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On the spectral densities of hot gluons

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The first moments of the spectral densities of the resummed thermal gluon propagator are shown to have the same values as in the full interacting theory. The sum rules of the latter are derived from the equal-time commutator within the class of covariant gauges.

1. Introduction

The idea, which this note is based on, was stated in 1989 in a paper of Pisarski [1]. There he studied the spectral functions of the resummed thermal gluon propagator, noticed the exact sum rule for the transversal density, and asked for an equivalent in the longitudinal case. In short, under certain circumstances and restricting to very special quantities, one can estimate the value of perturbative results through contrasting them with non-perturbative ones.

Meanwhile, the high temperature limit of the gluon plasma has been well understood, since Braaten and Pisarski [2] exhibited the systematics to be followed in order to obtain consistent results [3]. Also, the direct use of sum rules has become an obvious need in calculations, which include inner lines at soft momenta [4].

In this note, we shall first collect the more or less known facts on the "perturbative" spectral densities and its sum rules. For this we focus on covariant gauges rather than Coulomb (section 2). Then, we start anew on the non-perturbative side (sections 3 and 4), end up with the first moments and compare.

2. The spectral densities at one-loop order

The system under consideration is a volume V containing blackbody radiation of the "other eight sorts of light". Its lagrangian is

$$\mathscr{L}=-\tfrac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu a}-\frac{1}{2\alpha}(\partial^{\mu}A^{a}_{\mu})^{2}+\text{ghost term}\,. \tag{1}$$

Within covariant gauges the resummed gluon propagator reads

$$G_{\mu\nu}(Q)=\frac{A_{\mu\nu}(Q)}{Q^{2}-\Pi_{\mathrm{t}}(Q)}+\frac{B_{\mu\nu}(Q)}{Q^{2}-\Pi_{\ell}(Q)}+D_{\mu\nu}(Q)\,\frac{\alpha}{Q^{2}}\,, \tag{2}$$

where $Q=(\mathrm{i}\omega_n,\boldsymbol{Q})$ and $\omega_n=2\pi nT$. The indices t, ℓ stand for "transversal" and "longitudinal", respectively. We use the Matsubara contour and Minkowski metric $+---$. Thus, $Q^2=(\mathrm{i}\omega_n)^2-q^2$ with $q\equiv|\boldsymbol{Q}|$. (2) is obtained when solving $G=G^0+G^0\Pi G$ by means of the matrix basis A–D [5]. We shall need these matrices here with all their details:

$$A=g-B-D\,,\quad B=\frac{V\circ V}{V^2}\,,\quad C=\frac{Q\circ V+V\circ Q}{\sqrt{2Q^2q}}\,,\quad D=\frac{Q\circ Q}{Q^2}\,, \tag{3}$$

where $V=Q^2U-(U\cdot Q)Q$, and $U=(1,\mathbf{0})$ is the four-velocity of the thermal bath at rest. Hence, $U^2=1$ and $V^2=-Q^2q^2$. The properties of (3) are collected as

$$\begin{aligned}
&A^2=A\,,\quad B^2=B\,,\quad D^2=D\,,\quad C^2=-\tfrac{1}{2}(B+D)\,,\\
&AB=AC=AD=BD=0\,,\quad BC=CD=V\circ Q/\sqrt{2Q^2q}\,,\\
&\tfrac{1}{2}\,\mathrm{Tr}\,A^2=\mathrm{Tr}\,B^2=-\mathrm{Tr}\,C^2=\mathrm{Tr}\,D^2=1\,,\\
&\mathrm{Tr}\,XY=0\,,\quad \textit{if}\quad X\neq Y\quad\text{and}\quad X,Y=A\text{–}D\,.
\end{aligned} \tag{4}$$

The functions $\Pi_{\rm t}=\frac{1}{2}\mathrm{Tr}\,A\Pi$ and $\Pi_\ell=\mathrm{Tr}\,B\Pi$ in (2) are obtained from the leading high temperature term [6,7]

$$\Pi^{\mu\nu}=3m^2(U^\mu U^\nu-\langle(U\cdot Q)Y^\mu Y^\nu/(Y\cdot Q)\rangle)\,,\quad Y=(1,\boldsymbol{e})\,, \tag{5}$$

of the polarization function at one-loop order. In (5), $\langle\;\;\rangle$ is an average over the directions of $\boldsymbol{e}$, and $m^2=\frac{1}{9}g^2NT^2$ is the well known gluon plasma frequency at zero three momentum (with N the number of colours). To complete the listing:

$$\Pi_{\rm t}=\tfrac{3}{2}m^2 g\left(\frac{\Omega}{q}\right),\quad \Pi_\ell=3m^2\left[1-g\left(\frac{\Omega}{q}\right)\right],\quad g(z)=z^2-\tfrac{1}{2}z(z^2-1)\ln\left(\frac{z+1}{z-1}\right). \tag{6}$$

At large z: $g(z)=\frac{2}{3}+\frac{2}{15}z^{-2}+\frac{2}{35}z^{-4}+\dots$.

The spectral densities $\rho_j(x,q)$ of each of the partial propagators in (2), $\Delta_j(Q)$, $j=$ t, ℓ, c, d, are defined by

$$\Delta_j(Q)=\frac{\langle j\rangle}{Q^2-\Pi_j(Q)}=\frac{\langle j\rangle}{\Omega^2-q^2-\Pi_j(\Omega,q)}=\int \mathrm{d}x\,\frac{\rho_j(x,q)}{\Omega-x}\,, \tag{7}$$

where the notation has changed to $Q=(\Omega,q)$ to indicate analytical continuation from the set of discrete values $Q_0=\mathrm{i}\omega_n$ to the complex Ω-plane. The numerators $\langle j\rangle$ are $\langle{\rm t}\rangle=\langle\ell\rangle=1$, $\langle{\rm c}\rangle=0$ and $\langle{\rm d}\rangle=\alpha$. Furthermore $\Pi_{\rm d}=0$. Hence, $\rho_{\rm d}(x,q)$ is just α times the spectral density $(1/2q)[\delta(x-q)-\delta(x+q)]$ of the bare propagator.

Probably the easiest way of deriving sum rules is by expanding both sides of (7) into power series with respect to $1/\Omega$, the $(n+1)$th power giving the nth moment:

$$n=1\qquad \int \mathrm{d}x\,x\,\rho_j(x,q)=1\quad (j={\rm t},\ell)\,, \tag{8}$$

$$n=3\qquad \int \mathrm{d}x\,x^3\rho_j(x,q)=m^2+q^2\quad (j={\rm t},\ell)\,, \tag{9}$$

$$n=-1\quad \int \mathrm{d}x\,\frac{1}{x}\rho_{\rm t}(x,q)=\frac{1}{q^2}\,,\quad \int \mathrm{d}x\,\frac{1}{x}\rho_\ell(x,q)=\frac{1}{3m^2+q^2}\,. \tag{10}$$

(8)–(10) are in essence the sum rules given by Braaten and Yuan [4], translated into our notation and applied to our propagators (note that Δ_ℓ is defined differently). To derive (8), the leading term $\Delta(\Omega,q)\sim 1/\Omega^2$ at $|\Omega|^2\to\infty$ is sufficient for the LHS of (7). On the RHS of (7) the densities $\rho_{\rm t}$ and ρ_ℓ are required to be odd functions of x, as will be seen in (11), (12) below. To derive (9), one more term of Δ at large Ω is required. Finally, (10) derives from the small Ω asymptotics of both sides of (7) together with $\Delta_{\rm t}\to -q^{-2}$ and $\Delta_\ell\to -1/(3m^2+q^2)$ in this limit. In this way, moments of arbitrary order ($n=-1,1,3,5,\dots$) can be generated, since the densities are non-zero only in a finite x-interval. However, only the first moment ($n=1$) will be accessible also non-perturbatively.

The explicit forms of the densities ρ, at one-loop order, have been worked out by Pisarski [1] for the propagator in Coulomb gauge. Here we like to point to the little differences made by the metric and by our "covariant" propagator definitions. The weights ρ can be obtained directly [8] from the definitions (7) by taking the difference $[\Delta_j(x-\mathrm{i}\epsilon, q)-\Delta_j(x+\mathrm{i}\epsilon, q)]/2\pi\mathrm{i}$. The results read

$$\rho=\rho^{\text{pole}}+\rho^{\text{cut}}, \quad \rho^{\text{pole}}=r\delta(x-\omega)-r\delta(x+\omega), \quad \rho^{\text{cut}}=\theta(q^2-x^2)\frac{1}{m^2}\frac{N}{D^2+C^2}, \tag{11}$$

where the quantities ρ, r, ω, N, D and C are understood to carry an index $j=\mathrm{t}$, ℓ and are given by

$$r_\mathrm{t}=\frac{\omega_\mathrm{t}(\omega_\mathrm{t}^2-q^2)}{3m^2\omega_\mathrm{t}^2-(\omega_\mathrm{t}^2-q^2)^2}, \quad r_\ell=\frac{\omega_\ell}{3m^2-\omega_\ell^2+q^2}, \quad N_\mathrm{t}=12\xi\eta \quad \left(\xi\equiv\frac{x}{q}, \quad \eta\equiv1-\xi^2\right), \quad N_\ell=-6\frac{\xi}{\eta},$$

$$C_\mathrm{t}=3\pi\xi\eta, \quad C_\ell=3\pi\xi, \quad D_\mathrm{t}=4q^2\eta+6\xi^2+3\xi\eta\ln\left(\frac{1+\xi}{1-\xi}\right), \quad D_\ell=2q^2+6-3\xi\ln\left(\frac{1+\xi}{1-\xi}\right). \tag{12}$$

The frequency spectra $\omega_j=\omega_j(q)$ cannot be made explicit. They are the positive solutions ($\neq q$) of $\omega^2=q^2+\Pi_j(\omega, q)$, see (6).

Note that both spectral densities ρ are odd functions of x (in view of section 4 this is not entirely trivial). On the positive x axis both pole contributions are positive, and so is the transversal cut part. But the longitudinal density has a negative cut-contribution. Nevertheless, the sum rule (8) holds true (as one can also obtain with (11), (12) by contour integration). In passing, there is *no* "second pole" of the longitudinal propagator at $Q^2=0$ (i.e. at $\xi=1$, $\eta=0$). The denominator has a zero three, but the residue vanishes due to the logarithmic singularity in D_ℓ. The area under ρ_ℓ^{cut} is finite.

3. Non-perturbative approach

For a general Bose system the spectral representation (or "Lehmann representation") of the thermal Green's function is well introduced in the textbook of Fetter and Walecka [8], § 31 and § 32. Here we consider the peculiarities when applied to a non-abelian gauge field $A_\mu^a(x)$ with lagrangian (1). We must distinguish between the former "perturbative" quantities and the following "non-perturbative" ones, even if they have the same physical meaning. Accordingly, the full gluon propagator is now denoted by D (instead of G). Its partial propagators read D_j (instead of Δ_j) and have spectral densities σ_j (instead of ρ_j). We supply our black box V with periodic boundary conditions. Let $x=(-\mathrm{i}\tau, \boldsymbol{r})$, $Q=(\mathrm{i}\omega_n, \boldsymbol{Q})$, and, to exploit translational invariance, let $y=(0, \boldsymbol{r}')$ be a toy variable, the propagator D does not depend on. Then, the spectral density $\sigma_{\mu\nu}$ of the full interacting theory is obtained along the following three lines:

$$\begin{aligned}
D_{\mu\nu}^{ab}(Q) &= \int_V \mathrm{d}^3r \int_0^\beta \mathrm{d}\tau \exp(\mathrm{i}Qx)\langle\mathscr{T}A_\mu^a(x+y)A_\nu^b(y)\rangle \\
&= \int_0^\beta \mathrm{d}\tau\exp(\mathrm{i}\omega_n\tau)\int_V \mathrm{d}^3r\exp(-\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{r}) \\
&\times\left\langle \exp(H\tau)\frac{1}{V}\sum_{\boldsymbol{P}}\exp(\mathrm{i}\boldsymbol{P}\cdot\boldsymbol{r}+\mathrm{i}\boldsymbol{P}\cdot\boldsymbol{r}')\tilde{A}_\mu^a(\boldsymbol{P})\exp(-H\tau)\frac{1}{V}\sum_{\boldsymbol{K}}\exp(\mathrm{i}\boldsymbol{K}\cdot\boldsymbol{r}')\tilde{A}_\nu^b(\boldsymbol{K})\right\rangle \\
&= \int \mathrm{d}x\frac{1}{\mathrm{i}\omega_n-x}\sigma_{\mu\nu}^{ab}(x, \boldsymbol{Q}),
\end{aligned} \tag{13}$$

with

$$\sigma^{ab}_{\mu\nu}(x, Q) = \sum_{fg} \exp[\beta(F-E_f)][1-\exp(\beta\omega_{fg})]\delta(x-\omega_{fg})\frac{1}{V}\langle f|\tilde{A}^b_\nu(-Q)|g\rangle\langle g|\tilde{A}^a_\mu(Q)|f\rangle . \tag{14}$$

$\langle \quad \rangle$ is the thermal average $\mathrm{Tr}\{\exp[\beta(F-H)]...\}$ with F the free energy and H the hamiltonian of the system (1). H governs the Heisenberg time dependence of the field operators A. Its many-particle eigenstates, $H|f\rangle = E_f|f\rangle$, are used to perform the trace and to put a unit operator $|g\rangle\langle g|$ into (14). $\omega_{fg} \equiv E_f - E_g$. In the step to the last line of (13) the RHS was integrated over the toy variable $\boldsymbol{r}'$ and multiplied by $1/V$.

In passing, there is, of course, also a response function χ having the spectral representation (13) with $\omega + \mathrm{i}\epsilon$ in place of $\mathrm{i}\omega_n$ [8]. In the case at hand, it relates the answer $\langle A^a_\mu(x)\rangle$ of the system to an external source $J^b_\nu(x)$ to be included in (1) as $+J^{\nu b}A^b_\nu$. $\chi(\omega) = D(\mathrm{i}\omega_n \to \omega + \mathrm{i}\epsilon)$ is the reason behind the common analytical continuation.

There is more symmetry than the translational one already exploited. Colours do not mix. At $a \neq b$ there is no state $|g\rangle$ giving a non-zero double matrix element in (14). This also follows from perturbation expansion. We refer to the latter in stating symmetry among the Lorentz indices also. Thus,

$$\sigma^{ab}_{\mu\nu}(x, Q) = \sigma_{\mu\nu}(x, Q)\delta^{ab} \quad \text{and} \quad \sigma_{\mu\nu}(x, Q) = \sigma_{\nu\mu}(x, Q) . \tag{15}$$

As a nice little exercise one derives from (14) that

$$-\sigma_{\mu\nu}(-x, -Q) = \sigma_{\nu\mu}(x, Q) = \sigma_{\mu\nu}(x, Q) . \tag{16}$$

There is also rotational symmetry. To make it transparent, we expand $\sigma_{\mu\nu}$ with respect to the non-orthogonal matrices g, $U \circ U$, $T \circ T$ and $U \circ T + T \circ U \equiv S$, where $T = (0, Q)$. In essence, these matrices are A–D, see (3), taken at $Q_0 = 0$ (only A is a linear combination). Now, rotational symmetry tells us that the coefficient functions φ, χ, ψ, ϑ can only depend on $|Q| = q$:

$$\sigma_{\mu\nu}(x, Q) = g_{\mu\nu}\varphi(x, q) + U_\mu U_\nu \chi(x, q) + T_\mu T_\nu \psi(x, q) + S_{\mu\nu}\vartheta(x, q) . \tag{17}$$

Note that $S_{\mu\nu}$ changes sign under $Q \to -Q$. (17) and (16) can now be combined to learn all about the symmetry properties under $x \to -x$:

$$\begin{aligned} &\varphi(x, q) = -\varphi(-x, q) , \quad \chi(x, q) = -\chi(-x, q) , \\ &\psi(x, q) = -\psi(-x, q) , \quad \text{but } \vartheta(x, q) = \vartheta(-x, q) . \end{aligned} \tag{18}$$

4. Sum rules

The first moment of $\sigma_{\mu\nu}$ is related to the thermal average ov an equal time commutator. To realize this, multiply (14) with x and integrate over. In front of the delta function x is converted to $\omega_{fg} = E_f - E_g$, and (at suitable places) the energies E_f, E_g become H. With the definition $\dot{\tilde{A}}_\mu(Q) \equiv \mathrm{i}[H, \tilde{A}_\mu(Q)]$ one obtains

$$\int \mathrm{d}x\, x\, \sigma_{\mu\nu}(x, Q) = \frac{\mathrm{i}}{V}\langle [\tilde{A}_\nu(-Q), \dot{\tilde{A}}_\mu(Q)]\rangle . \tag{19}$$

It is now irresistible to confront the commutator in (19) with the quantization rules. But note that the latter must be those of the full interacting theory. They are written as

$$[A^a_\nu(0, \boldsymbol{r}+\boldsymbol{r}'), \Pi^b_\mu(0, \boldsymbol{r}')] = \mathrm{i}g_{\mu\nu}\delta^{ab}\delta(\boldsymbol{r}) , \tag{20}$$

where $\Pi^b_\mu = \partial_{\dot{A}^{\mu b}}\mathscr{L}$. With (1) one obtains $\Pi^b_0 = -(1/\alpha)\dot{A}^b_0 - (1/\alpha)\partial^k A^b_k$ and $\Pi^b_k = F^b_{k0} = \partial_k A^b_0 - \dot{A}^b_k - gf^{bcd}A^c_0 A^d_k$. There are also the vanishing equal time commutators between two A fields. Thus, inserting Π^b_μ into (20), we retain only the $\dot{A}$ terms of Π^b_0, Π^b_k. Nevertheless, the A^a_μ are now Heisenberg operators, and $\dot{A}^a_\mu = \mathrm{i}[H, A^a_\nu]$. Operating on (20) with $\int_V \mathrm{d}^3r \exp(\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{r})$ as well as with $-(\mathrm{i}/V)\int_V \mathrm{d}^3r'$ and taking the thermal average, we obtain

$$\frac{\mathrm{i}}{V}\langle [\tilde{A}^a_\nu(-\boldsymbol{Q}), W_\mu{}^\rho \dot{\tilde{A}}^b_\rho(\boldsymbol{Q})] \rangle = \delta^{ab} g_{\mu\nu}\,, \quad W_\mu{}^\rho \equiv g_\mu{}^\rho + \left(\frac{1}{\alpha} - 1\right) U_\mu U^\rho\,.$$

With view to (19) we set $a=b$, apply $(W^{-1})_\lambda{}^\mu = g_\lambda{}^\mu + (\alpha-1)U_\lambda U^\mu$, and end up with

$$\int \mathrm{d}x\, x\, \sigma_{\mu\nu}(x, \boldsymbol{Q}) = g_{\mu\nu} + (\alpha - 1) U_\mu U_\nu\,. \tag{21}$$

Inserting (17) into (21) we obtain the first moments of φ, χ and ψ. The ϑ term of (17) does not contribute, because it is an even function of x. Instead, one learns about ϑ by just integrating over (14) without the weight x. This leads to an equal time commutator of two undotted fields, i.e. to a vanishing result. To summarise,

$$\int \mathrm{d}x\, x\, \varphi(x, q) = 1\,, \quad \int \mathrm{d}x\, x\, \chi(x, q) = \alpha - 1\,, \quad \int \mathrm{d}x\, x\, \psi(x, q) = 0\,, \quad \int \mathrm{d}x\, \vartheta(x, q) = 0\,. \tag{22}$$

The information on $\sigma_{\mu\nu}(x, \boldsymbol{Q})$, obtained so far, must now be converted into statements on the spectral densities $\sigma_j(x, q)$ of the partial propagators $D_j(Q)$. To formulate this problem, we recall $Q = (\Omega, \boldsymbol{Q})$ and use (13) at $\mathrm{i}\omega_n \to \Omega$:

$$\int \mathrm{d}x \frac{1}{\Omega - x} \sigma_{\mu\nu}(x, \boldsymbol{Q}) = A_{\mu\nu}(Q) D_t(Q) + \ldots\,, \quad D_j(Q) = \int \mathrm{d}x \frac{\sigma_j(x, q)}{\Omega - x}\,. \tag{23}$$

Our (humble) exact information is on the left end of this line, where (17) can be inserted. The densities we like to learn about are defined at the right end. In a first step we invert the left equation (23) by taking traces and using (4). This gives

$$\int \mathrm{d}x \frac{\sigma_j(x, q)}{\Omega - x} = \int \mathrm{d}x \frac{1}{\Omega - x} M_j(\Omega, x, q)\,, \tag{24}$$

where the functions M_j are linear combinations of $\varphi(x, q)$ to $\vartheta(x, q)$ with coefficients depending on Ω, q. These coefficients are 1, 0, 0, 0 (j=t); 1, $-q^2/Q^2$, $-q^2\Omega^2/Q^2$, $2q^2\Omega/Q^2$ (j=ℓ); 0, $\sqrt{2}q\Omega/Q^2$, $\sqrt{2}q^3\Omega/Q^2$, $-\sqrt{2}q(\Omega^2+q^2)/Q^2$ (j=c); 1, Ω^2/Q^2, q^4/Q^2, $-2q^2\Omega/Q^2$ (j=d). Note that, for j=t, ℓ, d, the odd functions φ, χ, ψ have a prefactor, which is even in Ω, while the even function ϑ has an Ω-odd prefactor. We conclude from (24) that σ_t, σ_ℓ and σ_d are odd functions of x. Things are reversed for j=c, and, therefore, σ_c is an even function of x. Using these properties we now make the Ω dependence in (24) fully explicit. But let us omit the obvious arguments x, q of σ_j and of φ to ϑ:

$$\int \mathrm{d}x \frac{x}{\Omega^2 - x^2} \sigma_t = \int \mathrm{d}x \frac{x}{\Omega^2 - x^2} \varphi\,, \tag{25}$$

$$\int \mathrm{d}x \frac{x}{\Omega^2 - x^2} \sigma_\ell = \int \mathrm{d}x \frac{1}{\Omega^2 - x^2} \left[x\left(\varphi - \frac{q^2}{\Omega^2 - q^2}\chi - \frac{\Omega^2 q^2}{\Omega^2 - q^2}\psi\right) + \frac{2q^2\Omega^2}{\Omega^2 - q^2}\vartheta \right], \tag{26}$$

$$\int \mathrm{d}x \frac{1}{\Omega^2 - x^2} \sigma_c = \int \mathrm{d}x \frac{1}{\Omega^2 - x^2} \left[x\left(\frac{\sqrt{2}q}{\Omega^2 - q^2}\chi + \frac{\sqrt{2}q^3}{\Omega^2 - q^2}\psi\right) - \frac{\sqrt{2}q(\Omega^2 + q^2)}{\Omega^2 - q^2}\vartheta \right], \tag{27}$$

$$\int \mathrm{d}x \frac{x}{\Omega^2 - x^2} \sigma_d = \int \mathrm{d}x \frac{1}{\Omega^2 - x^2} \left[x\left(\varphi + \frac{\Omega^2}{\Omega^2 - q^2}\chi + \frac{q^4}{\Omega^2 - q^2}\psi\right) - \frac{2q^2\Omega^2}{\Omega^2 - q^2}\vartheta \right]. \tag{28}$$

(25) allows for the conclusion that $\sigma_t(x,q)=\varphi(x,q)$. But we may also consider (25), as well as (26)–(28), at $\Omega\to\infty$, equate the Ω^{-2} terms, and use the sum rules (22). This immediately leads to the following results:

$$\int \mathrm{d}x\, x\, \sigma_t(x,q)=1\,, \quad \int \mathrm{d}x\, x\, \sigma_\ell(x,q)=1\,, \tag{29,30}$$

$$\int \mathrm{d}x\, \sigma_c(x,q)=0\,, \quad \int dx\, x\, \sigma_d(x,q)=\alpha. \tag{31,32}$$

In view of (31), it is tempting to consider also the Ω^{-4} terms in (27):

$$\int \mathrm{d}x\, x^2\sigma_c(x,q)=\sqrt{2}q\left(\alpha-1-\int \mathrm{d}x\, x^2\vartheta(x,q)\right)=0\,. \tag{33}$$

At this point, and in order to explain the zero to the right of (33), we remember that there is a third equal time commutator, namely $[\Pi,\Pi]=0$. Exploiting it by following, in essence, the lines through (19)–(22), one obtains $\int \mathrm{d}x\, x^2\vartheta(x,q)=\alpha-1$, indeed. As soon as we go to higher order moments, however, our lack of knowledge on the non-perturbative side will become appearent. Possibly, the function $\sigma_c(x,q)$ vanishes identically; we do not know.

5. Discussion

The first moments (8) of the "perturbative" spectral densities of the transverse and longitudinal gluon propagators are equal to the exact ones (29), (30). Even the gauge dependent part is confirmed by (32).

Since we know [2], however, that (a) the resummed gluon propagator (2) contains *all* contributions to leading order, when the outer momentum Q is soft, (b) the bare propagator is sufficient at hard Q and (c) with increasing Q (2) turns into the bare propagator automatically, this agreement is no surprise. The resummed propagator, and moreover the corresponding gauge invariant effective action [7,9–11], is the exact first term of a high temperature asymptotics. As the first moments do neither depend on temperature nor on the coupling, non-agreement would have been a disaster. From this point of view, we have merely seen here that two quite different procedures both work well.

On the other hand, as the spectral properties of the full theory are detailed here to some extent, any gluon propagator improving over the one-loop approximation can now be tested in this respect.

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