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Screening of large exotic scales in the light Higgs-boson mass bound for a general class of supersymmetric models

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We consider the class of supersymmetric models whose Higgs sector contains a "conventional" pair of $SU(2)_L$ doublets and an arbitrary number of extra "exotic" $SU(2)_L \times U(1)_Y$ neutral Higgs singlets (possibly related to an extra symmetry group G), with *a priori* arbitrarily large vacuum expectation values (VEV's). We show that, as a consequence of the functional structure of the scalar potential (dictated by gauge and supersymmetry invariance), a general property exists in the scalar mass matrix which decouples the lightest Higgs-boson mass from the large VEV effects at the tree level. At one loop, we show that, if the fermion-sfermion sector is considered, the analogous property "screens" the lightest Higgs-boson mass under very general assumptions for the symmetry-breaking mechanism.

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A well-known property of a class [1] of "minimal" supersymmetric models with two $SU(2)_L$ doublets of Higgs fields is the existence of one light scalar$\equiv H_1$ for whose mass the famous bound exists at the tree level,

$$M_{H_1} \le M_Z \; , \tag{1}$$

which can be also expressed as the statement that the bound on M_{H_1} is of $O(v)$, where $v=(v_1^2+v_2^2)^{1/2}$ and $v_{1,2}$ are the vacuum expectation values (VEV's) responsible for the $SU(2)_L \times U(1)_Y$ breaking.

The problem of the existence of a similar bound has also been investigated for nonminimal supersymmetric cases. In particular, models where one extra Higgs singlet acquires a VEV$\equiv x$ have been recently and extensively considered, both for the canonical situation of a conventional fermionic spectrum [2] and for the case where the extra Higgs singlet is related to a larger gauge group which could be the low-energy relic of an original symmetry suggested by superstring models [3]. In both cases, it has been remarked that the light Higgs-boson mass remains, at the tree level, of $O(v)$ rather than becoming of $O(x)$. This means that in the large-x limit (formally, in the limit $x\to\infty$) the nonminimal VEV effect decouples from the light Higgs-boson mass bound. The latter contains, though, in addition to the "minimal" term, Eq. (1), an extra piece depending on the new couplings that appear in the potential. To fix qualitatively a bound requires now therefore the use of renormalization-group arguments, from which one derives in general a "correction" of the same size $=O(v)$ as that of the minimal bound.

The previous considerations are valid at the lowest order of perturbation theory. Since the considered models are renormalizable ones, the question naturally arises of whether this fundamental feature is retained, or lost, at the next one-loop order. In fact, the presence of radiative corrections of the kind $\delta M^2_{H_1}=O(\alpha x^2)$ would destroy the previous formal decoupling property and affect dramatically the low-energy spectrum of the theory. Therefore, a general investigation of this problem (that, to our knowledge, is still lacking) seems to us rather strongly motivated.

The aim of this paper is to show the following.

(i) For a class of supersymmetric models whose Higgs sector contains, in addition to the two minimal doublets, an arbitrary number of $SU(2)_L \times U(1)_Y$ singlets [not necessarily related to extra $U(1)_i$ although this might be the case] with possible "large" VEV's $x_i \gg v$, $i=1,\ldots,N$, a special property of the mass matrix exists that automatically produces the phenomenon of the tree-level x_i decoupling from the light Higgs-boson mass bound.

(ii) For the previous class of supersymmetric models, assuming that the fermion-sfermion spectrum is made of the "conventional" standard model multiplets with the possible addition of an "exotic" sector [not necessarily $SU(2)_L \times U(1)_Y$ neutral] whose masses are only generated by the extra x_i, an analogous property of the mass matrix exists that automatically eliminates, in the formal $x_i\to\infty$ limit, the contributions to the bound on $M^2_{H_1}$ from this sector that are of $O(\alpha x_i^2)$, $O(\alpha x_i)$ and only allows the presence of logarithmic terms $\simeq \alpha \ln x_i^2$.

To prove our statement, we consider a supersymmetric model for which the part of the superpotential that only contains Higgs fields has the form

$$W(h_1,h_2,\phi_i)=\lambda_i\phi_i h_1 h_2+\overline{W}(\phi_i) \; , \tag{2}$$

where $h_{1,2}$ are the "minimal" $SU(2)_L$ doublets and

$$\overline{W}(\phi_i)=k_{ijz}\phi_i\phi_j\phi_z+\mu_{ij}\phi_i\phi_j \tag{3}$$

is the most general function of the extra Higgs fields. The choice of the various couplings will be dictated by the specific considered model. In particular, it might be possible that the new Higgs fields are associated with an extra gauge symmetry group G (typically, whenever in the low-energy phenomenology a new Z appears). In this case, we shall require that the full superpotential is a scalar with respect to the extended gauge group $SU(2)_L \times U(1)_Y \times G$, and this will automatically select the number of possibly nonvanishing λ_i, k_{ijz}, μ_{ij}.

Starting from the general expression Eq. (2) we are led to the scalar potential

$$V(h_1,h_2,\phi_i)=V_F+V_D+V_{SB} \tag{4}$$

where, in the minimal-energy configuration,

$$V_F=\sum_i \left|\frac{dW}{dx_i}\right|^2 =\sum_i \left|\lambda_i v_1 v_2+\frac{d\overline{W}}{dx_i}\right|^2+|\lambda_j x_j|^2[v_1^2+v_2^2] , \tag{5}$$

$$V_D=D^2=D_{gL}^2+D_{gY}^2+D_{g1}^2 =\frac{g_L^2+g_Y^2}{8}|v_1^2-v_2^2|^2+\frac{g_1^2}{2}|v_1^2T_1^a+v_2^2T_2^a+x_i^2T_i^a|^2 . \tag{6}$$

In Eq. (6) T^a are the generators of the extra group G with coupling g_1; $T_i^a=\langle\phi_i T^a\phi_i\rangle/\langle\phi_i\phi_i\rangle$ (no sum on the index i); $T_{1(2)}^a=\langle H_{1(2)}T^aH_{1(2)}\rangle/\langle H_{1(2)}H_{1(2)}\rangle$; and we have assumed the usual expression of the soft supersymmetry breaking term:

$$V_{SB}=m_1^2v_1^2+m_2^2v_2^2+m_i^2x_i^2-2A\lambda_i x_i v_1 v_2-2A\overline{W}(x) , \tag{7}$$

where $A\overline{W}(x)$ could be, in principle, still split into two different (bilinear and trilinear) terms (since this will not affect our conclusions, we retained this simplified inclusive notation). Starting from the potential Eq. (4) it would be straightforward to derive the nondiagonal mass matrix of the scalar sector; this will be a $(2+N)$-dimensional quantity, whose eigenvalues will provide the masses of the physical scalar Higgs particles. But for the specific purpose of the determination of a general bound for the lightest Higgs boson, the problem is much simpler. In fact, a known property of any Hermitian matrix is that its minimum eigenvalue must be smaller than that of its upper-left-corner 2×2 submatrix [we choose the ordering so that the (1,2) indices correspond to $h_{1,2}$]. Calling m_{ij}^2 the matrix elements of the squared mass matrix $[M^2]$, and denoting by $M_{H_1}^2$ the lightest physical Higgs-boson mass, the following rigorous bound will therefore obtain:

$$M_{H_1}^2\le\frac{m_{11}^2+m_{22}^2}{2}\left[1-\left[1-4\frac{m_{11}^2m_{22}^2-m_{12}^4}{(m_{11}^2+m_{22}^2)^2}\right]^{1/2}\right] . \tag{8}$$

Starting from the expressions Eqs. (4)–(7) of the potential and imposing the minimum conditions $dV/dh_{1,2}=0$, the following expressions for m_{11}^2, m_{22}^2, m_{12}^2 are obtained:

$$m_{11}^2=\left[-\lambda_i\tan\beta\left[\frac{d\overline{W}}{dx_i}-Ax_i\right]\right]+\left[\frac{g_Z^2v_1^2}{2}+2g_1^2v_1^2T_1^aT_1^a\right] , \tag{9}$$

$$m_{22}^2=\left[-\lambda_i\cot\beta\left[\frac{d\overline{W}}{dx_i}-Ax_i\right]\right]+\left[\frac{g_Z^2v_2^2}{2}+2g_1^2v_2^2T_2^aT_2^a\right] , \tag{10}$$

$$m_{12}^2=\left[\lambda_i\left[\frac{d\overline{W}}{dx_i}-Ax_i\right]\right]+\left[\sum_i 2\lambda_i^2v_1v_2-\frac{g_Z^2v_1v_2}{2}+2g_1^2v_1v_2T_1^aT_2^a\right] , \tag{11}$$

where $\tan\beta=v_2/v_1$ and $g_Z^2=g_Y^2+g_L^2$. As one can see, all the three matrix elements are separately *a priori* of $O(x^2)$. To make this fact evident, we have rewritten them in the form

$$m_{ij}^2=\hat{m}_{ij}^2+\overline{m}_{ij}^2 \tag{12}$$

where the first terms on the right-hand side (RHS) contain all the (quadratic and linear) x dependence, and correspond to the first set of square brackets in Eqs. (9)–(11), while $\overline{m}_{ij}$ contains terms of $O(v)$ multiplied by dimensionless constants that can be treated by renormalization-group (RG) arguments. Thus, *a priori,* one might expect that the bound of Eq. (8) is of $O(x^2)$ as well, and as such not particularly useful. The surprising fact that invalidates this argument is that, while the trace of the 2×2 matrix $m_{11}^2+m_{22}^2$ is actually of $O(x^2)$, its determinant $m_{11}^2m_{22}^2-m_{12}^4$ is in fact not of $O(x^4)$ but only of $O(x^2)$. This "miraculous" cancellation of the two highest orders is a pure consequence of the functional structure of the scalar potential, whose form is dictated by the constraints of supersymmetry and gauge invariance. In fact, the general expression of the determinant reads

$$m_{11}^2m_{22}^2-m_{12}^4=\hat{m}_{11}^2\hat{m}_{22}^2-\hat{m}_{12}^4+\hat{m}_{11}^2\overline{m}_{22}^2+\hat{m}_{22}^2\overline{m}_{11}^2 -\hat{m}_{12}^2\overline{m}_{12}^2+\overline{m}_{11}^2\overline{m}_{22}^2-\overline{m}_{12}^4 \tag{13}$$

and one sees that the cancellation of the leading power is automatic in our case, since

$$\hat{m}_{11}^2\hat{m}_{22}^2-\hat{m}_{12}^4=0 \tag{14}$$

is verified irrespectively of the explicit form of $\overline{W}(x_i)$. Therefore,

$$m_{11}^2m_{22}^2-m_{12}^4=\hat{m}_{11}^2\overline{m}_{22}^2+\overline{m}_{11}^2\hat{m}_{22}^2-2\hat{m}_{12}^2\overline{m}_{12}^2 +\overline{m}_{11}^2\overline{m}_{22}^2-\overline{m}_{12}^4$$

is of $O(x^2)$, and its ratio with $(m_{11}^2+m_{22}^2)^2$ is of $O(1/x^2)$. In the (formal) limit $x_i\to\infty$ this allows us to develop the square root in Eq. (8) and to obtain the formal result

$$M_{H_1}^2(x_i\to\infty)\le\frac{1}{v^2}[\overline{m}_{11}^2v_1^2+\overline{m}_{22}^2v_2^2+2\overline{m}_{12}^2v_1v_2] , \tag{15}$$

which contains terms of $O(v^2)$ multiplied by dimensionless couplings. We can reformulate Eq. (15) as the statement that for the class of supersymmetric models derivable from the superpotential of Eq. (2) there exists a tree-level bound on the lightest Higgs-boson mass that remains of $O(v)$ in the formal limit $x_i \to \infty$. Thus, the extra introduced VEV's decouple from the latter bound in this limit.[1]

Equation (15) represents the generalization to a more general class of models of a property that was previously stated in some specific cases [2,3].[2] The new property that we shall show is that, even when one considers one-loop corrections, an almost analogous phenomenon is met. As a consequence of this, the two leading x powers in the bound disappear in the formal limit $x_i \to \infty$, leaving only a logarithmic term that will be, in general, nonzero. This is somehow reminescent of Veltman's "screening" theorem [5] of electroweak radiative corrections at one loop in the minimal standard model, which only retain a logarithmic dependence on the Higgs-boson mass in the (formal) limit $M_H \to \infty$, since the $\sim M_H^2$ term is exactly canceled. Contrary to what happens at the tree level, this screening property of the bound is only possible if a supersymmetric scenario is assumed, and as such it appears as a genuine feature of models of this type.

To demonstrate the previous statement, a specification of the full particle-sparticle spectrum, in particular of the fermion-sfermion sector, is now needed. We shall consider in this paper a fermionic sector consisting of the conventional standard model families, whose masses are generated by $v_{1,2}$, and of a possible "exotic" sector of particles whose masses are only generated by x_i (in this way, as a subcase of special interest, models originated by a previous E_6 symmetry will be incorporated in our treatment). This corresponds to adding to the Higgs superpotential Eq. (2) some extra terms, leading to the complete expression

$$W = W_{SM}(S,h_{1,2}) + W(h_1,h_2,\phi_i) + \tilde{W}(E,\phi) + \ddot{W}(S,E) \tag{16}$$

where W_{SM} is the superpotential of the conventional matter superfield (S) of the minimal supersymmetric (SUSY) standard model; $W(h_1,h_2,\phi_i)$ is the superpotential of Eq. (2) involving only Higgs superfields; $\tilde{W}(E,\phi)$ and $\ddot{W}(S,E)$ are the terms which give mass to the exotic fields and describe the possible interactions with the standard fields. From Eq. (16) one easily derives by standard techniques the complete form of the full scalar potential. Rather than giving that, we write the expressions of those field-dependent masses that will be relevant for our purposes. More specifically, we shall be first concerned with the fermion-sfermion sector, where the (potentially dangerous) Yukawa couplings are contained. For a general sfermion system of two physical states, we shall write, in full generality,

$$M_{1(2)}^2 = M^2 \pm \Delta^2 , \tag{17}$$

$$M^2 = \tfrac{1}{2}(M_{LL}^2 + M_{RR}^2), \quad \Delta^2 = \tfrac{1}{2}\sqrt{(M_{LL}^2 - M_{RR}^2)^2 + 4M_{LR}^4} , \tag{18}$$

$$M_{LL}^2 = m_{\mathrm{soft},L}^2 + M_\psi^2 - \left[\frac{g_L^2}{2} T_{3L} - \frac{g_Y^2}{2} Y_L \right] (v_2^2 - v_1^2) + \frac{g_1^2}{2} T_L (T_1^a v_1^2 + T_2^a v_2^2 + T_i^a x_i^2) , \tag{19}$$

$$M_{RR}^2 = M_{LL}^2 \quad \text{with} \quad (L \Longleftrightarrow R) . \tag{20}$$

Here $m_{\mathrm{soft},L(R)}^2$ and the squared SUSY-breaking masses generally not equal at the M_Z scale, M_ψ is the mass of the corresponding fermion, and $T_{3L,3R}$, $Y_{L,R}$, $T_{L,R}^a$ are the charges of the left and right sfermion with respect to $SU(2)_L$, $U(1)_Y$, and G; M_{LR}^2 is the mixing term of the sfermion matrix, which has to be computed separately for each specific case.

The strategy for computing radiative corrections at one loop to the Higgs scalar mass matrix is well known [6]. Starting from the effective potential at one loop [7]

$$V^{(1)} = (1/64\pi^2)\mathrm{STr}M^4(\ln M^2/Q^2 - \tfrac{3}{2}) ,$$

and using the given expressions of the various masses, one is led to an expression for the mass matrix at one loop that contains the contribution of all field-dependent masses. In particular, we shall be interested here in the 2×2 submatrix m_{ij}^2, $i,j=1,2$. Writing the corresponding matrix elements as

$$m_{ij}^2 = m_{ij}^{(0)2} + \delta m_{ij}^2 , \tag{21}$$

where $m_{ij}^{(0)2}$ is formally analogous to the tree-level expressions but depends on the renormalized running couplings (evaluated at $Q^2 = M_Z^2$ in our case), and using the definitions of Eqs. (17) and (18), it is easy to derive (taking the minimum conditions into account) the following expressions of the sfermion contributions:

$$\delta m_{11}^2 = \frac{n_f}{32\pi^2} \left\{ Z \left[\frac{d^2\Delta^2}{d^2 v_1^2} - \frac{1}{v_1}\frac{d\Delta^2}{dv_1} \right] + \left[\left[\frac{dM^2}{dv_1} \right]^2 + \left[\frac{d\Delta^2}{dv_1} \right]^2 \right] V + 2\frac{d\Delta^2}{dv_1}\frac{dM^2}{dv_1} R \right\} , \tag{22}$$

$$\delta m_{22}^2 = \delta m_{11} \quad (1 \leftrightarrow 2) , \tag{23}$$

$$\delta m_{12}^2 = \frac{n_f}{32\pi^2} \left[Z \frac{d^2\Delta^2}{dv_1 dv_2} + \left[\frac{d\Delta^2}{dv_1}\frac{d\Delta^2}{dv_2} + \frac{dM^2}{dv_1}\frac{dM^2}{dv_2} \right] V + \left[\frac{d\Delta^2}{dv_2}\frac{dM^2}{dv_1} + \frac{d\Delta^2}{dv_1}\frac{dM^2}{dv_2} \right] R \right] , \tag{24}$$

where

[1] One easily verifies that this property remains true if the model is such that the terms of $O(x_i^2)$ coming from $\tilde{W}$ are absent, such as, e.g., in the models with extra Z's of Ref. [3]. In this case, $m_{11,22,12}$ are separately of order x and the determinant is of order x as well owing to the same mechanism, so that the previous conclusion still applies.

[2] Strictly speaking, a similar result could be obtained even without imposing supersymmetry and starting from a scalar potential whose form is constrained (e.g., by gauge invariance of some kind) to be as in Eq. (4) (an example of this statement can be found in previous works where two nonsupersymmetric doublets were considered [4]).

$$R = \ln(M_1^2/M_2^2) , \quad (25)$$

$$V = \ln(M_1^2/Q^2) + \ln(M_2^2/Q^2) , \quad (26)$$

$$Z = M^2 R + \Delta^2(V-2) , \quad (27)$$

and n_f is the number of degrees of the fermion ($n_f = 3$ for a colored particle, $n_f = 1$ for a lepton). To compute the fermion contribution is trivial, but unnecessary for our purposes of isolating the leading-x behavior of Eqs. (22)–(24), since from our previous discussion it is clear that no fermion state will be able to produce x-dependent terms. Thus, Eqs. (22)–(24) are all we need for the discussion of the fermion-sfermion sector.

The large-x behavior of the light Higgs-boson mass bound at one loop will still be determined by the properties of the 2×2 determinant $m_{11}^2 m_{22}^2 - m_{12}^4$ at the corresponding order, since the bound Eq. (8) is valid at every order of perturbation theory. Using the notations Eqs. (19) and (12) to isolate the terms of $O(x^2)$, $O(x)$ from the remaining part of the matrix elements [which as we shall see will include at one loop smooth logarithmic $O(\ln x^2)$ terms], the constraint that the leading x powers be canceled in the bound at one loop leads to the "screening equation"

$$\cot\beta\, \delta\hat{m}_{11}^2 + \tan\beta\, \delta\hat{m}_{22}^2 + 2\delta\hat{m}_{12}^2 = 0 \quad (28)$$

as one easily sees from Eqs. (9)–(11) and (14).

To verify whether this is actually the case, a separate analysis has to be performed for different fermion-sfermion systems since the properties of the related Δ^2 terms are generally different. We shall consider here the two most characteristic and relevant cases.

(i) *The exotic fermion-sfermion system.* Here the expansion of Δ^2 [Eqs. (17) and (18)] reads

$$\Delta^2 = \tfrac{1}{2}\sqrt{[\Delta m_s^2 + k_i x_i^2 + q_1 v_1^2 + q_2 v_2^2]^2 + 4M_{LR}^4} , \quad (29)$$

where

$$k_i = \tfrac{1}{2} g_1^2 [T_L^a - T_R^a] T_i^a , \quad (30)$$

$$q_j = \tfrac{1}{2} g_Y^2 [Y_L - Y_R] - \tfrac{1}{2} g_L^2 [T_{3L} - T_{3R}] + \tfrac{1}{2} g_1^2 [T_L^a - T_R^a] T_j^a, \quad j = 1,2 , \quad (31)$$

and Δm_s^2 is the difference $m_{\mathrm{soft},L}^2 - m_{\mathrm{soft},R}^2$, which will be, in general, nonzero at the relevant M_Z^2 scale and $v_{1,2}$ independent. The mixing term M_{LR}^2 has the form

$$M_{LR}^2 = \sum_i h_{Ei} \left[\lambda_i v_1 v_2 + \frac{d\overline{W}}{dx_i} - A x_i \right] , \quad (32)$$

where

$$\frac{d^2 \widetilde{W}}{d\widetilde{E}\, d\widetilde{E}^c} = h_{Ei} x_i . \quad (33)$$

Quite generally, Δ^2 is of $O(x^2)$ in the large-x limit and, in principle, the radiative corrections to m_{ij}^2 $(i,j=1,2)$ will be of the same order. The crucial point for our analysis is the fact that all the various Δ^2 derivatives that appear in Eqs. (22)–(24) are now finite in the limit. Thus, the only dangerous terms will come in this case from the quantity Z, Eq. (27). The expressions of the Z coefficients can be easily computed, and from them one can isolate the most divergent components of the matrix elements at one loop, which read

$$\delta\hat{m}_{11}^2 = -(n_f/32\pi^2) Z h_{Ei} \lambda_i \tan\beta\, m_{12}^2/\Delta^2 , \quad (34)$$

$$\delta\hat{m}_{22}^2 = -(n_f/32\pi^2) Z h_{Ei} \lambda_i \cot\beta\, m_{12}^2/\Delta^2 , \quad (35)$$

$$\delta\hat{m}_{12}^2 = (n_f/32\pi^2) Z h_{Ei} \lambda_i m_{12}^2/\Delta^2 . \quad (36)$$

These equations allow us to confirm our statement for this contribution, since the cancellation Eq. (27) turns out to be automatically satisfied. Thus, the only possible corrections to the light Higgs mass bound will come from logarithmic terms, which are actually present at one loop in the $\delta\overline{m}_{ij}^2$ "smooth" components.

(ii) *The top-quark–top-squark system.* In this case (an identical treatment can be given for the bottom-quark–bottom-squark or lepton-slepton case with obvious $t \to b,l$ replacements) the expression of Δ^2 is analogous to that of Eq. (29), with the only difference that M_{LR}^2 now reads

$$M_{LR}^2 = h_t [v_1 \lambda_i x_i - A v_2] . \quad (37)$$

A proper treatment of this term requires a separation of three different subcases, corresponding to three different situations. We discuss them in order of decreasing simplicity.

(a) An extra group G exists, and the quantum numbers of the left-handed and right-handed top system under the group are different, $T_L^a \neq T_R^a$. In this case, the treatment is similar to that of case (i). All the Δ^2 derivatives are finite, and the formal expressions of $\delta\hat{m}_{ij}^2$ become

$$\delta\hat{m}_{11}^2 = (3/32\pi^2) Z h_t^2 (A\lambda_i x_i/\Delta^2) \tan\beta , \quad (38)$$

$$\delta\hat{m}_{22}^2 = (3/32\pi^2) Z h_t^2 (A\lambda_i x_i/\Delta^2) \cot\beta , \quad (39)$$

$$\delta\hat{m}_{12}^2 = -(3/(32\pi^2) Z h_t^2 (A\lambda_i x_i/\Delta^2) , \quad (40)$$

from which one derives immediately the screening condition Eq. (28).

(b) A gauge group G exists, but now $T_L^a = T_R^a$. In this case, the quantity[3] Δ^2 is still of $O(x^2)$ if M^2 is of $O(x^2)$ and the difference $m_{\mathrm{soft},L}^2 - m_{\mathrm{soft},R}^2$ is of $O(x^2)$ as well. In this situation, the treatment is again similar to that of case (i) and one comes to the same conclusion. If, as a particular case, M^2 is of $O(x^2)$ and $m_{\mathrm{soft},L}^2 = m_{\mathrm{soft},R}^2$, the quantity Δ^2 is of $O(x)$ rather than being of $O(x^2)$, and its relevant expression is

$$\Delta^2 = h_t [v_1 \lambda_i x_i - A v_2] . \quad (41)$$

The proof of Eq. (28) is more elaborate, since all the three terms in Eqs. (22)–(24) are separately contributing $\delta\hat{m}_{ij}^2$ due to the fact that $d\Delta^2/dv_1$ is now of $O(x)$. To demonstrate our statement, a full expansion of all the various terms in the large-x limit is needed. In this procedure, an

[3]Actually, this is not only a formal consequence of the presence of the D terms in Eqs. (17) and (18) but is also motivated from the fact that the scale of soft SUSY breaking is actually of the same order as that of the extra group breaking $\sim x$ in a class of relevant models [3].

essential ingredient is the fact that one can write

$$\ln M_1^2/M_2^2 = 2(\Delta^2/M^2) + O(1/x^3) \ . \tag{42}$$

When Eq. (42) is taken into account, the final expressions of the relevant quantities become

$$\delta\hat{m}_{11}^2 = (3/32\pi)Vh_t^2 A\lambda_i x_i \tan\beta \ , \tag{43}$$

$$\delta\hat{m}_{22}^2 = (3/32\pi^2)Vh_t^2 A\lambda_i x_i \cot\beta \ , \tag{44}$$

$$\delta\hat{m}_{12}^2 = -(3/32\pi^2)Vh_t^2 A\lambda_i x_i \ , \tag{45}$$

and the screening condition Eq. (28) is again recovered.

(c) No extra gauge group exists. This would be the case of "minimal-nonminimal" SUSY models such as those treated in Ref. [2]. This case is formally analogous to the previous one (b). Therefore, our procedure will automatically guarantee the screening of the large-x effects under the assumption that, in the large-x limit,[4] M^2 is of $O(x^2)$. If this condition is not met, our formal proof fails, and we cannot guarantee, in this simple and general way, the absence of dangerous radiative corrections to the bound. This would require separate and specific analyses (e.g., if they turned out to be sizable but negative they would not affect the final result in any case).

In conclusion, we can say that, for the fermion-sfermion sector, for a rather general class of models, the large-x effect on the light Higgs-boson mass bound is screened to one loop.[5] Since this happens for each single contribution separately, no kind of fine-tuning is requested between the different couplings. In other words, the requests of SUSY and gauge invariance protect at one loop the light Higgs-boson mass from the potentially dangerous effects of exotic scales in this sector. This is, in fact, the main result of our paper.

The next important step would be to verify whether the previous nice property remains true for the remaining contributions to the one-loop potential, i.e., those coming from the Higgs-Higgsino-gauge-gaugino sector. In the case in which an extra gauge group G is present, an answer to this question requires a specification of the considered group. This would imply a detailed analysis of the various possible realistic cases [extra U(1), extra SU(2), . . .] that is clearly beyond the purposes of this preliminary investigation. We have, though, performed a partial analysis in the simplest relevant case, that is for the "minimal-nonminimal" model examined in Ref. [2]. For this model, we have performed an analysis of the charged scalar, pseudoscalar, chargino, and neutralino sector using some recently proposed approximate expressions for the neutral masses [8]. We have verified that even in these cases the separate contributions satisfy the screening condition Eq. (28). This leads us to conjecture that the property will remain true for a more general case, although at the moment we cannot provide a rigorous proof of this statement (that is, in fact, under investigation).

We want to conclude this paper with the derivation of a particularly simple formulation of our result in the (possibly meaningful) situation of large $\tan\beta$ values, $\tan\beta \gg 1$. In this limit $\overline{m}_{11}^2 = \overline{m}_{12}^2 = 0$ [see Eqs. (9)–(12)], and the one-loop bound on the light Higgs-boson mass assumes the compact expression

$$M_{H_1}^2(\tan\beta \gg 1) \le \overline{m}_{22}^2 + \delta\overline{m}_{22}^2 = 2v^2 \sum_n g_n^2 (T_2^n)^2 + \delta\overline{m}_{22}^2 \ , \tag{46}$$

where now n denotes the general symmetry group of the model [$n=1,2$ correspond to $U(1)_Y$, $SU(2)_L$; higher n values correspond to the remaining possible components of the extra group G]. Thus the full extra information on the bound at one loop from the fermion-sfermion sector is contained in the finite part of the (2,2) matrix element. Quite generally, we obtain from our formulas that the latter is dominated by the top-quark–top-squark system. Therefore, in this situation of large $\tan\beta$, the full contribution of the exotic sector can be safely neglected, which is the generalization of a result already found for the special case of the so-called η model in a previous work [9].

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[4]This seems to be, at least, a natural assumption in the model if the minimum conditions of the potential have to be valid in the large-x limit without resorting to special "fine-tuning" conditions.

[5]Note that the cancellation of the unwanted effects is valid including a formal $O(\alpha^2)$ term that was neglected in Eq. (26), since their factorization in the overall one-loop quantities reproduces rigorously the tree-level situation of Eqs. (9)–(11).

[1] See, e.g., for a discussion of this point, L. E. Ibáñez, Report No. CERN-TH.5982/91 (unpublished), and references therein.

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