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Unitarity corrections to the Lipatov pomeron and the small-x region in deep inelastic scattering in QCD

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We present the first corrections to the BFKL pomeron and discuss some of their properties, in particular their application to the small-x region of deep inelastic scattering. We find evidence that the fan diagrams of the Gribov–Levin–Ryskin equation are not the most dominant ones.

1. It is now more and more being realized that the small-x region in deep inelastic scattering is one of the new frontiers in perturbative QCD. HERA will soon allow to measure, for the first time, structure functions for Bjorken-x values down to 10^{-4}. Later on, also the hadron colliders LHC and the SSC will face the low-x behavior of structure functions. From the theoretical side, the small-x region represents the challenge of studying the interface between perturbative and nonperturbative QCD, with the peculiar feature that this transition is taken in a kinematic region where α_s is small.

It has been pointed out in several papers and articles [1,2] that the breakdown in the low-x region of conventional perturbative QCD can be made visible, for example, in the operator expansion: when $\ln(1/x_B)$ is of the order of $\ln(Q^2/\Lambda^2)$ (for illustration see fig. 1), terms of higher twist which in the canonical treatment of the Bjorken limit are neglected because of the $1/Q^2$ suppression, become as important as the leading-twist contribution. In this kinematic region the parton model becomes more and more complicated and requires serious modifications (e.g. parton recombination and annihilation terms). Alternatively, the small-x region of deep inelastic scattering can be seen as the analytic continuation of the Regge limit ($1/x \approx s/Q^2$: it is the limit $s \to \infty$ at large Q^2). In contrast to the usual Regge limit at low Q^2 where the use of perturbation theory could never by justified, one is now in a much better position: since

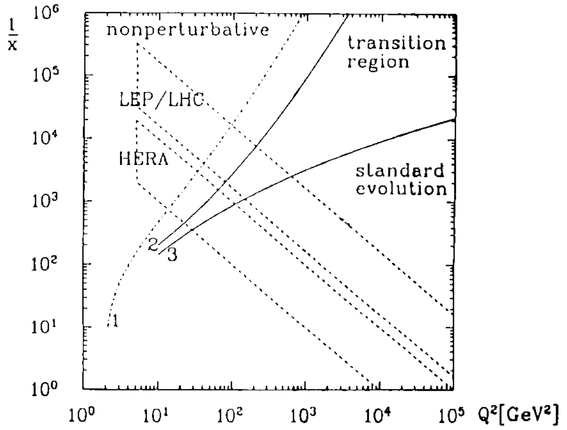

Fig. 1. The $\ln Q^2$–$\ln(1/x_B)$ plane. The line marks the expected end of perturbative QCD (from ref. [3]).

$\alpha_s(Q^2)$ is small at least the beginning (i.e. the region where x is not yet so small) lies in the region of validity of perturbation theory. The Regge limit feels most strongly unitarity. It is therefore natural to expect that the small-x limit of DIS first witnesses a reordering of the relevant terms in the perturbation expansion: instead of the operator expansion in inverse powers of Q^2, unitarity in both the x-channel and the t-channel now take over as the guiding principle. Ultimately, i.e. at very small x, nonperturbative contributions must come in. One might hope, however, that a careful investigation of perturbative unitarity will give insight into this nonperturbative dynamics.

Pioneering work on QCD in the low-x region has been done more than ten years ago by Gribov, Levin and Ryskin [4]. Starting from the standard QCD framework in DIS, they investigated which set of Feynman diagrams have to be added in the small-x region, and they suggested to modify the usual linear QCD evolution equations by simply adding a nonlinear term. This can be interpreted as a first step towards unitarizing the usual one-ladder approximation. The effect of this new term is a lowering of the increase of the structure at small Bjorken-x. In the limit $x_B\to\infty$, the structure function approaches a constant limit (saturation). Because of the importance of this equation, in particular for practical applications, further studies of its validity are urgent.

In this letter we report on an attempt to approach the small-x limit of DIS coming from above, i.e. from the Regge limit. The leading-ln s approximation, the so-called Balitzky–Fadin–Kuraev–Lipatov [5] (BFKL) pomeron, has been known by now since more than 15 years. With its fixed angular momentum plane cut to the right of 1 it violates s-channel unitarity. Its connection with DIS is well-understood [4]. We have calculated the first corrections to this approximation, observing both t-channel (reggeon unitarity) and (asymptotic) s-channel unitarity. The main motivation comes from the hope that, because of the reggeization of the gluon, it should be possible to derive a full reggeon field theory which satisfies unitarity in both the direct and the exchange channel. The results presented in this paper in fact support this hope, although a few essential steps are still missing. A solution to such a (effective) field theory should bring in nonperturbative contributions.

As a first application, we have analyzed the limit of large transverse momenta of these new corrections to the BFKL pomeron. When this limit is taken in the BFKL pomeron, it leads to the standard evolution equation of the gluon structure function (in the double logarithmic approximation). In the same way one would expect that the first corrections should lead to the first fan diagram of the GLR equation. Our result does not quite confirm this expectation: the matrix of anomalous dimensions which governs the Q^2-evolution of the four gluon operator is found to have an eigenvalue larger than predicted by the GLR equation. Numerically the difference is not large, thus giving rise to the hope that the numerical estimates which are all based upon the GLR equation will not change by too much. The appealing simple form of the equation, however, can survive, if at all, only as an approximation. Even in the restricted transition region, QCD seems to require a more complicated evolution scheme. How this scheme will look in detail, is presently unknown and requires more work.

2. Since the high energy behavior in the Regge limit is generally most easily described in terms of cross channel partial waves, we need to calculate the partial wave for the transition 2-particles→4-(reggeized) gluons. The point of interest is angular momentum $l=1$ (or, in the language of moments, $n=1$). One starts from a six-point amplitude in the triple Regge limit (fig. 2a): $s_{12}=(p_1+p_2)^2$, $s_{1'3'}=(p_{1'}+p_{3'})^2$, $s_{122'}=(p_1+p_2-p_{2'})^2\to\infty$; $s_{122'}/s_{12}$, $s_{122'}/s_{1'3'}\ll 1$; $t_{12}=(p_1-p_{1'})^2=0$; $t_{22'}$, $t_{33'}=\mathrm{O}(1)$. The corresponding partial wave is given by a triple energy discontinuity (in s_{12}, $s_{122'}$ and $s_{1'3'}$) (fig. 2b) which is calculated from unitarity integrals. Its phase space is restricted to the multiperipheral region, and for the amplitudes $T_{2\to n}$, $T_{n\to m}$ we use the (real-valued) leading-ln s approximations. Signature in all three t-channels is introduced by adding (or subtracting) the

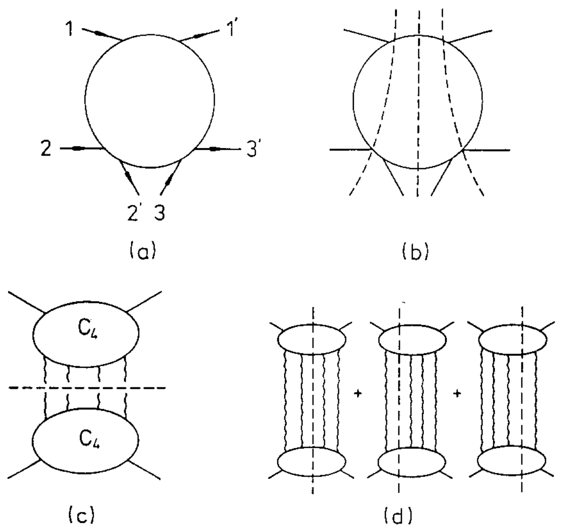

Fig. 2. (a) Six point amplitude in the triple Regge limit; (b) the triple energy discontinuity; (c) reggeon unitarity equation in the partial wave of the elastic scattering amplitude; (d) the AGK cutting rules.

proper s–u crossing terms. Finally, we move onto the nonsense pole at $l_{11'}=l_{22'}+l_{33'}-1$ and take the two-reggeon cut in the $t_{22'}$ and $t_{33'}$ channel. Inserting these "residues" C_4 into the reggeon unitarity equations [6] for the partial wave of the $2\to2$ scattering amplitude (at the tip of the four-reggeon cut), and inserting the proper signature factors, one arrives at the partial wave (fig. 2c). We mention that this construction is in agreement with the Abramovsky–Gribov–Kancheli [7] cutting rules: the resulting scattering amplitudes could also have been obtained from s-channel unitarity directly (fig. 2d). For the present purposes, however, we do not need this $2\to2$ scattering amplitudes, but only the "residue" function C_4.

Finally, a remark is in place about the completeness of these new contributions. Clearly, any set of diagrams that goes beyond the BFKL ladders leaves the leading $\log s$ approximation. The new contributions discussed in this letter present only a "minimal subset" of nonleading terms, in the sense that they are needed to satisfy unitarity equations. One of their most important is their selfconsistency. Nevertheless, this cannot be the whole story: there also exist other nonleading contributions to the elements $K_{2\to2}$ etc. (part of which will be responsible for changing the fixed coupling into a running one), and they can be obtained only by calculating new real parts. This program is being persued by V. Fadin and L. Lipatov. A similar effort will be needed, in order to include fermions.

3. Results for D_2, D_3 and D_4 are illustrated in fig. 3. At the upper end the gluon lines couple to the photon via a closed fermion loop; properties of such coupling functions have been studied in ref. [2], and further details will be presented elsewhere [8]. Moving

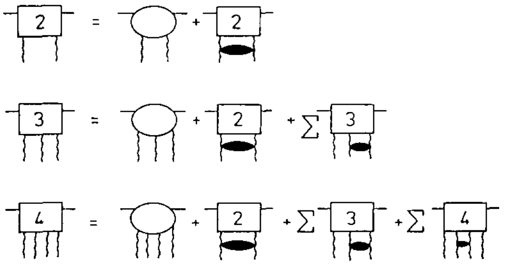

Fig. 3. The coupled equations for D_2, D_3 and D_4.

downwards, the number of gluon lines never decreases; in addition to the interaction of two gluon lines (the BFKL kernel $K_{2\to2}$), there are two new elements, $K_{2\to3}$ and $K_{2\to4}$. They have been derived first in ref. [9]. In order to write equations for these new terms, it is useful to define vertex functions D_n for 2-particles$\to n$-gluons (with $n=2, 3, 4$) (in contrast to the functions C_n from above, the D_n include a reggeon propagator, e.g. $D_4=C_4/[\omega-\Sigma_{i=1}^4\alpha(k_i)+3)]$. For the group structure we choose the coupling scheme where line 1 is combined with line 2, 3 with 4. Since the total color content is that of a singlet, both pairs of gluons must be in the same representation (1, 8_A, 8_S, $10+\overline{10}$, or 27). Furthermore, it is necessary to introduce signature in both subsystems, defined as symmetry or antisymmetry under the combined exchange of group indices and momenta. For $n=2$ we have only one possibility, D_2; this is the BFKL pomeron. For $n=3$, one pair of gluons, say (23), must be in the 8_A representation, but two signatures are possible: $D_3^{(+)}$ and $D_3^{(-)}$. The equations are

$$\begin{aligned}\omega D_3^{(-)}(k_1,k_2,k_3;\omega)\\ =D_{30}^{(-)}+\tfrac{3}{2}\sqrt{3}\,D_2\otimes K_{2\to3}(k_1\{k_2k_3\})\\ +D_3^{(-)}\otimes[-\tfrac{3}{2}K_{2\to2}(k_2k_3)-\tfrac{3}{2}K_{2\to2}(k_1k_2)\\ -\tfrac{3}{2}K_{2\to2}(k_1k_3)+\alpha(k_1)+\alpha(k_2)+\alpha(k_3)-3]\,,\\ \omega D_3^{(+)}(k_1,k_2,k_3;\omega)\\ =D_{30}^{(+)}D_2^{(+)}\otimes K_{2\to3}(k_1[k_2k_3])\\ +D_3^{(8A;+)}\otimes[-\tfrac{3}{2}K_{2\to2}(k_2k_3)-\tfrac{3}{2}K_{2\to2}(k_1k_2)\\ -\tfrac{3}{2}K_{2\to2}(k_1k_3)+\alpha(k_1)+\alpha(k_2)+\alpha(k_3)-3]\,.\end{aligned}\tag{1}$$

Finally, in the case of $n=4$, we have three sets of five-component vectors, $D_{4;i}^{(++)}$, $D_{4;i}^{(--)}$, and $D_{4;i}^{(+)}$ with $i\epsilon\{1,5\}$. The components are labelled by the different color representations. It turns out that the different signatures in D_4 decouple from each other, and we arrive at three sets of coupled equations. For the $D_4^{(++)}$:

$$\begin{aligned}\omega D_{4;i}^{(++)}=D_{40;i}^{(++)}+D_2\otimes K_{2\to4;i}^{(++)}\\ +\sum D_3\otimes K_{3\to4;i}^{(++)}+D_{4;j}^{(++)}\otimes K_{4\to4;ji}\,,\end{aligned}\tag{2}$$

where $\otimes=\mathrm{d}^2k/(2\pi)^3$. In D_4 and D_{40} the components are the color representations, e.g.

$$D_{4;1}^{(++)}=D_4^{(1;++)}(\omega;k_1,k_2,k_3,k_4)\,. \tag{3}$$

On the RHS of eq. (2), the first term stands for the coupling of gluon lines to a quark loop with two external photon lines with $(\text{mass})^2=-Q^2$. The second term has the form

$$K_{2\to4;1}^{\{++\}}=\frac{9}{\sqrt{8}}K_{2\to4}(\{k_1,k_2\}\{k_3,k_4\})\,,$$

$$K_{2\to4;2}^{\{++\}}=\tfrac{9}{4}K_{2\to4}([k_1,k_2][k_3,k_4])\,,$$

$$K_{2\to4;3}^{\{++\}}=\tfrac{9}{4}K_{2\to4}(\{k_1,k_2\}\{k_3,k_4\})\,,$$

$$K_{2\to4;4}^{\{++\}}=0\,,$$

$$K_{2\to4;5}^{\{++\}}=\frac{3\sqrt{3}}{\sqrt{8}}K_{2\to4}(\{k_1,k_2\}\{k_3,k_4\}) \tag{4}$$

(the explicit form of the kernels will be given below). In the third term, the $\sum$ includes two terms. In the first of them the five vector components are

$$-3\sqrt{\tfrac{3}{8}}\{D_3^+(k_a[k_bk_4])\otimes K_{2\to3}(\{k_1k_2\}k_3)$$
$$+D_3^+([k_1k_a]k_b)\otimes K_{2\to3}(k_2\{k_3k_4\})\}\,,$$
$$-\tfrac{3}{4}\sqrt{3}\{D_3^+(k_a[k_bk_4])\otimes K_{2\to3}([k_1k_2]k_3)$$
$$+D_3^+([k_1k_a]k_b)\otimes K_{2\to3}(k_2[k_3k_4])\}\,,$$
$$-\tfrac{3}{4}\sqrt{3}\{D_3^+(k_a[k_bk_4])\otimes K_{2\to3}(\{k_1k_2\}k_3)$$
$$+D_3^+([k_1k_a]k_b)\otimes K_{2\to3}(k_2\{k_3k_4\})\}\,,$$
$$-\frac{3}{\sqrt{8}}\{D_3^+(k_a[k_bk_4])\otimes K_{2\to3}(\{k_1k_2\}k_3)$$
$$+D_3^+([k_1k_a]k_b)\otimes K_{2\to3}(k_2\{k_3k_4\})\} \tag{5}$$

(k_a, k_b are the internal momenta which connect D_3 with the $2\to3$ transition vertex). The second term is obtained by interchanging k_1 with k_2 and k_3 with k_4 (in the second component we have to include a minus sign, since it belongs to the antisymmetric octet representation and hence is antisymmetric in the color indices), such that in total the third term has even signature in both pairs (12) and (34). Finally,

$$K_{4\to4;11}=-3Q_{(12)(34)}+\Sigma\,,$$

$$K_{4\to4;12}=\frac{3}{\sqrt{8}}(-Q_{(14)(23)}+Q_{(13)(24)})\,,$$

$$K_{4\to4;21}=\frac{3}{\sqrt{8}}(-Q_{(14)(23)}+Q_{(13)(24)})\,, \tag{6}$$

$$K_{4\to4;22}=-\tfrac{3}{2}Q_{(12)(34)}+\Sigma-\tfrac{3}{4}(Q_{(14)(23)}+Q_{(13)(24)})\,,$$

$$K_{4\to4;23}=\tfrac{3}{4}(-Q_{(14)(23)}+Q_{(13)(24)})\,,$$

$$K_{4\to4;25}=\sqrt{\tfrac{3}{8}}\,(-Q_{(14)(23)}+Q_{(13)(24)})\,,$$

$$K_{4\to4;32}=\tfrac{3}{4}(-Q_{(14)(23)}+Q_{(13)(24)})\,,$$

$$K_{4\to4;33}=-\tfrac{3}{2}Q_{(12)(34)}+\Sigma$$
$$-\tfrac{3}{4}(Q_{(14)(23)}+Q_{(13)(24)})\,,$$

$$K_{4\to4;34}=\frac{3}{\sqrt{10}}(-Q_{(14)(23)}+Q_{(13)(24)})\,,$$

$$K_{4\to4;43}=\frac{3}{\sqrt{10}}(-Q_{(14)(23)}+Q_{(13)(24)})\,,$$

$$K_{4\to4;44}=\Sigma+\tfrac{3}{2}(Q_{(13)(24)}+Q_{(14)(23)})\,,$$

$$K_{4\to4;45}=\sqrt{\tfrac{3}{5}}(-Q_{(14)(23)}+Q_{(13)(24)})\,,$$

$$K_{4\to4;52}=\sqrt{\tfrac{3}{8}}(-Q_{(14)(23)}+Q_{(13)(24)})\,,$$

$$K_{4\to4;54}=\sqrt{\tfrac{3}{5}}(-Q_{(14)(23)}+Q_{(13)(24)})\,,$$

$$K_{4\to4;55}=Q_{(12)(34)}+\Sigma$$
$$-2(Q_{(14)(23)}+Q_{(13)(24)})\,, \tag{6 cont'd}$$

The other matrix elements $K_{4\to4;ij}$ vanish. Here we have used the abbreviations

$$\Sigma=\sum_1^4[\alpha(k_i)-1]\,,$$

$$Q_{(12)(34)}=K_{2\to2}(k_1,k_2)+K_{2\to2}(k_3,k_4)\,,$$

$$\alpha(q)=1-\tfrac{3}{2}q^2\beta_2(q)\,,$$

$$\beta_2(q)=g^2\int\frac{d^2k}{(2\pi)^3}\frac{1}{k^2(k-q)^2}\,. \tag{7}$$

The kernels are

$$\frac{1}{g^2}K_{2\to2}(k_1,k_2;k_1',k_2')=q^2-\frac{k_1^2k_2'^2+k_2^2k_1'^2}{(k_1-k_1')^2}\,, \tag{8}$$

$$\frac{1}{g^3}K_{2\to3}(k_1k_2;k_1'k_2'k_3')=-q^2+\frac{(k_1'+k_2')^2k_2^2}{(k_2-k_3')^2}$$
$$+\frac{(k_2'+k_3')^2k_1^2}{(k_1-k_1')^2}-\frac{k_1^2k_2^2k_2'^2}{(k_1-k_1')^2(k_2-k_3')^2}\,, \tag{9}$$

and

$$\frac{1}{g^4} K_{2\to 4}(k_1 k_2; k'_1 k'_2 k'_3 k'_4) = q^2 - \frac{(k'_1+k'_2+k'_3)^2 k_2^2}{(k_2-k'_4)^2} - \frac{(k'_2+k'_3+k'_4)^2 k_1^2}{(k_1-k'_1)^2} + \frac{k_1^2 k_2^2 (k'_2+k'_3)^2}{(k_1-k'_1)^2 (k_2-k'_4)} . \quad (10)$$

For the other two signature configurations, $D_{4;i}^{(--)}$ and $D_{4;i}^{(+-)}$, the equations have exactly the same form as eq. (2). Changes are in the first three terms. In order to arrive at $K_{2\to 4}^{(--)}$, substitute in eq. (4) for $K_{2\to 4}^{(++)}$ { }→[ ] and [ ]→{ }. For $K_{2\to 4}^{(+-)}$, this substitution should be done only for the second pair of momenta. For the third term, substitute in eq. (4) { }→[ ] and [ ]→{ }, and then antisymmetrize (symmetrize) in (k_1, k_2) and in (k_3, k_4), such that we obtain negative signature in both pairs. For the mixed signature case, again start from eq. (4) and perform the substitutions { }→[ ] and [ ]→{ } in the following way: for each component, in the first term only in the argument of D_3, in the second term only in $K_{2\to 3}$. Then symmetrize or antisymmetrize in (k_1, k_2) and in (k_3, k_4), to obtain the correct signature properties.

4. Let us list a few properties of these equations:

(a) Although almost each term on the RHS of the equations contains infrared singularities, it can be shown that in the sum all singularities cancel: all vertex functions D_2, D_3, D_4 are infrared finite. This is the generalization of the celebrated infrared finiteness of the ladder graphs of BFKL.

(b) After some lengthy algebra one finds that the solutions to these equations can greatly be simplified. Starting with D_3 one finds

$$D_3^{(-)}(k_1, k_2, k_3; \omega) = g \cdot \tfrac{1}{2}\sqrt{3}\, D_2(k_1, k_2+k_3; \omega) \quad (11)$$

and

$$D_3^{(+)}(k_1, k_2, k_3; \omega) = g \cdot \tfrac{1}{2}\sqrt{3}\, [D_2(k_1+k_2, k_3; \omega) - D_2(k_1+k_3, k_2; \omega)] . \quad (12)$$

For D_4 we have

$$D_4^{(1;--)} = D_4^{(8s;--)} = D_4^{(10+\overline{10};--)} = D_4^{(27;--)} = 0 , \quad (13)$$

whereas

$$D_4^{(8A;--)}(k_1, k_2, k_3, k_4; \omega) = -g^2 \cdot \tfrac{3}{4} D_2(k_1+k_2, k_3+k_4; \omega) . \quad (14)$$

In other words, all amplitudes with "wrong statistics" vanish, except for the channel with the quantum numbers of the gluon which reggeizes. The same is also true for the mixed signature case:

$$D_4^{(1;+-)} = D_4^{(8s;+-)} = D_4^{(10+\overline{10};+-)} = D_4^{(27;+-)} = 0 , \quad (15)$$

$$D_4^{(8A;+-)}(k_1, k_2, k_3, k_4; \omega) = -g \cdot \tfrac{1}{2}\sqrt{3}\, D_3^{+}(k_1, k_2, k_3+k_4; \omega) . \quad (16)$$

For the double-even signature case $D_4^{(++)}$, we only remark that the intermediate state with three gluons can be eliminated by use of eqs. (11) and (12) (fig. 4). At the transition from the 2-gluon state to the 4-gluon state a new reggeon vertex appears which follows from eqs. (2) and (11), (12). Fig. 4 should be compared with the first fan diagram of GLR: in the 4-gluon state there is pairwise interaction between all four gluons, and there is no restriction on the group structure (except for an overall singlet). In contrast to this, the fan diagram consists of two isolated color singlet ladders.

The reggeization properties in eqs. (11)–(13), and (16) are generalizations of the famous "bootstrap" property of the BFKL pomeron. Together with the infrared finiteness in (a), this result indicates that the BFKL is just the first term in a systematic expansion, and what we are discussing in this paper (D_3 and D_4) are the second and third term.

(c) We have studied the limit of $D_4^{(++)}$ where

$$k_1^2 = k_2^2 = k_3^2 = k_4^2 = q^2 \ll Q^2 . \quad (17)$$

This is the region where fig. 4. should describe the small-x region of DIS: following the experience with the BFKL pomeron, one expects to reach the double-

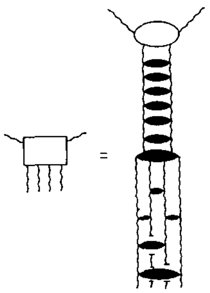

Fig. 4. Diagrammatic illustration of $D_4^{(++)}$.

logarithmic approximation of DIS. In particular, the leading behavior in $\ln(Q^2/q^2)$ should exhibit the singular (near $n=1$) part of the anomalous dimensions of the two and four gluon states, and the strength of the 2-gluon$\to$4-gluon transition can be calculated. Details of this calculation will be presented elsewhere. Here we only report that the anomalous dimension of the four gluon operator is derived from the singularity structure of a $12\otimes12$ matrix. The leading singularity lies at

$$\gamma_4(\omega)=12.115\frac{\alpha_s}{\omega\pi}, \tag{18}$$

and the corresponding four gluon state consists mainly (but not exclusively) of two color-zero two-gluon ladders.

This result may have serious implications for the GLR equation. Namely from this equation one easily deduces that the four gluon anomalous dimension should satisfy [10]

$$\gamma_4(\omega)=2\gamma_2(\omega/2)\,. \tag{19}$$

With the 2-gluon anomalous dimension $\gamma_2(\omega)=N_c(\alpha_s/\omega\pi)$ one concludes that the numerical coefficient should be 12. Although the numerical difference is small, it nevertheless implies that the fan diagrams in the GLR equation are not exactly the leading ones. The evolution of the four gluon state is not simply the product of two independent two gluon ladders, but proceeds through the pairwise interaction of all four gluon lines (also the strength of the 2-gluon$\to$4-gluon transition differs from that of GLR). In order to decide whether the GLR equation nevertheless remains a useful tool, we need to know whether (and by how much)

$$\gamma_n(\omega)>n\gamma(\omega/n)\,. \tag{20}$$

If the difference stays as small as in the present case, the GLR equation may remain a very good approximation (but not more!) to the true QCD prediction. Otherwise we will have to live with a much more complicated evolution scheme.

(d) Finally, no analytic method for determining the leading singularity in the angular momentum plane has been found yet. One may however speculate that the result in eq. (18) indicates the existence of a new bound state (fixed cut) of the four gluon state to the right of the "two-pomeron" cut. In the absence of anything better, one may try a computer analysis of eq. (2) and find numerically the location of the leading singularity.

It should be mentioned that there are immediate applications of such a calculation. Some time ago, Mueller and Navelet [11] pointed out that the BFKL pomeron could be tested in a hadron collider or at HERA by looking at certain jets in the final state. In context with HERA, these jets probe "hot spots" [12] in the proton. Up to now it is not known whether the BFKL pomeron will become visible at all before the unitarity corrections take over. A similar word of caution applies to the hypothesis that the power behavior of the BFKL pomeron might describe the low-x behavior of the gluon distribution at low Q^2.

5. Clearly, the results presented in this letter should be considered only as a step towards improving our understanding of the small-x behavior of deep-inelastic scattering and, ultimately, also the nonperturbative Regge limit. As was said before, we strongly believe that a full reggeon field theory can be derived for the Regge limit of QCD. So far we have considered only particle$\to$reggeon amplitudes but not reggeon$\to$reggeon amplitudes. Ultimately, the small-x problem in DIS can be solved only by solving this reggeon field theory. The novel aspect of this "old problem" is the smallness of α_s: the transition from perturbative to nonperturbative QCD is taken in a region where the coupling is still small.

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