeV}^2$.

2. Extrapolation to LHC and SSC using black disk $T_{qq}(b)$

According to the spirit of the model, the wave function parameters μ and ν are independent of energy. With one eye on the proton charge radius and the other on the phenomenology, we set $\nu = -0.25$ and $\mu = 0.257\,\mathrm{GeV}$ as described above. The quark–quark black disk interaction distance D in eq. (3) can be determined at any given energy by fitting the total cross section. Consistency is then demonstrated by the fact that the model correctly describes the shape of $d\sigma/dt$.

Numerical results were obtained from the model by Monte Carlo integration of eq. (1) using a technique described in the Appendix, followed by numerical integration of eq. (2). In the region we need it, the cross section according to the model can be described conveniently by an empirical formula

$$\sigma_{tot} = 4.66\,\text{mb} \cdot D^2(D^2 + 25.1)/(D^2 + 6.00). \quad (9)$$

(This formula is not valid at extreme values of D, since it goes to $15.9\pi D^2$ instead of the exact $18\pi D^2$ for $D \to 0$, and to $3.8\pi D^2$ instead of the exact $2\pi D^2$ for $D \to \infty$.)

In order to have an adequate lever arm for extrapolation to higher energy, we begin with the ISR measurement $\sigma_{tot} = 43.55 \pm 0.31$ mb [11] for pp scattering at $\sqrt{s} = 62.3$ GeV. (Contributions from ordinary Regge poles, which fall as $s^{\alpha-1}$ with $\alpha \simeq 0.5$, are neglected here in σ_{tot}^{pp}, since they are expected to be small compared to $\sigma_{tot}^{\bar{p}p} - \sigma_{tot}^{pp}$ according to *exchange degeneracy*, and that difference is already quite small at this energy. The Regge contributions can be neglected even in $\sigma_{tot}^{\bar{p}p}$ at the other – much higher – energies we consider, because of their power-law decrease.) Fitting the cross section requires $D = 1.730 \pm 0.008\,\text{GeV}^{-1}$ in the model.

At $\sqrt{s} = 546$ GeV, the measured $\bar{p}p$ cross section is $\sigma_{tot} = 62.1 \pm 1.6$ mb [12] (assuming $\rho = \text{Re}\,M(0)/\text{Im}\,M(0) \simeq 0.14$). This is reproduced in the model by $D = 2.195 \pm 0.039\,\text{GeV}^{-1}$. The model can be tested by its predictions for the shape of $d\sigma/dt$. For the average of the slope parameter $B(t) = d(\ln d\sigma/dt)/dt$ over the region $0.03 < -t < 0.10$, it gives $\overline{B} = 14.7$, which is close to the measured 15.3 ± 0.3 [13]. The forward slope (ignoring Coulomb effects) according to the model is $B(0) = 16.7$, which agrees with a previous [14] extrapolation $B(0) = 16.8 \pm 0.2$ of these data to $t = 0$. The variation of $B(t)$ with t confirms the significant "curvature" of $\ln d\sigma/dt$ [14,15]. (A small change in the values of μ and ν could be used to make the slope agree precisely at any one energy. It would be further necessary to include the real part of the amplitude, and to make a small change in the parametrization, in order to fit the precise location of the "diffraction dip" and the height of the secondary maximum that appear at larger $-t$ [16].)

At $\sqrt{s} = 1800$ GeV, E710 has measured $\sigma_{tot} = 72.8 \pm 3.1$ mb [17]. (The true cross section is probably on the high side of this, because the analysis assumed zero curvature [14]. A CDF measurement 72.0 ± 3.6 mb [18] is consistent.) The model reproduces the E710 result with $D = 2.458 \pm 0.076\,\text{GeV}^{-1}$. The model then predicts an average slope $\overline{B} = 16.0$ over the region $0.0006 < -t < 0.142$, which is close to the value 16.99 ± 0.47 measured by E710 in that region. The predicted variation of $B(t)$ with t cannot be tested at present, since the E710 data analysis did not allow for that possibility. According to the model, the forward hadronic slope is $B(0) = 17.6$.

We can fit the energy dependence of D based on the above three measurements, and thereby extrapolate to higher energies. A simple logarithmic form

$$D = 0.837 + 0.108 \ln s \quad (10)$$

is plausible, and fits the three points extremely well. The resulting extrapolations to LHC and SSC energies are shown in table 1.

Only the central values are shown in table 1, since the uncertainty in extrapolation is dominated by the choice of formula for fitting $D(s)$. For example, two somewhat extreme choices which nevertheless fit the three low-energy measurements satisfactorily are $D = (-0.578 + 0.432 \ln s)^{1/2}$ and $D = 1.111 s^{0.0536}$. At the SSC energy $\sqrt{s} = 40\,000$, these lead to $\sigma_{tot} = 92$ mb, $B(0) = 19.3$ and $\sigma_{tot} = 115$ mb, $B(0) = 21.3$ respectively.

Table 1
Fit and extrapolation using black disk qq interaction.

$\sqrt{s}$ (GeV)	D (GeV^{-1})	σ_{tot} (mb)	$B(0)$ (GeV^{-2})	σ_{sd} (mb)
62.3	1.730	43.6	15.2	4.5
546	2.198	62.2	16.7	7.1
1 800	2.456	72.7	17.6	8.3
16 000	2.928	92.3	19.2	10.2
40 000	3.126	100.7	20.0	10.9

Table 2
Fit and extrapolation using gaussian qq interaction.

$\sqrt{s}$ (GeV)	R (GeV^{-1})	σ_{tot} (mb)	$B(0)$ (GeV^{-2})	σ_{sd} (mb)	$2\langle b_1^2\rangle$	$\langle b^2\rangle_{qq}$	$\langle b^2\rangle_{MI}$
62.3	1.887	43.6	15.0	2.4	23.2	3.6	3.3
546	2.332	62.1	16.7	3.4	23.2	5.4	4.7
1 800	2.577	73.1	17.7	3.8	23.2	6.6	5.5
16 000	3.024	94.6	19.7	4.5	23.2	9.1	7.1
40 000	3.212	104.1	20.7	4.7	23.2	10.3	7.9

3. Extrapolation to LHC and SSC using gaussian $T_{qq}(\boldsymbol{b})$

In this section, we replace the black disk assumption for the quark–quark interaction by the gaussian form of eq. (4). The presence of a "tail" in the interaction probability at larger $|\boldsymbol{b}|$ is more reasonable physically. Also, the presence of two interaction parameters (η and R) provides enough flexibility to fit the cross section and slope at any one energy. In view of this flexibility, it works to determine the quark probability distribution solely from $G_{EM}(t)$. Hence in this section we use the preferred parameters $\nu = 0.20$ and $\mu = 0.395$ GeV described in section 1.

At $\sqrt{s} = 546$ GeV, where the shape of $d\sigma/dt$ is well measured, we find that $\eta \simeq 0.7$ is needed in order to fit σ_{tot} and $B(0)$ simultaneously. If we assume $\eta = 0.7$ independently of energy, we can make an analysis similar to that of section 2. For numerical work, the cross section can be described this time by the empirical formula

$$\sigma_{tot} = 6.63\,\text{mb} \cdot R^2(R^2 + 16.3)/(R^2 + 7.18). \quad (11)$$

[Like eq. (9), this formula is not valid at extreme values of R. It goes to $12.30\pi R^2$ instead of the exact $12.60\pi R^2$ for $R \to 0$, and to $5.42\pi R^2$ instead of the exact $4.94\pi R^2$ for $R \to \infty$.]

Fitting the three σ_{tot} measurements of section 2 leads to

$$R = 1.036 + 0.1026 \ln s, \quad (12)$$

which extrapolates to $R = 3.210 \Rightarrow \sigma_{tot} = 104.1$ mb, $B(0) = 20.7$ at SSC. Further results from this parametrization are shown in table 2.

Other possible "extreme" forms for the extrapolation are $R = (-0.104 + 0.442 \ln s)^{1/2} \Rightarrow R = 3.044$, $\sigma_{tot} = 95.5$ mb, $B(0) = 19.8$; and $R = 1.270 s^{0.0477} \Rightarrow R = 3.490$, $\sigma_{tot} = 118.8$ mb, $B(0) = 22.3$ at SSC. These results are similar to those obtained using the black disk interaction in section 2.

4. Discussion

We have seen that a model in which protons contain a fixed distribution of three constituent quarks, whose interaction range is small and increases gradually with energy, can describe the observed features of the total cross section and small angle elastic scattering [#1]. We have used the model to extrapolate σ_{tot} and B to energies of the future colliders LHC and SSC. Results are presented in tables 1 and 2. Readers who prefer graphical displays can create them using eqs. (9), (10) and (11), (12).

It is interesting to compare our results at the SSC energy $\sqrt{s} = 40$ TeV with some other predictions. Assuming a black disk qq interaction whose range D grows as $\ln s$, we have found $\sigma_{tot} \approx 101$ mb and $B(0) \approx 20.0\,GeV^{-2}$ as shown in table 1. A gaussian qq interaction leads to similar values which are shown in table 2. Even allowing somewhat extreme parametrizations for $D(s)$ or $R(s)$ in these models, our extrapolated σ_{tot} always falls in the range 92–119 mb. On the other hand, the eikonal model of Bourrely, Soffer and Wu [2,14] predicts $\sigma_{tot} = 120$ mb and $B(0) = 21.1\,GeV^{-2}$ [#2]. Ref. [19] predicts $\sigma_{tot} > 125$ mb. The model of Barshay et al. [21] predicts $\sigma_{tot} = 144$ mb. *Our constituent quark prediction for*

[#1] A recent paper by Gotsman, Levin and Maor [19] claims that an additive quark model such as this cannot describe the data. However, they did not include the multiple scattering terms which are found here to be important.

[#2] An eikonal model which uses different parametrizations leads to similar results [20].

σ_{tot} is thus noticeably smaller than all of these other predictions.

In a more naive approach, simply fitting the three cross section measurements of section 2 to a power law yields an excellent fit with $\sigma_{\rm tot} = 22.7 s^{0.0791}$, which extrapolates to $\sigma_{\rm tot} = 121$ mb at SSC. On the other hand, a logarithmic dependence $\sigma_{\rm tot} = 8.0 + 4.30 \ln s$ also fits these three measurements, and extrapolates to $\sigma_{\rm tot} = 99$ mb. Our constituent quark prediction for $\sigma_{\rm tot}$ is near the lower "limit" of these naive extrapolations.

The total cross section for constituent qq scattering according to the model is $2\pi D^2 = 11.8$ mb (black disk $T_{\rm qq}$), or $2\pi\eta R^2 = 9.3$ mb (gaussian $T_{\rm qq}$) for pp scattering at $\sqrt{s} = 546$ GeV which corresponds to constituent qq scattering at $\sqrt{s} \simeq 61$ GeV. Thus $\sigma_{\rm pp} \simeq (5.3\text{–}6.7) \times \sigma_{\rm qq}$ at that energy. *This shows that multiple interactions (shadow effects) are important, since without them one would have $\sigma_{pp} = 9 \times \sigma_{qq}$.* Accordingly, the "additive quark model" [22] requires significant corrections.

Because of the possibility of multiple interactions, different constituent quark configurations of a proton have different probabilities for interacting with another proton. This leads to inelastic diffraction scattering ("diffractive dissociation") [23] in addition to elastic diffraction. The integrated cross sections can be calculated from

$$\frac{d\sigma_{\rm el}}{d^2\boldsymbol{b}} = \left| \int d\mathcal{P} \int d\mathcal{P}' \left(1 - \prod_{j,k} (1 - T_{\rm qq}) \right) \right|^2 , \tag{13}$$

$$\frac{d\sigma_{\rm el}}{d^2\boldsymbol{b}} + \frac{d\sigma_{\rm sd}}{d^2\boldsymbol{b}} = \int d\mathcal{P} \left| \int d\mathcal{P}' \left(1 - \prod_{j,k} (1 - T_{\rm qq}) \right) \right|^2 , \tag{14}$$

where eq. (13) expresses $d\sigma_{\rm el}/d^2\boldsymbol{b} = |\tilde{M}(b)|^2$ using a simplified notation for $\tilde{M}(b)$ of eq. (1). The predicted cross sections $\sigma_{\rm sd}$ for inelastic diffraction are shown in tables 1 and 2. The printed values are to be doubled to include dissociation of either proton. We note that the two models for $T_{\rm qq}$ lead to predictions which differ by more than a factor of 2, so measurements of the single diffractive cross section – particularly at the Tevatron – would be useful for choosing between those models. It has been suggested [8], however, that it will be necessary to include inelastic shadow effects in eq. (1) in order to properly describe the t-dependence of diffractive production. This point remains under investigation.

It is interesting to consider the origin of the range of the pomeron in impact parameter space. That range can be described by the mean squared radius, which is measured by the forward slope parameter

$$\langle b^2 \rangle = \frac{1}{\sigma_{\rm tot}} \int d^2\boldsymbol{b}\, \boldsymbol{b}^2 \frac{d\sigma_{\rm tot}}{d^2\boldsymbol{b}} = 2B(0). \tag{15}$$

In the constituent quark picture, there are three physical contributions to this range: the range associated with the quark wave functions of each proton, the range of interaction between the quarks, and the effect of multiple interactions. If we ignore multiple interactions for the moment, i.e., keep only first order terms in $T_{\rm qq}$ in eq. (1), our model gives $\langle b^2 \rangle = 2\langle b_1^2 \rangle + \langle b^2 \rangle_{\rm qq}$. In this formula, the size of the protons enters via $\langle b_1^2 \rangle = \int d\mathcal{P}(\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3)\, \boldsymbol{b}_1^2 = 11.6\,{\rm GeV}^{-2}$, which is exactly the quantity we have constrained to fit the EM charge radius. A factor of 2 appears because both beam and target particles contribute. The range of qq scattering enters via $\langle b^2 \rangle_{\rm qq} = \int d^2\boldsymbol{b}\, \boldsymbol{b}^2 T_{\rm qq}(|\boldsymbol{b}|) / \int d^2\boldsymbol{b}\, T_{\rm qq}(|\boldsymbol{b}|)$. Including the effect of multiple interactions reduces the amplitude mainly at small impact parameter, and hence further increases the average range. To compare the magnitudes of the three contributions to the range, let us *define* $\langle b^2 \rangle_{\rm MI}$ as the contribution from multiple interactions, using

$$\langle b^2 \rangle = 2\langle b_1^2 \rangle + \langle b^2 \rangle_{\rm qq} + \langle b^2 \rangle_{\rm MI} . \tag{16}$$

Numerical values are shown in table 2, using the gaussian $T_{\rm qq}(b)$ of section 3. It is striking that a large part of the range of interaction – even at SSC – results simply from the size of the protons. The contribution from $\langle b^2 \rangle_{\rm qq}$ is small. It is even smaller in the black disk $T_{\rm qq}(b)$ model of section 2. *An "Asymptopia" in which the expanding qq interaction range is large compared to the effect of the proton size thus remains very far away.*

As table 2 shows, all previous measurements of elastic scattering and total cross sections have been made in a regime where the range of the quark–quark interaction is short compared to the range associated with the wave functions. As the interaction range expands at higher energies, it is therefore entirely possible that novel effects will appear at energies of the LHC, SSC, and beyond.

Acknowledgement

I thank E. Levin and P. Landshoff for valuable discussions. I thank DESY for supporting my sabbatical visit.

Appendix

Monte Carlo integration of eq. (1) requires generating quark configurations according to the probability distribution

$$\mathrm{d}\mathcal{P} = \prod_{j=1}^{N} [\,\mathrm{d}^2\boldsymbol{b}_j f(\boldsymbol{b}_j)]\delta^{(2)}\left(\sum_{k=1}^{N}\boldsymbol{b}_k\right), \tag{17}$$

where $N = 3$ and

$$f(\boldsymbol{b}) \propto \int \mathrm{d}^2\boldsymbol{p}\, \exp(\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{b})\,(\boldsymbol{p}^2 + \mu^2)^{-\nu-1}. \tag{18}$$

The integral representation

$$(\boldsymbol{p}^2 + \mu^2)^{-\nu-1} = \frac{1}{\Gamma(\nu+1)} \times \int_0^\infty \mathrm{d}x\, x^\nu \exp[-x(\boldsymbol{p}^2 + \mu^2)] \tag{19}$$

gives

$$\mathrm{d}\mathcal{P} \propto \prod_{j=1}^{N}\left[\mathrm{d}x_j\, x_j^{\nu-1}\exp(-\mu^2 x_j)\,\mathrm{d}^2\boldsymbol{b}_j \times \exp\left(\frac{-\boldsymbol{b}_j^2}{4x_j}\right)\right]\delta^{(2)}\left(\sum_{k=1}^{N}\boldsymbol{b}_k\right). \tag{20}$$

Performing the $\boldsymbol{b}_j$ integrals gives

$$\mathrm{d}\mathcal{P} \propto \int \mathrm{d}^2\boldsymbol{q} \prod_{j=1}^{N}\{\mathrm{d}x_j\, x_j^\nu \exp[-x_j(\boldsymbol{q}^2 + \mu^2)]\}. \tag{21}$$

Performing the x_j integrals gives

$$\mathrm{d}\mathcal{P} \propto \int \mathrm{d}^2\boldsymbol{q}\,(\boldsymbol{q}^2 + \mu^2)^{-(\nu+1)N}. \tag{22}$$

The desired configurations for Monte Carlo integration are obtained by generating a random $\boldsymbol{q}$ according to eq. (22), then random $x_1, \cdots, x_N$ according to eq. (21), then random $\boldsymbol{b}_1, \cdots, \boldsymbol{b}_N$ according to eq. (20).

References

[1] T.T. Chou and C.N. Yang, Phys. Lett. B 244 (1990) 113.
[2] C. Bourrely, J. Soffer and T.T. Wu, Z. Phys. C 37 (1988) 369; Phys. Lett. B 252 (1990) 287.
[3] L. Durand and H. Pi, Phys. Rev. D 40 (1989) 1436; B. Margolis, P. Valin, M. Block, F. Halzen and R.S. Fletcher, Phys. Lett. B 213 (1988) 221; Phys. Rev. D 41 (1990) 978.
[4] A. Manohar and H. Georgi, Nucl. Phys. B 234 (1984) 189.
[5] S. Weinberg, Phys. Rev. Lett. 65 (1990) 1181.
[6] J.D. Bjorken, A full-acceptance detector for SSC physics, preprints SLAC-PUB-5545 (May 1991) and SLAC-PUB-5692 (November 1991).
[7] V. Franco and R. Glauber, Phys. Rev. 142 (1965) 1195.
[8] V. Braun, Sov. J. Nucl. Phys. 41 (1985) 259.
[9] P. Bosted et al., Phys. Rev. C 42 (1990) 38.
[10] G. Höhler et al., Nucl. Phys. B 114 (1976) 505.
[11] N. Amos et al., Nucl. Phys. B 262 (1985) 689.
[12] UA4 Collab., M. Bozzo et al., Phys. Lett. B 147 (1984) 392.
[13] UA4 Collab., M. Bozzo et al., Phys. Lett. B 147 (1984) 385.
[14] J. Pumplin, Phys. Lett. B 276 (1992) 517.
[15] A. Belyakov, A. Grashin and M. Lepeshkin, Phys. Lett. B 233 (1989) 493.
[16] UA4 Collab., M. Bozzo et al., Phys. Lett. B 155 (1985) 197.
[17] E710 Collab., N. Amos et al., Phys. Lett. B 243 (1990) 158; B 247 (1990) 127; Phys. Rev. Lett. 68 (1992) 2433.
[18] CDF Collab., S. White et al., preprint FERMILAB-CONF-91-268-E (October 1991).
[19] E. Gotsman, E. Levin and U. Maor, DESY preprint 92-040 (March 1992).
[20] A. Belyakov, A. Grashin and M. Lepeshkin, Phys. Lett. B 249 (1990) 539.
[21] S. Barshay, P. Heiliger and D. Rein, Z. Phys. C 52 (1988) 415; Aachen preprint PITHA 92/8 (March 1992).
[22] A. Donnachie and P. Landshoff, Nucl. Phys. B 267 (1986) 690.
[23] J. Pumplin, Phys. Scr. 25 (1982) 191, and references therein.