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Radiative rare B decays into higher K-resonances

Ahmed Ali, Thorsten Ohl [1,2]
Deutsches Elektronen-Synchrotron DESY, W-2000 Hamburg, FRG

and

Thomas Mannel
Technische Hochschule Darmstadt, W-6100 Darmstadt, FRG

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We estimate the contributions of higher K-resonances to the radiative rare decays $b \to s\gamma$. We use the spin symmetry for heavy quarks to reduce the number of independent form factors to four Isgur–Wise functions, which are estimated from the wave function model of Isgur, Scora, Grinstein, and Wise. Our results suggest a substantially larger branching fraction for the decay $B \to K_2^*(1430)\gamma$ than previous investigations.

1. Introduction

The heavy quark limit has proven to be a useful tool to obtain model independent information on systems containing heavy quarks [1–3]. In the limit $m_Q \to \infty$ (m_Q being the mass of the heavy quark) two additional symmetries beyond those of QCD arise. The first one is the so-called heavy flavor symmetry, the masses of the heavy quarks may be scaled out and the lagrangian is the same for all heavy flavors. Thus there is an $SU(N_f)$ symmetry of rotations among the heavy quarks. The second symmetry is the spin symmetry. In the limit of infinitely heavy quarks the spin degrees of freedom of the heavy quark decouple and the SU(2) rotations of the heavy quark spin become a symmetry. This means that the spectrum of heavy hadrons should show degenerate spin symmetry doublets; for the case of heavy mesons built of a heavy quark and a light antiquark the pseudoscalar (0^-) and the corresponding vector mesons (1^-) form such a spin symmetry doublet.

These additional symmetries allow many interesting predictions. In particular, they imply model independent relations between form factors for weak decays. For instance, the semileptonic B decays into the lowest spin symmetry doublet of D-mesons (i.e. the D and D^*) are described in terms of only one independent form factor; in addition one obtains a model independent statement about the normalization of this form factor at maximum momentum transfer. Also the excited states of heavy hadrons fall into spin symmetry doublets in the heavy quark limit [4]. The corresponding group theory has been elaborated in ref. [5]. The main result is that for mesonic transitions from a heavy 0^- ground state meson to a heavy excited meson only one independent form factor per spin symmetry doublet appears.

Heavy to heavy transitions of a 0^--meson into excited states have been considered in refs. [4,6] for the case of semileptonic $B \to D^{**}$ decays. Although the s-quark is not particularly heavy and very substantial corrections to the Isgur–Wise functions are to be anticipated, particularly at the symmetry point, yet we feel that the heavy quark symmetry can be used gainfully to relate various form factors. The heavy quark limit has been applied to

[1] Supported by a grant of Deutsche Forschungsgemeinschaft.
[2] Address after 1 September 1992: Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA.

Table 1
Spectrum of excited neutral K-mesons. Notation, masses, and assignment of quantum numbers are taken from ref. [9], except for the quantities in brackets. The systematical and statistical errors on the masses have been added in quadrature

Name	State	J^P	$n^{2s+1}L_J$	$[j_{light}]$	Mass/MeV
K	C	0^-	1^1S_0	[1/2]	497.67 ± 0.03
$K^*(892)$	C^*	1^-	1^3S_1	[1/2]	896.1 ± 0.3
$K_1(1270)$	E^*	1^+	$1^1P_1/1^3P_1$	[1/2]	1270 ± 10
$K_1(1400)$	F	1^+	$1^1P_1/1^3P_1$	[3/2]	1402 ± 7
$K^*(1410)$	C_2^*	1^-	2^3S_1	[1/2]	1412 ± 12
$K^*(1430)$	E	0^+	1^3P_0	[1/2]	1429 ± 7
$K_2^*(1430)$	F^*	2^+	1^3P_2	[3/2]	1425.4 ± 1.3
$K(1460)$	C_2	0^-	2^1S_0	[1/2]	≈ 1460
$K_2(1580)$	G^*	2^-	$[1^1D_2/1^3D_2]$	[3/2]	≈ 1580
$K_1(1650)$		1^+	$[2^1P_1]$	[1/2]	1650 ± 50
$K^*(1680)$	G	1^-	1^3D_1	[3/2]	1714 ± 20
$K_2(1770)$		2^-	$[1^1D_2/]1^3D_2$	[5/2]	1768 ± 14
$K_3^*(1780)$		3^-	1^3D_3	[5/2]	1770 ± 10
$K(1830)$		0^-	3^1S_0	[1/2]	≈ 1830
$K_0^*(1950)$		0^+	$[2^3P_0]$	[1/2]	1945 ± 32
$K_2^*(1980)$		2^+			1975 ± 22
$K_4^*(2045)$		4^+	1^3F_4		2045 ± 9
$K_2(2250)$		2^-			2247 ± 17
$K_3(2320)$		3^+			2324 ± 24
$K_5^*(2380)$		5^-			2382 ± 24
$K_4(2500)$		4^-			2490 ± 20

some exclusive rare $b \to s$ decays [7] in order to obtain an estimate of the corresponding branching fractions, where it has been determined that the branching ratios depend on the extrapolation of the Isgur–Wise function $\xi(vv')$ into the region of interest $vv' \gg 1$. This requires a reasonable model for the wave functions, for which we take the Grinstein–Isgur–Scora–Wise (GISW) model [8]. In the present paper we extend the work of ref. [7] to the higher resonances in the K system. We shall focus on the two body decays $B \to K^*\gamma$ and $B \to K^{**}\gamma$ since they are expected to have larger branching fractions than the corresponding short distance contributions to the leptonic decays $B \to K^{**}l^+l^-$.

A rich spectrum of states has been observed in the K system. In table 1 we summarize the present knowledge about the excited K-mesons [9]. We use the standard notation for the various quantum numbers. The radial excitation quantum number is denoted by $n = 1, 2, 3, \ldots$. $L = (S=0), (P=1), (D=2), \ldots$ is the orbital angular momentum, $s = 0, 1$ is the sum of the spins of the two quarks in the meson and finally $J = 1, 2, 3, \ldots$ is the total spin of the meson. The parity is given by the orbital angular momentum to be $P = (-1)^{L+1}$.

The paper is organized as follows. After a brief review of the spin symmetry formalism for higher resonances we shall assign spin symmetry doublets to the states listed in table 1. Based on the effective hamiltonian from ref. [10], we calculate the resulting branching ratios in section 3. The undetermined Isgur–Wise functions will be estimated from the GISW wave function model [8] in section 4. Finally we compare our estimates with a previous prediction from a wave function model [11], which does not take into account the spin symmetry.

2. Matrix representation of the states

Heavy quark symmetry will lead to relations between form factors which may be formulated in terms of a Wigner–Eckart theorem. The calculation of the relevant Clebsch–Gordan coefficients and the counting of the number of independent form factors is conveniently done in the framework of the trace formalism which was formulated in refs. [2,12,13] and generalized to excited states in ref. [5].

Following ref. [5] we write for the lowest lying states

$$C(v) = \tfrac{1}{2}\sqrt{m}\,(\not{v}+1)\gamma_5, \quad J^P = 0^-, \tag{1}$$

$$C^*(v,\epsilon) = \tfrac{1}{2}\sqrt{m}\,(\not{v}+1)\not{\epsilon}, \quad J^P = 1^-, \tag{2}$$

$$E(v) = \tfrac{1}{2}\sqrt{m}\,(\not{v}+1), \quad J^P = 0^+, \tag{3}$$

$$E^*(v,\epsilon) = \tfrac{1}{2}\sqrt{m}\,(\not{v}+1)\gamma_5\not{\epsilon}, \quad J^P = 1^+, \quad j_{\rm light} = \tfrac{1}{2}, \tag{4}$$

$$F(v,\epsilon) = \tfrac{1}{2}\sqrt{m}\sqrt{\tfrac{3}{2}}\,(\not{v}+1)\gamma_5[\epsilon^\mu - \tfrac{1}{3}\not{\epsilon}(\gamma^\mu - v^\mu)], \quad J^P = 1^+, \quad j_{\rm light} = \tfrac{3}{2}, \tag{5}$$

$$F^*(v,\epsilon) = \tfrac{1}{2}\sqrt{m}\,(\not{v}+1)\gamma_\nu\epsilon^{\mu\nu}, \quad J^P = 2^+, \quad j_{\rm light} = \tfrac{3}{2}, \tag{6}$$

$$G(v,\epsilon) = \tfrac{1}{2}\sqrt{m}\sqrt{\tfrac{3}{2}}\,(\not{v}+1)[\epsilon^\mu - \tfrac{1}{3}\not{\epsilon}(\gamma^\mu + v^\mu)], \quad J^P = 1^-, \quad j_{\rm light} = \tfrac{3}{2}, \tag{7}$$

$$G^*(v,\epsilon) = \tfrac{1}{2}\sqrt{m}\,(\not{v}+1)\gamma_5\gamma_\nu\epsilon^{\mu\nu}, \quad J^P = 2^-, \quad j_{\rm light} = \tfrac{3}{2}. \tag{8}$$

Here m and v are the mass and the velocity of the heavy meson, $j_{\rm light}$ denotes the total spin of the light degrees of freedom and the tensor $\epsilon^{\mu\nu}$ is the polarization of a spin 2 object with $\epsilon^{\mu\nu}=\epsilon^{\nu\mu}$ and $\epsilon^{\mu\nu}v_\nu=0$. These eight states fall into four spin symmetry doublets. The two ground state mesons $C(v)$ and $C^*(v,\epsilon)$ form the first one, $E(v)$ and $E^*(v,\epsilon)$ the second, $F(v,\epsilon)$ together with $F^*(v,\epsilon)$ is the third, and $G(v,\epsilon)$ forms the fourth with $G^*(v,\epsilon)$.

Matrix elements of bilinear currents of two heavy quarks are calculated by taking the trace:

$$\langle \mathscr{H}'(v')|\bar{h}'_{v'}\Gamma h'_v|\mathscr{H}(v)\rangle = {\rm Tr}\{\mathscr{H}'(v')\Gamma\mathscr{H}(v)\mathscr{M}(v,v')\}, \tag{9}$$

where $\mathscr{H}$ denotes any of the representation matrices (1)–(8). The matrix $\mathscr{M}$ represents the light degrees of freedom. As discussed in ref. [5] this matrix may be expressed in terms of only one independent form factor for each spin symmetry doublet. We define the Isgur–Wise functions for the transitions of a 0^- ground state meson into an excited meson by

$$\mathscr{M}(v,v') = \xi_C(vv'), \quad \text{for } 0^- \to (0^-, 1^-) = (C, C^*), \tag{10}$$

$$\mathscr{M}(v,v') = \xi_E(vv'), \quad \text{for } 0^- \to (0^+, 1^+) = (E, E^*), \tag{11}$$

$$\mathscr{M}(v,v') = \xi_F(vv')v_\mu, \quad \text{for } 0^- \to (1^+, 2^+) = (F, F^*), \tag{12}$$

$$\mathscr{M}(v,v') = \xi_G(vv')v_\mu, \quad \text{for } 0^- \to (1^-, 2^-) = (G, G^*), \tag{13}$$

where the vector index in the last two equations will be contracted with the one in the representations of the excited mesons, cf. eqs. (5)–(8). Note that $\xi_C=\xi$ is the usual Isgur–Wise function with $\xi(vv'=1)=1$. We shall also consider a decay into a radially excited K-meson which has the same spin parity quantum numbers as the ground state. The corresponding Isgur–Wise function will be denoted by ξ_{C2}.

Only for the lowest lying spin symmetry doublet the normalization of the corresponding Isgur–Wise function is known. The value at the normalization point $vv'=1$ of the Isgur–Wise functions for the higher spin symmetry doublets may be related to the slope of the Isgur–Wise function of the lowest doublet using the Bjorken sum rule techniques [2,14]. However, the applications discussed below involve the Isgur–Wise functions at values of vv' much in excess of 1, and hence the information at and near the normalization point is not very useful in the present context.

We shall discuss rare B decays into excited K-mesons assuming that the s-quark may be treated in the static limit. Thus we have to assign the states listed in table 1 to the members of the spin symmetry doublets (1)–(8). The assignment of the resonances to spin symmetry doublets is not unique. We shall choose our assignment of the excited K-mesons to the spin symmetry doublets in the following way: we put the $K_1(1270)$ and the $K_2^*(1430)$ into the spin symmetry doublet (E, E^*) and correspondingly the $K_1(1400)$ and the $K(1430)$ into

(F, F^*). In particular, this assignment of the states $K_1(1270)$ and $K_1(1400)$ means that due to spin symmetry these will be a mixture of the quark model states 1^1P_1 and 1^3P_1 with a mixing angle of 45°. In fact, this value of the mixing angle is consistent with experimental results [15], which in turn supports our assignment of the states to the spin symmetry doublets. A further motivation for our assignment is the fact that in the D system the lowest lying 1^+ state is the spin symmetry partner of the lowest 2^+ state. We shall put the $K^*(1680)$ and the $K_2(1580)$ into a fourth spin symmetry doublet (G, G^*), assuming that they have both orbital angular momentum $L=2$. Finally we assume that the $K^*(1460)$ and $K^*(1410)$ are radially excited states, which form a fifth spin symmetry doublet $(C, C^*)_2$.

Two more spin symmetry doublets could be formed by $K_0^*(1950)$, $K_1(1650)$ $(E, E^*)_2$ and $K_2(1770)$, $K_3^*(1780)$ (F, F^*). However, we shall neither consider decays into radially excited states (except for the low lying $(C, C^*)_2$ doublet), nor decays into states with $J\geqslant 3$.

3. Radiative rare *B* decays into excited *K*-meson states

The radiative rare decays are mediated by the effective hamiltonian [10]

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* C_7(m_b)\mathcal{O}_7(m_b) , \tag{14}$$

where the operator $\mathcal{O}_7$ is given by

$$\mathcal{O}_7 = \frac{e}{32\pi^2} F_{\mu\nu}[m_b \bar{s}\sigma^{\mu\nu}(1+\gamma_5)b + m_s\bar{s}\sigma^{\mu\nu}(1-\gamma_5)b] , \tag{15}$$

and the Wilson coefficient $C_7(m_b)$ is

$$C_7(m_b) = \eta^{-16/23}[C_7(M_W) + \tfrac{58}{135}(\eta^{10/23}-1) + \tfrac{29}{189}(\eta^{28/23}-1)] , \tag{16}$$

with $\eta=\alpha_s(m_b)/\alpha_s(M_W)$ and

$$C_7(M_W) = \frac{x}{2}\left(\frac{2x^2/3+5x/12-7/12}{(x-1)^3} - \frac{3x^2/2-x}{(x-1)^4}\ln x\right), \quad x = \frac{m_t^2}{M_W^2} . \tag{17}$$

The matrix element of $\mathcal{O}_7$ between a B-meson in the initial state and a photon and a generic K^{**}-meson in the final state may be written as

$$\langle \mathcal{K}(v'), \gamma(q,\eta)|\mathcal{O}_7|B(v)\rangle = \frac{e}{16\pi^2}\eta_\mu\langle \mathcal{K}(v'), \gamma(q,\eta)|\Omega^\mu|B(v)\rangle , \tag{18}$$

where $\mathcal{K}$ denotes any of the six states represented by (1)–(8),

$$\Omega_\mu = m_B\bar{s}\sigma_{\mu\nu}(1+\gamma_5)bq^\nu + m_{K^{**}}\bar{s}\sigma_{\mu\nu}(1-\gamma_5)bq^\nu, \quad q = m_Bv - m_{K^{**}}v' , \tag{19}$$

and η_μ is the polarization vector of the photon.

Using the mass shell condition of the photon,

$$q^2 = m_B^2 + m_{K^{**}}^2 - 2vv'm_Bm_{K^{**}} = 0 , \tag{20}$$

and the polarization sums for spin-1 and spin-2 particles,

$$M_{\mu\nu}^{(1)}(v) = -g_{\mu\nu} + v_\mu v_\nu , \tag{21}$$

$$M_{\mu\nu,\rho\sigma}^{(2)}(v) = \tfrac{1}{2}M_{\mu\rho}^{(1)}(v)M_{\nu\sigma}^{(1)}(v) + \tfrac{1}{2}M_{\mu\sigma}^{(1)}(v)M_{\mu\sigma}^{(1)}(v) - \tfrac{1}{3}M_{\mu\nu}^{(1)}(v)M_{\rho\sigma}^{(1)}(v) , \tag{22}$$

we arrive at the following branching ratios:

$$\Gamma(B\to K^*\gamma) = \Omega|\xi_C(vv')|^2 \frac{1}{y}[(1-y)^2(1+y)^4(1+y^2)] , \tag{23}$$

$$\Gamma(B\to K_1(1270)\gamma) = \Omega|\xi_E(vv')|^2 \frac{1}{y}[(1-y)^4(1+y)^2(1+y^2)] , \tag{24}$$

$$\Gamma(B\to K_1(1400)\gamma) = \Omega|\xi_F(vv')|^2 \frac{1}{24y^3}[(1-y)^4(1+y)^6(1+y^2)] , \tag{25}$$

$$\Gamma(B\to K_2^*(1430)\gamma) = \Omega|\xi_F(vv')|^2 \frac{1}{8y^3}[(1-y)^4(1+y)^6(1+y^2)] , \tag{26}$$

$$\Gamma(B\to K^*(1680)\gamma) = \Omega|\xi_G(vv')|^2 \frac{1}{24y^3}[(1-y)^6(1+y)^4(1+y^2)] , \tag{27}$$

$$\Gamma(B\to K_2(1580)\gamma) = \Omega|\xi_G(vv')|^2 \frac{1}{8y^3}[(1-y)^6(1+y)^4(1+y^2)] , \tag{28}$$

$$\Gamma(B\to K^*(1410)\gamma) = \Omega|\xi_{C_2}(vv')|^2 \frac{1}{y}[(1-y)^2(1+y)^4(1+y^2)] . \tag{29}$$

The argument vv' of the Isgur–Wise functions is fixed by the condition (20)

$$vv' = \frac{m_B^2 + m_{K^{**}}^2}{2m_B m_{K^{**}}} , \tag{30}$$

and we have used the following abbreviations:

$$\Omega = \frac{\alpha}{128\pi^4} G_F^2 m_b^5 |V_{tb}|^2 |V_{ts}|^2 |\mathscr{C}_7(m_b)|^2, \quad y = \frac{m_{K^{**}}}{m_B} . \tag{31}$$

Since the decays into states in the same spin symmetry doublet are described by a single Isgur–Wise function, spin symmetry relates these decays. From (23)–(29) we find

$$\Gamma(B\to K_2^*(1430)\gamma) = 3\Gamma(B\to K_1(1400)\gamma), \quad \Gamma(B\to K_2(1580)\gamma) = 3\Gamma(B\to K^*(1680)\gamma) . \tag{32}$$

In the heavy quark limit the two members of a spin symmetry doublet should be degenerate in mass, which means that the corresponding Isgur–Wise functions would be taken at the same value of vv' and the ratio y would be the same for both states. However, for the s-quark we expect a large breaking of the spin symmetry, which means that the two relations (32) will be only approximate. From the numerical estimates presented in the next section one finds that the relations (32) indeed hold within the indicated accuracy.

4. Wave function model for the Isgur–Wise functions

Since we are dealing with two body decays, the product vv' is fixed by kinematics and hence the Isgur–Wise functions have to be taken at this point. Due to the large mass difference between the B and the excited K^{**}-mesons, the vv' are not close to unity and we shall use a model to extrapolate from the point where the normalization of the usual Isgur–Wise function is known.

We shall employ the wave function model of Grinstein, Scora, Isgur and Wise [8] to determine the Isgur–Wise functions in (23)–(29). In the context of this model the Isgur–Wise functions $\xi_{C,E,F,G}$ may be extracted from the overlap integral

$$I(v') = \int \frac{d^3p}{(2\pi)^3} \tilde{\Phi}_F^*(p+\Lambda v')\tilde{\Phi}_I(p) = \int d^3x\, \Phi_F^*(x)\Phi_I(x)\exp(-i\Lambda v'\cdot x) , \qquad (33)$$

where the labels I and F denote the wave function of the initial and final meson, respectively, and the "inertia parameter" Λ corresponds to the mass of the light degrees of freedom. In fact, since the K-mesons are not particularly heavy, we shall use for Λ the expression [11]

$$\Lambda = \frac{m_{K^{**}} m_d}{m_s + m_d} , \qquad (34)$$

which accounts for the kinematic effects of a finite s-quark mass. The quark masses are taken to be

$$m_d = 330\ \mathrm{MeV}, \quad m_s = 550\ \mathrm{MeV} . \qquad (35)$$

The wave functions are chosen to be eigenfunctions of orbital angular momentum L, where the initial meson will have $L=0$; thus the wave functions are given by

$$\Phi_I(x) = Y_0^0\left(\frac{x}{|x|}\right)\phi_I(|x|), \quad \Phi_F(x) = Y_L^m\left(\frac{x}{|x|}\right)\phi_F(|x|) , \qquad (36)$$

with the normalization

$$1 = \int d^3x\, \Phi_{I/F}^*(x)\Phi_{I/F}(x) = \int dr\, r^2 \phi_{I/F}^*(r)\phi_{I/F}(r) . \qquad (37)$$

Inserting the wave functions (36) into the overlap integral (33) and choosing the quantization axis of the orbital angular momentum in the direction of the velocity v, the overlap (33) vanishes for $m\neq 0$. For $m=0$ the overlap integral becomes

$$I(v') = \sqrt{2L+1}\, i^L \int r^2 dr\, \phi_F^*(r)\phi_I(r) j_L(\Lambda r|v'|) , \qquad (38)$$

where j_L is the spherical Bessel function of order L. This expression holds in the rest frame of the heavy meson; in a general frame (38) becomes

$$I(vv') = \sqrt{2L+1}\, i^L \int r^2 dr\, \phi_F^*(r)\phi_I(r) j_L[\Lambda r\sqrt{(vv')^2-1}] . \qquad (39)$$

For $L=0$ the overlap integral is the usual Isgur–Wise function, which is correctly normalized at $v\cdot v'=1$, since $j_0(0)=1$. The overlap integrals for $L=1$ and $L=2$ are identified with the remaining Isgur–Wise functions $\xi_{E,F,G}$:

$$\xi_E(v\cdot v') = \xi_F(v\cdot v') = \sqrt{3}\, i \int r^2 dr\, \phi_F^*(r)\phi_I(r) j_1[\Lambda r\sqrt{(v\cdot v')^2-1}] , \qquad (40)$$

$$\xi_G(v\cdot v') = -\sqrt{5} \int r^2 dr\, \phi_F^*(r)\phi_I(r) j_2[\Lambda r\sqrt{(v\cdot v')^2-1}] . \qquad (41)$$

Note that these functions will vanish at $v\cdot v'=1$ like $[(v\cdot v')^2-1]^{L/2}$, since the final state wave function is orthogonal to the one of the initial state.

To calculate the overlap integrals we insert the radial wave functions of the GISW model [8] into eq. (39). These correspond to harmonic oscillator wave functions with oscillator strengths β_K and β_B. The values fitted in ref. [8] to the semileptonic B and D decays (β_K=0.34 GeV, β_B=0.41 GeV) are not equal. This breaking of the heavy flavor symmetry causes a violation of the normalization condition $\xi_C(1)=1$. To estimate the model dependence and the effect of the breaking of heavy quark symmetry we use the same parameter β for both wave functions and vary it between β_K and β_B. The resulting ranges for the Isgur–Wise functions are shown in the shaded bands of fig. 1. For comparison, we have also plotted the Isgur–Wise function ξ_C as calculated in ref. [16] from the model of ref. [17]. In the region of interest for the decays $B\to K^*\gamma$, $B\to K^{**}\gamma$ discussed in this

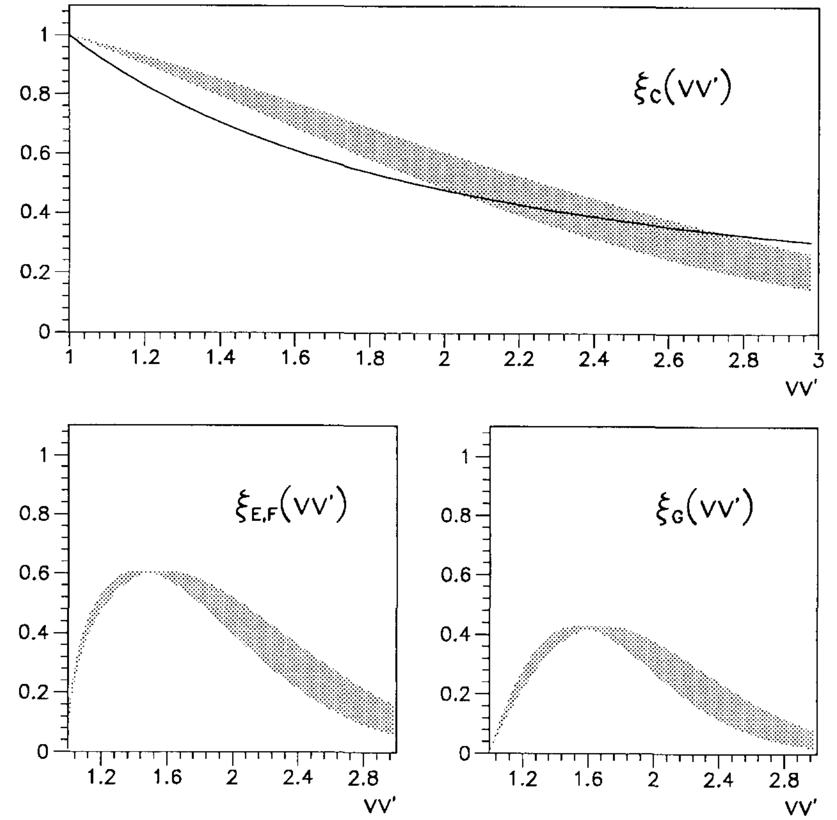

Fig. 1. Plot of the Isgur–Wise functions ξ_C, $\xi_{E,F}$, ξ_G as a function of the variable vv'. The shaded bands give the range of our estimate as obtained from the model of ref. [8] by varying the parameter β as discussed in the text. For comparison, the Isgur–Wise function ξ_C calculated from the model of ref. [17] is shown by the solid curve.

paper, the variable vv' lies in the range $1.70 \leqslant vv' \leqslant 3.05$. We see from fig. 1 that the Isgur–Wise functions ξ_C determined from the two models are very similar in this range, particularly for $vv' \geqslant 2$.

Table 2 shows our estimates for the exclusive $B \to K^{**}\gamma$ branching ratios. Our results are given in units of the inclusive branching ratio $B \to X_s\gamma$, which is estimated by the QCD improved $b \to s\gamma$ quark decay rate

$$\Gamma(B \to X_s\gamma) \approx \Gamma(b \to s\gamma) = 4\Omega(1-y)^3(1+y)^3(1+y^2)\,, \tag{42}$$

which corresponds to an inclusive branching ratio $\mathrm{BR}(B \to X_s\gamma) = (3\text{–}5) \times 10^{-4}$ [18]. For the numerical values of the $B \to K^{**}\gamma$ branching ratios in table 2 we have used $\mathrm{BR}(B \to X_s\gamma) = 4 \times 10^{-4}$, corresponding to a top quark mass of about 150 GeV. The ranges are obtained from the variation of the model parameters as mentioned above.

To compare the numbers in table 2 with the results of ref. [11], we have to note first that in the meantime the experimental information on the K^{**}-resonances has increased considerably. Taking into account updated mass values, our results are generally in agreement with ref. [11]. However, there is a noticeable difference in our prediction for the branching fraction into $K_2^*(1430)$, which is from our estimate about a factor of three larger compared to ref. [11]. Our result is consistent with spin symmetry, because it satisfies the general relations (32) which hold without model dependent input for the Isgur–Wise functions.

In quark model calculations [11], the decay into the $1\,^1P_1$ state (K_{1B}) is forbidden, because $\mathcal{O}_7$ is a spin-flip operator. In our analysis both $K_1(1270)$ and $K_1(1400)$ contribute since they are states with good angular mo-

Table 2
Values of the Isgur–Wise function at the indicated value of vv', the ratio $R=\Gamma(B\to K^{**}\gamma)/\Gamma(B\to X_s\gamma)$, and the branching ratio $BR(B\to K^{**}\gamma)$ for the various excited K^{**}-mesons with the indicated mass and J^P.

Name	State	J^P	vv'	ξ	R(%)	BR$\times 10^5$
K	C	0^-	5.346	0.125–0.239	forbidden	forbidden
K^*	C^*	1^-	3.030	0.136–0.253	3.5–12.2	1.4–4.9
$K^*(1430)$	E	0^+	1.982	0.309–0.453	forbidden	forbidden
$K_1(1270)$	E^*	1^+	2.198	0.296–0.441	4.5–10.1	1.8–4.0
$K_1(1400)$	F	1^+	2.015	0.307–0.451	6.0–13.0	2.4–5.2
$K_2^*(1430)$	F^*	2^+	1.987	0.309–0.452	17.3–37.1	6.9–14.8
$K^*(1680)$	G	1^-	1.702	0.359–0.420	1.1–1.5	0.4–0.6
$K_2(1580)$	G^*	2^-	1.820	0.350–0.417	4.5–6.4	1.8–2.6
$K(1460)$	C_2	0^-	1.946	−0.242– −0.293	forbidden	forbidden
$K^*(1410)$	C_2^*	1^-	2.005	−0.240– −0.292	7.2–10.6	2.9–4.2
sum					44.1–90.9	17.6–36.4

mentum of the light degrees of freedom and thus they both contain a 1^3P_1 component. The two channels $K^*(892)$ and $K_2^*(1430)$ are the easiest channels to observe, since the efficiency for the reconstruction of the decaying K^*-meson is high in both cases [19]. In particular the $K_2^*(1430)$ is a very interesting candidate, since our model calculation yields a very high branching fraction into this channel.

5. Conclusions

We have studied the predictions of the heavy quark symmetry for the rare decays of B-mesons into higher K-resonances $B\to K^{**}\gamma$. In contrast to earlier calculation, we find that a substantial fraction ((17–37)%) of the inclusive $b\to s\gamma$ branching ratio goes into the $K_2^*(1430)$ channel, which should be relatively easy to observe.

Not much is known about the Isgur–Wise functions for the different spin symmetry doublets. To obtain more information one needs to use model dependent input. We have employed a specific model [8] to calculate the Isgur–Wise functions for the various spin symmetry doublets. To estimate the uncertainties introduced by the model we have varied the model parameters within a range corresponding to the breaking of heavy quark symmetry. In defense of the GISW model, we can add that its predictions for the D^* and higher resonances in the semileptonic B decays $B\to D^*l\nu_l$ and $B\to D^{**}l\nu_l$ are in reasonable agreement with recent data [20,21].

Since the decays studied here are all two body decays, the corresponding Isgur–Wise functions are taken at a single point, as vv' is fixed by kinematics. Due to the large mass difference between the B-meson and all the K-resonances the values for vv' range between 2 and 3 and thus a considerably large extrapolation off the normalization point $vv'=1$ is necessary. It remains to be tested whether the representation of the Isgur–Wise functions assumed here is trustworthy. Radiative rare B decays $B\to K^*\gamma$, $B\to K^{**}\gamma$ provide interesting avenues for checking the predictions of the heavy quark symmetry combined with meson wave function models.

We expect that the estimates presented in this paper should hold within the indicated range; in particular we expect a relatively large branching fraction for the channel $K_2^*(1430)$ which is according to our estimate a factor of 3 to 4 larger than the one for the $K^*(892)$. The resulting hadron mass spectra are also in qualitative agreement with the estimates for the inclusive radiative B decays $B\to X_s\gamma$ [18].

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