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Hadronization of quark flavour dynamics including three-quark forces

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We investigate an effective chiral quark model containing besides a two-quark Nambu–Jona-Lasinio force also three-quark forces. By means of path-integral techniques this model is, at first, converted into a quark–diquark–meson–baryon theory and afterwards hadronized into a meson–baryon theory. This way, we have performed an approximative step-by-step reduction of a three-body problem to a two-body problem in the collective field approach.

1. Introduction

QCD as the theory of strong interactions should describe hadrons. However, it is given in terms of coloured quarks and gluons, which are supposed to be unobservable objects and, therefore, should be confined. It is a challenge to derive effective chiral lagrangians describing low-energy hadron physics from the microscopic QCD. Some progress in this direction has been reached, especially concerning the "derivation" of effective meson lagrangians. Here intermediate QCD-motivated quark models such as the Nambu–Jona-Lasinio (NJL) model [1] play an important role. A lot of papers devoted to the path-integral bosonization of QCD and NJL models has been published [2–6]. In this way, the microscopic theories are rewritten as effective theories given in terms of composite bosonic objects, mesons. Let us mention that in a similar way diquarks as effective degrees of freedom have been introduced both in two-dimensional QCD [2] and in four-dimensional QCD-type models [6–8]. The concept of diquarks is also successfully used for the understanding of nonleptonic weak decays at low energies [9]. Furthermore, there are first attempts in the direction of path-integral hadronization, intended to describe QCD in terms of hadrons (mesons and baryons) [10–12].

Our purpose consists in further investigating path-integral hadronization using the method applied in ref. [8] for meson–diquark bosonization but extending the underlying NJL model by including in addition to the two-quark NJL force a three-quark force. As usual, the two-current NJL interaction term in the lagrangian models quark–antiquark and quark–quark interactions via gluon exchange. The novel introduced six-quark interaction terms describe in addition gluon-mediated quark–diquark interactions. From a methodical point of view these terms require the development of sophisticated functional methods for reducing in an approximative way the three-body problem to a two-body problem of interacting quarks and diquarks. Extra complications due to

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additional many-body interactions are neglected. These techniques could be useful for other applications, too. It is worth mentioning that in our model the baryon fields must be introduced from the very beginning as dynamical fields. This is in contrast to ref. [10] where they are required only in a later stage in order to handle closed loops appearing in the evaluation of the fermion determinant.

In the present paper the effective meson–diquark–baryon action is derived and presented in a complete form, including all interaction terms. After integration over the diquark fields we obtain the effective meson–baryon action with explicit expressions for the meson and baryon propagators. Moreover, the inhomogeneous Bethe–Salpeter equation for the quark–diquark Green function is quoted. Finally, the effect of the quark–diquark interaction on the baryon spectrum is qualitatively estimated.

The paper is organized as follows. In section 2 we introduce the model including some motivations for the six-quark interaction term. Section 3 is devoted to the derivation of the meson–diquark–baryon lagrangian. Its final hadronized form is presented in section 4.

2. The model

Throughout this paper we consider the following effective quark model with the lagrangian:

$$\mathscr{L}=\mathscr{L}_0+\mathscr{L}^{(4)}_{\mathrm{NJL}}+\mathscr{L}^{(6)}\,. \tag{1}$$

Here $\mathscr{L}_0$ is the free lagrangian,

$$\mathscr{L}_0=\bar{q}S_0^{-1}q\,,\quad S_0^{-1}(x)=\mathrm{i}\partial\!\!\!/_x-m_0\,, \tag{2}$$

and $\mathscr{L}^{(4)}_{\mathrm{NJL}}$ denotes the extended NJL four-quark interaction term taken from ref. [8]. It consists of two parts with quantum numbers of meson and diquark bound states:

$$\mathscr{L}^{(4)}_{\mathrm{NJL}}=\sum_{i=1}^{4}\left[G_i(\bar{q}\mathscr{M}^{ie}_{\mathrm{M}}q)(\bar{q}\mathscr{M}^{ie}_{\mathrm{M}}q)+\tilde{G}_i(\bar{q}\mathscr{M}^{\alpha ig}_{\mathrm{D}}q^{\mathrm{c}})(\bar{q}^{\mathrm{c}}\mathscr{M}^{\alpha ig}_{\mathrm{D}}q)\right]\,. \tag{3}$$

Here q^{c} and $\bar{q}^{\mathrm{c}}$ are charge-conjugated fields,

$$q^{\mathrm{c}}=C\bar{q}^{\mathrm{T}}\,,\quad \bar{q}^{\mathrm{c}}=q^{\mathrm{T}}C\,,$$

with C being the charge conjugation matrix, and T means transposition. The meson $\mathscr{M}^{\tau}_{\mathrm{M}}$ and diquark $\mathscr{M}^{\theta}_{\mathrm{D}}$ projection matrices in eq. (3) are defined by

$$\mathscr{M}^{\tau}_{\mathrm{M}}\equiv\mathscr{M}^{ie}_{\mathrm{M}}=\frac{1}{\sqrt{3}}\mathscr{K}^i\mathbf{1}_{\mathrm{c}}\mathscr{F}^e\,,\quad \mathscr{M}^{\theta}_{\mathrm{D}}\equiv\mathscr{M}^{\alpha A}_{\mathrm{D}}\equiv\mathscr{M}^{\alpha ig}_{\mathrm{D}}=\frac{\mathrm{i}}{\sqrt{6}}\epsilon^{\alpha}\mathscr{K}^i\mathscr{H}^g\,,$$

where $\mathscr{K}^i$ are Dirac matrices

$$\{\mathscr{K}^i,\,i=1,2,3,4\}=\left\{\mathbf{1},\,\mathrm{i}\gamma^5,\,\frac{1}{\sqrt{2}}\gamma^{\mu},\,\frac{1}{\sqrt{2}}\gamma^{\mu}\gamma^5\right\}\,,$$

and ϵ^{α} is the antisymmetric Levi-Civita tensor defined in colour space for the group $\mathrm{SU}(3)_{\mathrm{c}}$. For simplicity, we consider the case of isospin flavour symmetry $\mathrm{SU}(2)_{\mathrm{f}}$, so that the flavour generators $\mathscr{F}^e$, $\mathscr{H}^g$ are given by the relations

$$\{\mathscr{F}^e,\,e=0,1,2,3\}=\left\{\frac{1}{\sqrt{2}}\mathbf{1}_{\mathrm{f}},\,\sqrt{2}\,T^1,\,\sqrt{2}\,T^2,\,\sqrt{2}\,T^3\right\}\,,\quad \{\mathscr{H}^g,\,g=1,2,3,4\}=\{\mathscr{F}^e,\,e=2,0,1,3\}\,,$$

with $T^n=\frac{1}{2}\sigma^n$ and σ^n being Pauli matrices. In eq. (3) summation over repeated indices τ, θ is understood. The coupling constants for mesons (G_i) and diquarks ($\tilde{G}_i$) have dimension $(\mathrm{mass})^{-2}$. For scalar/pseudoscalar channels we take $G_1=G_2\equiv G$, $\tilde{G}_1=\tilde{G}_2\equiv\tilde{G}$ and for vector/axial-vector channels $G_3=G_4\equiv -G'$, $\tilde{G}_3=\tilde{G}_4\equiv -\tilde{G}'$. In this way chiral symmetry takes still place [#1].

Before discussing the last term on the RHS of (1) let us mention, that the effective lagrangian $\mathscr{L}_0+\mathscr{L}^{(4)}_{\rm NJL}$ can be motivated from QCD by using adequate Fierz identities [8]. Thereby, the simultaneous decomposition into colour singlet ($\bar{q}q$) and colour antitriplet (qq) channels is physically attractive. The diquark sector of this model has been studied in ref. [8], whereas the meson sector has been extensively investigated earlier in ref. [4].

Now, the six-quark interaction term $\mathscr{L}^{(6)}$ included into (1) reads

$$\mathscr{L}^{(6)}=\sum_N\sum_{i=1}^4 G_{6N}[\tilde{G}_i\bar{q}^a_\alpha(\bar{q}\mathscr{M}^{\alpha ig}_{\rm D}q^c)]O^N_i[\tilde{G}_i(\bar{q}^c\mathscr{M}^{\alpha' ig}_{\rm D}q)q^a_{\alpha'}]\,. \tag{4}$$

Here the index a refers to quark spin and flavour. The new coupling constants G_{6N} are of dimension (mass)$^{-2}$, and the operators O^N_i have dimension of mass (the coupling constants $\tilde{G}_i$ are included for convenience). In this paper we will work with unspecified operators O^N_i, $i=1, 2, 3, 4$, entering $\mathscr{L}^{(6)}$ in eq. (4). The lagrangian $\mathscr{L}^{(6)}$ can be motivated phenomenologically as describing gluon-mediated quark–diquark interactions in the local approximation for the gluon propagator (cf. section 3). Note, that an n-quark interaction term is for large colour number N_c proportional to N_c^{1-n}. Thus the three-quark interaction (4) with $n=3$ is a $1/N_c$ correction to the two-particle interactions (3).

3. Path-integral approach with collective fields

Let us consider the generating functional for Green functions of mesons and baryons for the model (1),

$$Z[\eta_{\rm M},\bar{\eta}_{\rm B},\eta_{\rm B}]=\int\mathscr{D}q\,\mathscr{D}\bar{q}\exp{\rm i}\int{\rm d}^4x\,(\mathscr{L}+\mathscr{L}_\eta)\,,$$

$$\mathscr{L}_\eta=(\bar{q}\mathscr{M}^\tau_{\rm M}q)\eta^\tau_{\rm M}+[\tilde{G}_i\bar{q}^a_\alpha(\bar{q}\mathscr{M}^{\alpha A}_{\rm D}q^c)]\eta^{aA}_{\rm B}+\bar{\eta}^{aA}_{\rm B}[\tilde{G}_i(\bar{q}^c\mathscr{M}^{\alpha A}_{\rm D}q)q^a_\alpha]\,, \tag{5}$$

where $\eta_{\rm M}$, $\eta_{\rm B}$ and $\bar{\eta}_{\rm B}$ are meson and baryon sources, respectively [#2]. Next, we introduce collective fields analogously to ref. [8], including from the very beginning also baryon degrees of freedom in addition to meson and diquark ones. To this end, use of the following identities is made:

$$1=\int\prod_\tau\mathscr{D}M^\tau\,\delta(\bar{q}\mathscr{M}^\tau_{\rm M}q-M^\tau)=\int\mathscr{D}M\,\mathscr{D}\phi\exp{\rm i}\int{\rm d}^4x\,[(\bar{q}\mathscr{M}^\tau_{\rm M}q-M^\tau)\phi^\tau]\,,$$

$$1=\int\prod_{\theta,\theta'}\mathscr{D}D^{\dagger\theta}\,\mathscr{D}D^{\theta'}\,\delta(\bar{q}\mathscr{M}^\theta_{\rm D}q^c-D^{\dagger\theta})\delta(\bar{q}^c\mathscr{M}^{\theta'}_{\rm D}q-D^{\theta'})$$

$$=\int\mathscr{D}D^\dagger\,\mathscr{D}D\,\mathscr{D}\omega^\dagger\,\mathscr{D}\omega\exp{\rm i}\int{\rm d}^4x\,[(\bar{q}\mathscr{M}^\theta_{\rm D}q^c-D^{\dagger\theta})\omega^\theta+\omega^{\dagger\theta}(\bar{q}^c\mathscr{M}^\theta_{\rm D}q-D^\theta)]\,,$$

$$1=\int\prod_{a,A,a',A'}\mathscr{D}\bar{B}^{aA}\,\mathscr{D}B^{a'A'}\,\delta(\tilde{G}_i\bar{q}^a_\alpha(\bar{q}\mathscr{M}^{\alpha A}_{\rm D}q^c)-\bar{B}^{aA})\delta(\tilde{G}_i(\bar{q}^c\mathscr{M}^{\alpha' A'}_{\rm D}q)q^{a'}_{\alpha'}-B^{a'A'})$$

$$=\int\mathscr{D}\bar{B}\,\mathscr{D}B\,\mathscr{D}\bar{\psi}\,\mathscr{D}\psi\exp{\rm i}\int{\rm d}^4x\,\{[\tilde{G}_i\bar{q}^a_\alpha(\bar{q}\mathscr{M}^{\alpha A}_{\rm D}q^c)-\bar{B}^{aA}]\psi^{aA}+\bar{\psi}^{aA}[\tilde{G}_i(\bar{q}^c\mathscr{M}^{\alpha A}_{\rm D}q)q^a_\alpha-B^{aA}]\}\,, \tag{6}$$

[#1] The relative minus sign between the couplings of scalar/pseudoscalar and vector/axial-vector channels is due to the Fierz rearrangement of Dirac matrices for the t-channel vector exchange.

[#2] From now on in the source terms and other formulae the summation over i is assumed. To understand this summation we should remind the reader that we use the earlier introduced short-hand notations A for the indices $\{i, g\}$, τ for $\{i, e\}$ and θ for $\{\alpha, i, g\}$. Therefore, beside the couplings G_i, $\tilde{G}_i$ all quantities depending on A, τ, θ carry the index i.

where M, D, $D^\dagger$, B, $\bar{B}$ are auxiliary fields to be integrated away and $\mathscr{D}M \equiv \prod_\tau \mathscr{D}M^\tau$, $\mathscr{D}D \equiv \prod_\theta \mathscr{D}D^\theta$ etc. The δ-functionals have been rewritten as functional Fourier integrals over meson (ϕ), diquark (ω, $\omega^\dagger$) and baryon (ψ, $\bar{\psi}$) fields, respectively. The indices A and a refer both to spin and flavour of the diquark and quark constituents, respectively. Notice that the baryons should be in a completely antisymmetric state. Concerning the colour antisymmetric diquark fields according to the Pauli principle for flavour antisymmetric $\mathbf{1}_f$ and flavour symmetric $\mathbf{3}_f$ states we must have respective antisymmetric or symmetric spin states.

Inserting the identities (6) into (5) and integrating over the auxiliary fields yields

$$Z[\eta_M, \bar{\eta}_B, \eta_B] = C_1 \int \mathscr{D}\bar{q}\, \mathscr{D}q\, \mathscr{D}\phi\, \mathscr{D}\omega\, \mathscr{D}\omega^\dagger\, \mathscr{D}\psi\, \mathscr{D}\bar{\psi} \exp \mathrm{i} \int \mathrm{d}^4x \bigg[\tfrac{1}{2} \int \mathrm{d}^4y\, (\bar{q}(x), q^{\mathrm{T}}(x))\, \hat{L}(x,y) \begin{pmatrix} q(y) \\ \bar{q}^{\mathrm{T}}(y) \end{pmatrix}$$
$$+ \omega^{\dagger\alpha A} \bar{q}^a_\alpha \psi^{aA} + \bar{\psi}^{aA} q^a_\alpha \omega^{\alpha A} - \left(\frac{1}{4G_i} (\eta^\tau_M - \phi^\tau)^2 + \frac{1}{\tilde{G}_i} \omega^{\dagger\theta}\omega^\theta + (\bar{\eta}^{aA}_B - \bar{\psi}^{aA}) O_i^{-1} (\eta^{aA}_B - \psi^{aA}) \right) \bigg], \quad (7)$$

where for simplicity we have introduced the short-hand notations

$$O_i = \sum_N G_{6N} O_i^N . \quad (8)$$

In (7) $\hat{L}$ is the 2×2 matrix

$$\hat{L}(x,y) = \begin{pmatrix} S^{-1}_{\Phi\psi}(x,y) & 2\Omega(x)\delta^{(4)}(x-y) \\ 2\Omega^\dagger(x)\delta^{(4)}(x-y) & -(S^{-1}_{\Phi\psi})^{\mathrm{T}}(x,y) \end{pmatrix},$$

with

$$S^{-1}_{\Phi\psi}(x,y) = [S_0^{-1}(x) + \tilde{G}_i \bar{\psi}^A(x)\psi^A(x) + \mathscr{M}^\tau_M \phi^\tau(x)]\delta^{(4)}(x-y)$$

and

$$\Omega(x) \equiv \mathscr{M}^\theta_D C\omega^\theta(x), \quad \Omega^\dagger(x) \equiv \omega^{\dagger\theta}(x) C\mathscr{M}^\theta_D .$$

We shall stress that a nontrivial result of the integration over the auxiliary baryon fields B, $\bar{B}$ has become only possible because of the presence of a term $\bar{B}^{aA} O_i B^{aA}$ resulting from the six-quark interaction (4). Without (4) one would get a trivial δ-functional $\delta(\eta_B - \psi)$ fixing the dynamical baryon field. In eq. (7) the baryon field ψ couples to the fields of a quark q and a diquark ω. Phenomenologically, the interaction term $\mathscr{L}^{(6)}$ is suggested to arise from a quark–diquark interaction mediated by gluon exchange (figs. 1a, b) in the local approximation for the gluon propagator. Thereby, in dependence on the Lorentz structure of the diquark system several types of operators O_i^N in (8) are possible,

$$O_i^1 = \gamma\cdot\overleftrightarrow{\partial}_- \quad \text{for } i=1,2,$$
$$= \gamma\cdot \mathrm{d}_+ \quad \text{for } i=3,4,$$
$$O_i^2 = c \quad \text{for } i=1,2,$$
$$= c\cdot\gamma_\mu\gamma_\nu \quad \text{for } i=3,4,$$

Fig. 1. Quark–diquark interactions mediated by (a) one-gluon and (b) two-gluon exchange.

where c denotes a constant, and the derivatives

$$\overleftrightarrow{\partial}^{\mu}_{-} \equiv \overleftarrow{\partial}^{\mu} - \overrightarrow{\partial}^{\mu} ,$$

$$\overleftrightarrow{d}^{\mu}_{+} \equiv \overleftarrow{d}^{\mu} + \overrightarrow{d}^{\mu} ,$$

with

$$\vec{d}_{\mu}\omega^{\theta}_{\nu} \equiv \partial_{\mu}\omega^{\theta}_{\nu} - \partial_{\nu}\omega^{\theta}_{\mu} , \quad \text{etc.} ,$$

are applied only to the diquark fields ω^{θ}, $\omega^{\dagger\theta}$, $\theta \equiv \{\alpha, i, g\}$, when acting on expressions $\sim (q\omega)$.

Let us next integrate over the quark fields. The result reads

$$Z[\eta_{\rm M}, \bar{\eta}_{\rm B}, \eta_{\rm B}] = C_2 \int \mathcal{D}\phi\, \mathcal{D}\omega^{\dagger}\, \mathcal{D}\omega\, \mathcal{D}\bar{\psi}\, \mathcal{D}\psi \exp \mathrm{i}(W^{\phi\omega\psi}_{\rm eff} + W_{\eta}) , \tag{9}$$

with

$$W^{\phi\omega\psi}_{\rm eff} = -\mathrm{i}\, \mathrm{Tr} \ln S^{-1}_{\phi\psi} - \tfrac{1}{2}\mathrm{i}\, \mathrm{Tr} \ln (1 + 4S^{\rm T}_{\phi\psi}\Omega^{\dagger}S_{\phi\psi}\Omega)$$
$$+ \int \mathrm{d}^4x \left[-\tfrac{1}{2} \int \mathrm{d}^4y\, (\bar{\beta}, -\beta^{\rm T}) \hat{L}^{-1} \begin{pmatrix} \beta \\ -\bar{\beta}^{\rm T} \end{pmatrix} - \left(\frac{1}{4G_i} (\phi^{\tau})^2 + \frac{1}{\tilde{G}_i} \omega^{\dagger\theta}\omega^{\theta} + \bar{\psi}^{aA} O_i^{-1} \psi^{aA} \right) \right], \tag{10}$$

$$W_{\eta} = \int \mathrm{d}^4x \left(-\frac{1}{4G_i} [(\eta^{\tau}_{\rm M})^2 - 2\phi^{\tau}\eta^{\tau}_{\rm M}] - \bar{\eta}^{aA}_{\rm B} O_i^{-1} \eta^{aA}_{\rm B} + \bar{\psi}^{aA} O_i^{-1} \eta^{aA}_{\rm B} + \bar{\eta}^{aA}_{\rm B} O_i^{-1} \psi^{aA} \right),$$

and

$$\beta^{\alpha a} = \omega^{\dagger\alpha A} \psi^{aA} , \quad \bar{\beta}^{\alpha a} = \bar{\psi}^{aA} \omega^{\alpha A} .$$

In the derivation of (10) we have used the fact that

$$\det \hat{L} = -S^{-2}_{\phi\psi} (1 + 4S^{\rm T}_{\phi\psi}\Omega^{\dagger}S_{\phi\psi}\Omega) .$$

The trace Tr in (10) runs over internal and spinor indices and includes an integration over space–time variables. Eqs. (9), (10) represent the result of the meson–diquark bosonization. A further integration over the intermediate diquark fields leads to a "hadronization" of the model, i.e., one obtains an expression for Z in terms of fields of observable physical hadrons (mesons and baryons). This integration has to be done rather carefully because of the presence of the diquark fields $\omega^{\dagger}$, ω up to all orders due to the second term in (10).

As usual let us perform a stationary phase approximation and restrict ourselves to the consideration of the sector with $\omega^{\theta} = \omega^{\dagger\theta} = 0$, $\psi^{A\alpha} = \bar{\psi}^{A\alpha} = 0$, but $\phi_0^{\tau} \neq 0$. Here ϕ_0^{τ} satisfies the Schwinger–Dyson equation and is related to the constituent quark mass m by $\phi_0^{ae} = \sqrt{6}\,(m_0 - m)\delta^{a1}\delta^{e0}$, where

$$m - m_0 = 8G\mathrm{i} \int^{\Lambda} \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{m}{k^2 - m^2} .$$

Shifting the integration variable $\phi^{\tau} \to \Phi^{\tau} = \phi^{\tau} - \phi_0^{\tau}$ and expanding $W^{\phi\omega\psi}_{\rm eff}$ in eq. (10) in terms of fields around this solution, one obtains

$$W^{\Phi\omega\psi}_{\rm eff} = W^{\Phi}_{\rm free\ 1} + W^{\omega}_{\rm free} + W^{\psi}_{\rm free\ 1} + W^{\Phi\omega\psi}_{\rm int} . \tag{11}$$

Here

$$W^{\omega}_{\rm free} = \iint \mathrm{d}^4x\, \mathrm{d}^4y\, \omega^{\dagger\theta}(x) (\Delta^{-1}_{\omega})^{\theta\theta'}(x, y) \omega^{\theta'}(y) , \tag{12}$$

$$W^{\Phi}_{\rm free\ 1} = \tfrac{1}{2} \iint \mathrm{d}^4x\, \mathrm{d}^4y\, \Phi^{\tau}(x) (\Delta^{-1}_{\Phi 1})^{\tau\tau'}(x, y) \Phi^{\tau'}(y) , \tag{13}$$

$$W^{\psi}_{\text{free 1}} = \int d^4x\, \bar{\psi}^{aA}(-O_i^{-1}\delta^{aa'}\delta^{AA'} - \delta^{AA'}\mathrm{i}\tilde{G}_i\, \mathrm{tr}\, S_m^{aa'})\psi^{a'A'} \tag{14}$$

are the free effective action for the fields of diquarks ω, $\omega^\dagger$ and parts (given before the integration over the diquark fields has been performed) of the free effective actions for the fields of mesons Φ and baryons ψ, $\bar{\psi}$, respectively. The inverse free composite field propagators in (12) and (13) are defined by the relations

$$(\Delta_{\Phi}^{-1})^{\tau\tau'}(x,y) = -\frac{1}{2G_i}\delta^{\tau\tau'}\delta^{(4)}(x-y) + \mathrm{i}\,\mathrm{tr}[S_m(x-y)\mathcal{M}^{\tau}_{\mathrm{M}}S_m(y-x)\mathcal{M}^{\tau'}_{\mathrm{M}}]\,, \tag{15}$$

$$(\Delta_{\omega}^{-1})^{\theta\theta'}(x,y) = -\frac{1}{\tilde{G}_i}\delta^{\theta\theta'}\delta^{(4)}(x-y) + 2\mathrm{i}\,\mathrm{tr}[S_m(x-y)\mathcal{M}^{\theta}_{\mathrm{D}}S_m(y-x)\mathcal{M}^{\theta'}_{\mathrm{D}}]\,, \tag{16}$$

with $S_m(x) = (\mathrm{i}\partial\!\!\!/ - m)^{-1}\delta^{(4)}(x)$. The trace tr means summation over internal and spinor indices. In the derivation of (16) we have used the fact that $CS_m^{\mathrm{T}}C = -S_m$. The inverse meson and diquark propagators (15) and (16) were obtained within the meson–diquark bosonization scheme already earlier in ref. [8] [#3]. The complete expression for the interaction part $W^{\Phi\omega\psi}_{\text{int}}$ of the effective action is given by the sum

$$W^{\Phi\omega\psi}_{\text{int}} = W^{\Phi}_{\text{int}} + W^{\psi}_{\text{int}} + W^{\Phi\psi}_{\text{int 1}} + W^{\Phi\psi}_{\text{int 2}}[\omega^\dagger, \omega]\,, \tag{17}$$

with

$$W^{\Phi}_{\text{int}} = \mathrm{i}\sum_{p\geqslant 3}\frac{1}{p}\mathrm{Tr}(S_m\Phi)^p\,, \quad \Phi \equiv -\mathcal{M}^{\tau}_{\mathrm{M}}\Phi^{\tau}\,, \tag{18}$$

$$W^{\psi}_{\text{int}} = \mathrm{i}\sum_{q\geqslant 2}\frac{1}{q}\mathrm{Tr}(-\tilde{G}_i S_m^{a'a}\bar{\psi}^{a'A}\psi^{bA})^q\,, \tag{19}$$

$$W^{\Phi\psi}_{\text{int 1}} = \mathrm{i}\sum_{p,q\geqslant 1}\frac{1}{p+q}\mathrm{Tr}[(S_m\Phi)^p(-\tilde{G}_i S_m^{a'a}\bar{\psi}^{a'A}\psi^{bA})^q + \ldots]\,, \tag{20}$$

$$W^{\Phi\psi}_{\text{int 2}}[\omega^\dagger,\omega] = -\bar{\psi}^{aA}\omega^{\alpha A}\sum_{n=0}^{\infty}[(-4S_{\Phi\psi}\Omega S^{\mathrm{T}}_{\Phi\psi}\Omega^\dagger)^n S_{\Phi\psi}]^{aa'}\omega^{\dagger\alpha A'}\psi^{a'A'}$$

$$+\tfrac{1}{2}\mathrm{i}\,\mathrm{Tr}\left(\sum_{n=1}^{\infty}\frac{1}{n}(-4S^{\mathrm{T}}_{\Phi\psi}\Omega^\dagger S_{\Phi\psi}\Omega)^n + 4S_m^{\mathrm{T}}\Omega^\dagger S_m\Omega\right). \tag{21}$$

Here $W^{\Phi\omega\psi}_{\text{int}}$ describes the meson–diquark–baryon interaction, W^{Φ}_{int} and W^{ψ}_{int} are the higher-order self-interactions of the corresponding fields. $W^{\Phi\psi}_{\text{int 1}}$ is the first part of the meson–baryon interaction, where the dots in the RHS of (20) denote analogous terms with all possible orderings of the p and q factors in brackets. Notice, that everywhere in (18)–(21) the space–time arguments for the fields ψ^{aA}, $\omega^{\alpha B}$ and Φ^{τ} have been included into the indices A, B and τ, respectively. A similar recipe holds for the propagators. (An integration over space–time arguments is understood.)

4. The effective meson–baryon lagrangian

So far we have obtained the generating functional Z in the representation (9). Now we want to derive in the final step the effective meson–baryon action. To do this we perform now the diquark integration. To this end, let us rewrite (9), (11) in the form

[#3] Notice, that some misprints in eqs. (16) and (17) of ref. [8] concerning signs and numerical factors have been corrected here.

$$Z[\eta_M, \bar{\eta}_B, \eta_B] = C_3 \int \mathscr{D}\Phi\, \mathscr{D}\bar{\psi}\, \mathscr{D}\psi \exp i(W^{\Phi}_{\text{free 1}} + W^{\psi}_{\text{free 1}} + W^{\Phi}_{\text{int}} + W^{\psi}_{\text{int}} + W^{\Phi\psi}_{\text{int 1}} + W_\eta)$$

$$\times \exp iW^{\Phi\psi}_{\text{int 2}}\left[\frac{\delta}{i\,\delta J_\omega}, \frac{\delta}{i\,\delta J^\dagger_\omega}\right] \int \mathscr{D}\omega^\dagger\, \mathscr{D}\omega \exp i\left(W^{\omega}_{\text{free}} + \int d^4x\, (J^\dagger_\omega \omega + \omega^\dagger J_\omega)\right)\Bigg|_{J^\dagger_\omega = J_\omega = 0} .$$

After the integration one has

$$Z[\eta_M, \bar{\eta}_B, \eta_B] = C_3 \int \mathscr{D}\Phi\, \mathscr{D}\bar{\psi}\, \mathscr{D}\psi \exp i(W^{\Phi}_{\text{free}} + W^{\psi}_{\text{free 1}} + W^{\Phi}_{\text{int}} + W^{\psi}_{\text{int}} + W^{\Phi\psi}_{\text{int1}} + W_\eta)$$

$$\times \exp iW^{\Phi\psi}_{\text{int 2}}\left[\frac{\delta}{i\,\delta J_\omega}, \frac{\delta}{i\,\delta J^\dagger_\omega}\right] \exp i\left(i\,\mathrm{Tr}\ln \Delta^{-1}_\omega - \iint J^{\dagger\theta}_\omega \Delta^{\theta\theta'}_\omega J^{\theta'}_\omega\right)\Bigg|_{J^\dagger_\omega = J_\omega = 0} .$$

Here the new effective meson action W^{Φ}_{free} differs from (13) only by the inclusion of self-energy correction terms associated to additional internal diquark propagators.

For simplicity we next neglect contributions in Z corresponding to diagrams of the type shown in figs. 2a, 2b and 2c as well as higher-order self-energy corrections to the baryon propagator [#4]. Then, in this approximation we obtain

$$Z[\eta_M, \bar{\eta}_B, \eta_B] \approx C_4 \int \mathscr{D}\Phi\, \mathscr{D}\bar{\psi}\, \mathscr{D}\psi \exp i\{W^{\Phi}_{\text{free}} + W^{\psi}_{\text{free}} + W_{\text{int}}[\Phi, \bar{\psi}, \psi] + W_\eta\} . \qquad (22)$$

Here the free effective baryon action reads

$$W^{\psi}_{\text{free}} = \iint d^4x\, d^4y\, \bar{\psi}^{aA}(x)(\Delta^{-1}_\psi)^{aAa'A'}(x,y)\psi^{a'A'}(y) , \qquad (23)$$

with the inverse baryon propagator being given by

$$(\Delta^{-1}_\psi)^{aAa'A'} = -\delta^{aa'}\delta^{AA'}O^{-1}_i - i\tilde{G}_i\,\mathrm{tr}\,(S^{aa'}_m)\delta^{AA'} - i\Delta^{\alpha A\alpha A'}_\omega S^{aa'}_m - iT^{aAa'A'}_m , \qquad (24)$$

$$T^{aAa'A'}_m = \Delta^{(\alpha A)\theta_1} \sum_{n=1}^{\infty} [(-4iS_m \mathscr{M}^{\theta_1}_{D} S_m \Delta^{\theta_1\theta'_2}_\omega \mathscr{M}^{\theta'_1}_{D})\ldots(-4iS_m \mathscr{M}^{\theta_n}_{D} S_m \Delta^{\theta_n(\alpha A')}_\omega \mathscr{M}^{\theta'_n}_{D})S_m]^{aa'} . \qquad (25)$$

The first two terms in (24) are taken from (14). The remaining terms result from the baryon–diquark interaction in the first term of the RHS of (21). (The second term of (21) leads to self-energy corrections only.)

Before giving the expression for $W_{\text{int}}[\Phi, \bar{\psi}, \psi]$ in (22) let us mention that on the RHS of (24) the second term is cancelled by an equivalent expression associated to the third term. Finally, this leads in short-hand notations to the following compact formula for the baryon propagator:

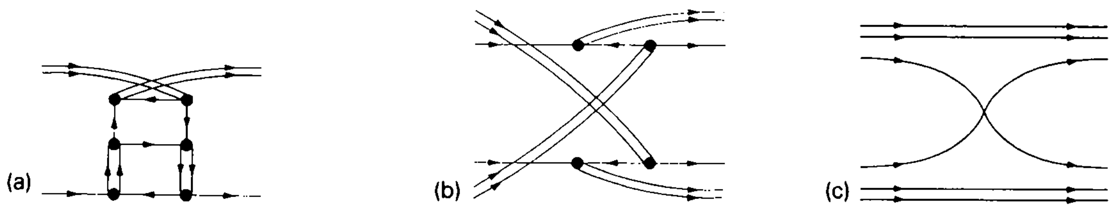

Fig. 2. Examples for complicated neglected diagrams in (22) (a) with internal diquark self-interactions and (b) baryon–baryon interactions; (c) Simplest box diagram contributing to baryon–baryon scattering.

[#4] Note that the simple box diagram contributing to baryon–baryon scattering (see fig. 2c) arises in perturbation theory in second order of $W^{\Phi\psi}_{\text{int 2}}$ with $n=0$ and $S_{\Phi\psi}$ replaced by S_m.

$$\Delta_\psi = -O_i[1 + \mathrm{i}(\Delta_\omega^{(1)} S_m + T_m) O_i]^{-1} ,$$

with

$$\Delta_\omega^{(1)} \equiv \Delta_\omega + \tilde{G}_i = -\tilde{G}_i \Pi_\omega \frac{1}{1-\tilde{G}_i \Pi_\omega} \tilde{G}_i , \quad \Pi_\omega = 2\mathrm{i}\, \mathrm{tr}(S_{m'} \mathcal{M}_{\mathrm{D}} S_{m'} \mathcal{M}_{\mathrm{D}}) .$$

The interaction part W_{int} of the effective action defined in eq. (22) can be represented as a sum,

$$W_{\mathrm{int}}[\Phi, \bar{\psi}, \psi] = W_{\mathrm{int}}^{\Phi} + W_{\mathrm{int}}^{\psi} + W_{\mathrm{int}}^{\Phi\psi} ,$$

where W_{int}^{Φ} and W_{int}^{ψ} are given by (18) and (19), respectively. In the approximation leading to eq. (22) the meson–baryon interaction part $W_{\mathrm{int}}^{\Phi\psi}$ now reads

$$W_{\mathrm{int}}^{\Phi\psi} = W_{\mathrm{int}\,1}^{\Phi\psi} + \iint \bar{\psi}^{aA} [\Delta_\omega^{(1)\alpha A\alpha A'} (S_{\Phi\psi}^{aa'} - S_m^{aa'}) + T_{\Phi\psi}^{aAa'A'} - T_m^{aAa'A'}] \psi^{a'A'} ,$$

with $W_{\mathrm{int}\,1}^{\Phi\psi}$ taken from (20). $T_{\Phi\psi}$ is given by (25) with all S_m substituted by $S_{\Phi\psi}$. Thus, the effective meson-baryon action in (22) is now defined.

In the second part of this section we want to derive the Bethe–Salpeter equation for the hadron spectrum. To do this we have to integrate in the generating functional (22) over the meson and baryon fields, taking as usual all interaction terms outside the functional integral. This yields

$$Z[\eta_{\mathrm{M}}, \bar{\eta}_{\mathrm{B}}, \eta_{\mathrm{B}}] \approx \exp \mathrm{i} W_{\mathrm{int}}\left[\frac{\delta}{\mathrm{i}\,\delta J_\Phi}, \frac{\delta}{\mathrm{i}\,\delta J_\psi}, \frac{\delta}{\mathrm{i}\,\delta \bar{J}_\psi}\right] Z_0[\eta_{\mathrm{M}}, \bar{\eta}_{\mathrm{B}}, \eta_{\mathrm{B}} | J_\Phi, \bar{J}_\psi, J_\psi]\Big|_{J_\psi = J_\psi = 0,\ J_\Phi = 0} ,$$

where

$$Z_0[\ ] = C_4 \exp \mathrm{i} \int \mathrm{d}^4x \left(-\frac{1}{4G_i} (\eta_{\mathrm{M}}^{\tau})^2 - \bar{\eta}_{\mathrm{B}}^{aA} O_i^{-1} \eta_{\mathrm{B}}^{aA}\right) \int \mathcal{D}\Phi \exp \mathrm{i}\left[W_{\mathrm{free}}^{\Phi} + \int \mathrm{d}^4x\, \Phi^{\tau}\left(\frac{1}{2G_i} \eta_{\mathrm{M}}^{\tau} + J_\Phi^{\tau}\right)\right]$$

$$\times \int \mathcal{D}\bar{\psi}\, \mathcal{D}\psi \exp \mathrm{i}\left(W_{\mathrm{free}}^{\psi} + \int \mathrm{d}^4x\, [\bar{\psi}^A (O_i^{-1} \eta_{\mathrm{B}}^A + J_\psi^A) + (\bar{\eta}_{\mathrm{B}}^A O_i^{-1} + \bar{J}_\psi^A) \psi^A]\right) .$$

After the integration over Φ, ψ, $\bar{\psi}$ one has for the free part [#5]

$$Z_0[\eta_{\mathrm{M}}, \bar{\eta}_{\mathrm{B}}, \eta_{\mathrm{B}} | 0, 0, 0] = C \exp\left(-\mathrm{i} \iint (\tfrac{1}{2} \eta_{\mathrm{M}} \Delta'_\Phi \eta_{\mathrm{M}} + \bar{\eta}_{\mathrm{B}} \Delta'_\psi \eta_{\mathrm{B}})\right) ,$$

where the new quantities Δ'_Φ and Δ'_ψ are given by

$$\Delta'_\Phi = \frac{-\Pi_\Phi}{1 - 2G_i \Pi_\Phi} , \quad \Pi_\Phi = \mathrm{i}\, \mathrm{tr}(S_{m'} \mathcal{M}_{\mathrm{M}} S_{m'} \mathcal{M}_{\mathrm{M}}) \tag{26}$$

and

$$\Delta'_\psi = \frac{\mathrm{i}(\Delta_\omega^{(1)} S_m + T_m)}{1 + \mathrm{i} O_i (\Delta_\omega^{(1)} S_m + T_m)} . \tag{27}$$

They describe quark–antiquark and quark–diquark Green functions, respectively (see figs 3, 4) [#6].

Δ'_ψ fulfils the inhomogeneous Bethe-Salpeter-equation

$$\Delta'_\psi = \mathrm{i}(\Delta_\omega^{(1)} S_m + T_m) - \mathrm{i}(\Delta_\omega^{(1)} S_m + T_m) O_i \Delta'_\psi . \tag{28}$$

[#5] Here the constant C contains the determinant $\det(\Delta'_\psi)^{-1}$.

[#6] For convenience, here and in other pictures quark and diquark lines in the initial and final states are not drawn together into a point.

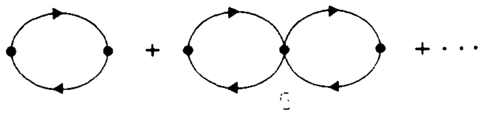

Fig. 3. Lowest order contributions to the meson Green function Δ'_Φ, eq. (26).

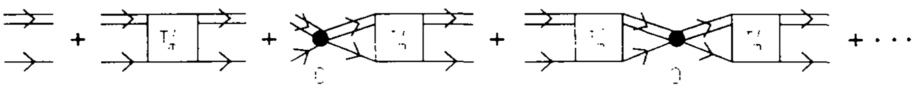

Fig. 4. Lowest order contributions to the baryon Green function Δ'_ψ, eq. (27), with T'_m defined in eq. (29).

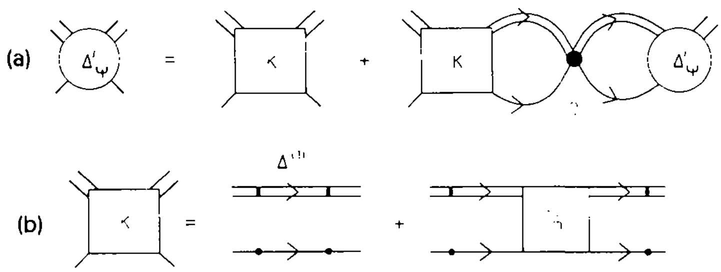

Fig. 5. (a) Inhomogeneous Bethe–Salpeter equation (28) for the quark–diquark Green function with kernel K; (b) Definition of the kernel K.

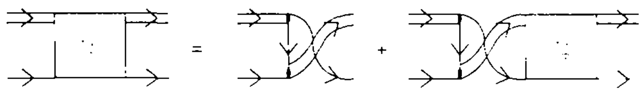

Fig. 6. Bethe–Salpeter equation for quark exchange defined according to eq. (29).

This equation is illustrated in figs. 5a, 5b. The quantity T_m defined by (25) can be represented in the form

$$T_m \equiv S_m \Delta_\omega T'_m S_m \Delta_\omega \,, \quad T'_m = \frac{V}{1-\Delta_\omega S_m V}\,, \tag{29}$$

with

$$V = -4\mathrm{i}\mathcal{M}_{\mathrm{D}} S_m \mathcal{M}_{\mathrm{D}}\,.$$

It describes a Bethe–Salpeter ladder of quark exchange diagrams shown in fig. 6. The corresponding bound state spectrum has been investigated in ref. [10]. Note, that the baryon pole of the quark-exchange amplitude T_m is not present in the quark–diquark Green function Δ'_ψ. Since the denominator in (27) will have a zero near the pole position of T_m, there should appear a shifted pole in Δ'_ψ. Thus, the effect of the interaction term containing O_i consists in shifting the baryon spectrum which is generated from quark-exchange diagrams alone. Numerical calculations of the shifted mass spectra for different O_i are under way. Besides one should calculate the coupling constants of the meson-baryon interaction for concrete couplings O_i.

In conclusion, we have studied an extended NJL-type model which includes as a novel feature a six-quark interaction term modelling quark–diquark interactions. Integrating over the intermediate diquarks an effective meson–baryon action with meson and baryon propagators was obtained.

References

[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345.
[2] D. Ebert and V.N. Pervushin, Theor. Math. Phys. 36 (1978) 759;
D. Ebert and L. Kaschluhn, Nucl. Phys. B 355 (1991) 123.
[3] H. Kleinert, Phys. Lett. B 62 (1976) 429; in: Understanding the fundamental constituents of matter, ed. A. Zichichi (Plenum, New York, 1978).
[4] D. Ebert and M.K. Volkov, Yad. Fiz. 26 (1982) 1265; Z. Phys. C 16 (1983) 205;
M.K. Volkov, Ann. Phys. 157 (1984) 282;
D. Eberg, Z. Phys. C 28 (1985) 433;
D. Ebert and H. Reinhardt, Nucl. Phys. B 271 (1986) 188; Phys. Lett. B 173 (1986) 453.
[5] N.I. Kartchev and A.A. Slavnov, Teor. Mat. Fiz. 65 (1985) 192;
A.A. Andrianov and Yu.V. Novozhilov, Phys. Lett. B 153 (1985) 422;
P. Simic, Phys. Rev. D 34 (1986) 1903;
D.W. McKay, H.J. Munczek and B.-L. Young, Phys. Rev. D 37 (1988) 195.
[6] R.T. Cahill, J. Praschifka and C.J. Burden, Aust. J. Phys. 42 (1989) 147, 161.
[7] D. Kahana and U. Vogl, Phys. Lett. B 244 (1990) 10;
U. Vogl, Z. Phys. A 337 (1990) 191.
[8] D. Ebert, L. Kaschluhn and G. Kastelewicz, Phys. Lett. B 264 (1991) 420.
[9] B. Stech, Phys. Rev. D 36 (1987) 975.
[10] R.T. Cahill, Aust. J. Phys. 42 (1989) 171.
[11] H. Reinhardt, Phys. Lett. B 244 (1990) 316.
[12] R.D. Ball, Intern. J. Mod. Phys. A 5 (1990) 4391.