

## A determination of the $Zb\bar{b}$ vertex correction from LEP data

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Received 10 August 1992

Using recent LEP measurements of the ratios of Z bosons widths  $\Gamma_{Z\rightarrow b\bar{b}}/\Gamma_{Z\rightarrow \text{hadrons}}$  and  $\Gamma_{Z\rightarrow \text{hadrons}}/\Gamma_{Z\rightarrow \text{leptons}}$ , we determine the size of the vertex radiative correction to the  $Z\rightarrow b\bar{b}$  partial decay width. This allows, in the context of the minimal standard model, to set the 95% confidence level upper bound on the top quark mass,  $m_t < 208$  GeV independently of the Higgs boson mass. This value is remarkably close to the completely independent available bounds, which are extracted from the measurement of the radiative correction to the  $\rho$  parameter. The implication of possible New Physics on this measurement is briefly commented upon.

1. It has been known for quite some time that within the minimal standard model (with three generations of fermions and one doublet of scalar fields) of the electroweak interactions, two independent radiative corrections can be used to pin down one of the still unknown parameters of the model, namely the top quark mass. The first correction is universal (in the sense that it enters all electroweak observables at one loop) and measures the deviation of the  $\rho$  parameter [1] from unity; it can be expressed in terms of the difference between the self-energies of the W and Z bosons at zero momentum transfer. The second correction is specific to  $b\bar{b}$  final states in Z decays and arises from the exchange of heavy top quarks in the  $Zb\bar{b}$  vertex [2]; it can be defined from the ratio of the partial decay widths of the Z boson into down quarks and massless bottom quarks [3]. In the limit of large top quark mass, the two corrections can be written as

$$\Delta\rho \equiv \frac{\Pi_{ZZ}(0)}{M_Z^2} - \frac{\Pi_{WW}(0)}{M_W^2} \simeq \frac{\alpha m_t^2}{\pi M_Z^2}, \quad (1)$$

$$\begin{aligned} \Delta_{bV} &\equiv \Gamma_{Z\rightarrow b\bar{b}}^{m_b=0} / \Gamma_{Z\rightarrow d\bar{d}} - 1 \\ &\simeq -\frac{20}{13} \frac{\alpha}{\pi} \left( \frac{m_t^2}{M_Z^2} + \frac{13}{6} \log \frac{m_t^2}{M_W^2} \right). \end{aligned} \quad (2)$$

For  $m_t$  close to 150 GeV, the two corrections enter

the one percent level which is now being probed by the high precision LEP experiments, and for  $\Delta\rho$ , by deep inelastic lepton–nucleon scattering experiments and by the measurement of the W mass at hadron colliders.

If the top mass were the only unknown parameter of the model, it could equivalently be determined from a measurement of either  $\Delta\rho$  or  $\Delta_{bV}$ ; the best determination being simply provided by the quantity which is measured with the best accuracy. However, in extensions of the minimal standard model (MSM), the two corrections receive in general completely different contributions as has been discussed thoroughly in refs. [4–6]. Therefore the measurement of both radiative corrections provides a fundamental consistency check of the model.

It should be noted that, already within the MSM,  $\Delta\rho$  receives a contribution from the other unknown parameter, the Higgs boson mass [7]. Although this contribution is only logarithmically dependent on  $M_H$  as a result of Veltman's screening theorem, the variation of  $M_H$  from the present experimental lower value of 60 GeV to the theoretical upper bound of  $\sim 1$  TeV leads to an error in  $m_t$  of typically  $\pm 20$  GeV. In contrast, because of the extremely weak coupling of the Higgs boson to b quarks, there is no Higgs correction to the  $Zb\bar{b}$  vertex and in this respect,  $\Delta_{bV}$  is a theoretically cleaner top indicator than  $\Delta\rho$ .

Several analyses [8] using recent experimental data have been performed to determine the value of the top quark mass from the measurement of the  $\Delta\rho$  correction and an upper bound of, typically, 200 GeV has been set on  $m_t$ . In this letter, using recent LEP measurements of  $\Gamma_{Z \rightarrow b\bar{b}}/\Gamma_{Z \rightarrow \text{hadrons}} \equiv \Gamma_b/\Gamma_{\text{had}}$  and  $\Gamma_{Z \rightarrow \text{hadrons}}/\Gamma_{Z \rightarrow \text{leptons}} \equiv \Gamma_{\text{had}}/\Gamma_{\text{lept}}$ , as well as those of the effective electroweak mixing angle and the strong coupling constant, we present a determination of the top quark mass from the  $Zb\bar{b}$  vertex correction  $\Delta_{bV}$ .

2. In the determination of the top quark mass, one would like to use observables which are free of strong interaction effects to avoid the uncertainties in the measurement of the QCD coupling constant,  $\alpha_s$ , which then translates into a potentially large error on  $m_t$ . In the case of  $\Delta\rho$ , this problem can simply be avoided by using only leptonic partial widths and asymmetries<sup>#1</sup>. In fact, the leptonic decay width of the Z boson  $\Gamma_{\text{lept}}$  provides an operational definition of  $\Delta\rho$  [10] via the expression

$$\Gamma_{\text{lept}} = \frac{G_F M_Z^3}{24\sqrt{2}} (1 + \Delta\rho) \left[ 1 + \left( \frac{g_V}{g_A} \right)^2 \right] \left( 1 + \frac{3}{4} \frac{\alpha}{\pi} \right), \quad (3)$$

where the last factor is due to the electromagnetic final state corrections and  $g_V/g_A \equiv 1 - 4 \sin^2 \theta_w^{\text{eff}}$  with  $\sin^2 \theta_w^{\text{eff}}$  being the effective electroweak mixing angle as defined from asymmetry measurements at the Z pole [10]. Similar operational definitions can be obtained from the muonic forward-backward asymmetry  $A_{FB}^{\mu}$  and the  $\tau$  lepton polarization asymmetry  $A_{\text{pol}}^{\tau}$ . In the case of the vertex correction  $\Delta_{bV}$  that has to be determined from  $\Gamma_b \equiv \Gamma_{Z \rightarrow b\bar{b}}$ , which is affected by final state QCD corrections, it is still possible to derive a similar operational definition that is (almost)  $\alpha_s$  free. Indeed, since the QCD correction is flavor independent, it completely cancels in the ratio  $\Gamma_b/\Gamma_{\text{had}}$  in the limit where b quark mass effects can be neglected<sup>#2</sup>. One can then write, to a very good approximation

$$\frac{\Gamma_b}{\Gamma_{\text{had}}} = \frac{13}{59} (1 + \frac{46}{59} \Delta_{bV}) (1 + c_b), \quad (4)$$

where  $c_b$  contains the residual small electromagnetic and vertex corrections (including non-leading top mass terms which are negligible for large  $m_t$ ) as well as term proportional to  $g_V/g_A$  which in this case con-

tains the residual oblique corrections. However, due to the similarity of the up and down type quark couplings to the Z boson, the coefficient of the latter term is extremely small ( $< \frac{1}{30}$ ) in the ratio  $\Gamma_b/\Gamma_{\text{had}}$  [6] leading to completely negligible oblique corrections. An important consequence of this feature is that, up to small calculable effects, the ratio  $\Gamma_b/\Gamma_{\text{had}}$  measures *only* the  $Zb\bar{b}$  vertex correction in much the same way as  $\Gamma_{\text{lept}}$  measures only the oblique correction  $\Delta\rho$ .

Measurements of  $\Gamma_b/\Gamma_{\text{had}}$  have been performed at LEP by the four experiments using a variety of methods, which can be grouped in three classes:

(1) High momentum and transverse momentum lepton tag of b events [12–14]. Here the systematic error comes from the understanding of the semi-leptonic decays of b-hadrons. For the sake of averaging we have assumed a common systematic error which is as large as the smallest one from the three experiments.

(2) Extraction of the fraction of  $b\bar{b}$  events in the sample of hadronic Z decays by fits of the distribution of boosted sphericity and other event shape variables [12,15]. The dominant systematic errors come from the modelling of inclusive b-hadron decays, from the simulation of the underlying event and from QCD parameters. Again, we chose the smallest systematic error as the common one.

(3) Recognition of  $b\bar{b}$  events using microvertex detectors. Preliminary results using this promising method, cross-checked in a double tag scheme with event shape variables, have been shown by DELPHI [16].

The experimental results are summarized in table 1. The systematic errors in these three methods originate from different details in the fragmentation and decay process of  $b\bar{b}$  events, so we assume here that they are independent. Given the large uncertainty involved in the averaging, this result should be taken with a grain of salt. The overall result is

<sup>#1</sup> Note that there is still an explicit  $\alpha_s$  dependence in the correction to the  $\rho$  parameter itself, due to gluonic exchange in the top and bottom quark loops [9]. This correction decreases the value of  $\Delta\rho$  by slightly more than 10%, but the associated error is small.

<sup>#2</sup> This property remains practically unaffected when the actual b mass is taken into account. Indeed, the additional contribution to the QCD correction associated with the finite b mass [11] is smaller than 0.5% and therefore, the associated error is completely negligible.

Table 1

$\Gamma_b/\Gamma_{\text{had}}$  measurements at LEP. The averages have been computed assuming a common systematic error which, for each of the three methods, is the smallest among the experiments of the systematic errors due to modelling. The systematic errors of the three methods are considered uncorrelated.

Method	Experiment	$\Gamma_b/\Gamma_{\text{had}}$		
		value	exp. error	modelling error
high $p$ , $P_{\perp}$ lepton tag	ALEPH	0.211	$\pm 0.007$	$\pm 0.008$
	L3	0.221	$\pm 0.004 \pm 0.006$	$\pm 0.011$
	OPAL	0.227	$\pm 0.007$	$\pm 0.018$
	average	0.2154	$\pm 0.0055$	$\pm 0.008$
event shape variables	DELPHI	0.219	$\pm 0.014$	$\pm 0.019$
	ALEPH	0.214	$\pm 0.003$	$\pm 0.012$
	average	0.214	$\pm 0.003$	$\pm 0.012$
microvertex tag	DELPHI	0.223	$\pm 0.004$	$\pm 0.013$
	average	0.2168		$\pm 0.0067$

$$\frac{\Gamma_b}{\Gamma_{\text{had}}} = 0.2168 \pm 0.0067, \quad (5)$$

from which one can derive

$$\Delta_{bV} = -0.011 \pm 0.039_{(\text{exp})} \pm 0.001_{(m_b)} \pm 0.0004_{(\alpha_s)}, \quad (6)$$

where the first error reflects the experimental error on  $\Gamma_b/\Gamma_{\text{had}}$ , the second one the not accurately known mass of the b quark obtained by varying  $m_b$  by  $\pm 0.5$  GeV around the central value of 4.9 GeV and the third one the uncertainty on  $\alpha_s = 0.117 \pm 0.007$  (a discussion of this point will be given later). Clearly, the uncertainties in  $m_b$  and  $\alpha_s$  are negligible compared to the experimental error.

This estimate of  $\Delta_{bV}$  leads to the 95% confidence level upper limit of  $\Delta_{bV} > -0.078$  and corresponds to the upper bound <sup>#3</sup> of  $m_t < 360$  GeV. The central value of  $\Delta_{bV}$  corresponds to  $m_t = 160$  GeV, and the one standard deviation limit would correspond to  $\Delta_{bV} = -0.050$  or  $m_t = 270$  GeV. To improve this unbiased determination of  $m_t$  to an accuracy of  $\pm 30$  GeV, one would need a determination of  $\Gamma_b/\Gamma_{\text{had}}$  to  $\pm 0.001$ , which does not seem an easy task. A more

realistic future measurement of  $\Gamma_b/\Gamma_{\text{had}}$  to, say, a precision of  $\pm 0.003$  will still be extremely relevant once the top quark is discovered. However, the precision or limit that will be obtained on the top quark mass from this measurement appears to be, at least for the near future, rather limited. Therefore, it is worthwhile to look for alternative and more precise possibilities.

3. The main motivation which determined our choice of measuring the  $\Delta_{bV}$  correction from  $\Gamma_b/\Gamma_{\text{had}}$ , despite of its relatively large experimental error, was that the latter observable is practically independent of the strong coupling constant  $\alpha_s$  and of the oblique correction  $\Delta\rho$ . However, these two quantities can be determined rather precisely from other observables that are not affected by the  $Zb\bar{b}$  vertex correction. For instance the value of  $\alpha_s$  has been measured with a very good accuracy from various observables in deep-inelastic lepton-nucleon scattering, event shape variables in hadronic Z decays and from the  $\tau$  lifetime. Furthermore, the value of  $\sin^2\theta_w^{\text{eff}}$  [and therefore the oblique corrections] has been determined with a very good precision from various asymmetry measurements at LEP. Therefore, using these two inputs the only unknown will be the  $Zb\bar{b}$  vertex correction and one can isolate it in all observables containing  $b\bar{b}$  final states.

An interesting observable which contains the  $\Delta_{bV}$  correction and which is measured with a much better

<sup>#3</sup> Here and in the following, the 95% CL upper limit is obtained following the Particle Data Group prescription, which requires integrating the probability densities above the physical bound. The bound  $m_t > 91$  GeV has been chosen. A lower physical bound would give more restrictive upper bounds on  $m_t$ .

accuracy than  $\Gamma_b/\Gamma_{had}$  is the ratio of the total hadronic to leptonic widths of the Z boson,  $R_h \equiv \Gamma_{had}/\Gamma_{lept}$ . Recent experimental values of  $R_h$  from LEP analyses [17,18] are shown in table 2. This ratio can be written as <sup>#4</sup>

$$R_h = \frac{59}{3} \left( 1 + \frac{20}{59} \frac{g_V}{g_A} + \frac{13}{59} A_{bv} \right) + \delta_{QCD} + \delta_{EW} + \delta_{m_b}, \quad (7)$$

where the additional corrections are as follows [19]:  $\delta_{QCD}$  includes all the QCD corrections to  $O(\alpha_s^3)$  for massless quarks, the b mass effects at  $O(\alpha_s)$  and the  $m_t^2$  dependence at  $O(\alpha_s^2)$  due to the top exchange in the  $Zb\bar{b}$  vertex;  $\delta_{EW}$  contains the small terms of order  $O(g_V^2/g_A^2)$  as well as the complete vertex corrections to  $\Gamma_{Z \rightarrow f\bar{f}}$  except  $A_{bv}$  in  $\Gamma_{Z \rightarrow b\bar{b}}$ ;  $\delta_{m_b}$  is for the kinematical corrections due to the b quark mass. All these corrections are included in our fitting program <sup>#5</sup>.

The choice of a value for the strong coupling constant is a critical issue here. Most discussion has taken place recently on this topic, and we refer to e.g. ref. [21] for a recent compilation. The conclusion ref. [21] is that, given the estimated theoretical systematic errors, the various measurements of  $\alpha_s$  are in good agreement with an average value of

$$\alpha_s(M_Z^2) = 0.117 \pm 0.004, \quad (8)$$

where the error is considered here as "realistic". A

<sup>#4</sup> Note that in the conventional approach,  $R_h$  is exploited to measure  $\alpha_s$  independently of the top mass. This is due to the fact that in the MSM the  $m_t$  contributions in the oblique and vertex corrections practically cancel each other. Here we will follow an orthogonal approach.

<sup>#5</sup> We have compared our results with various other calculations [20]; we found reasonable agreement in the results and in the  $m_t$  limits, within  $\pm 10$  GeV.

Table 2  
 $R_h$  measurements at LEP taken from refs. [17,18].

Experiment	$R_h$
ALEPH	$20.77 \pm 0.13$
DELPHI	$21.05 \pm 0.20$
L3	$20.88 \pm 0.17$
OPAL	$20.83 \pm 0.17$

more conservative estimate of the error is given by  $\pm 0.007$  [22] by allowing a larger theoretical error. We shall use the realistic error, but also mention the result obtained with the more conservative error  $\pm 0.007$ . One can rightfully object that this average includes the value of  $\alpha_s$  obtained from the measurement of  $R_h$  itself assuming the validity of the MSM; but the value of  $\alpha_s$  from  $R_h$  has a rather large experimental error, and we have verified that removing this measurement does not affect the average.

The value of  $\sin^2\theta_w^{eff}$  can be derived from asymmetry and  $\tau$  lepton polarization measurements at LEP. Using the latest results [18] of the lepton,  $q\bar{q}$  jets and  $b\bar{b}$  forward-backward asymmetries as well as the  $\tau$  polarization summarized in table 3, leads to an average value

$$\sin^2\theta_w^{eff} \equiv \frac{1}{4} \left( 1 - \frac{g_V}{g_A} \right) = 0.2326 \pm 0.0012. \quad (9)$$

Note that, in principle, the  $b\bar{b}$  forward-backward asymmetry also contains the vertex correction  $A_{bv}$ . However, as a consequence of an accidental cancellation among the contributions to the axial and vectorial  $Zb\bar{b}$  couplings, the vertex correction in this case is extremely small and it can be safely neglected in the derivation of  $\sin^2\theta_w^{eff}$  from the  $b\bar{b}$  asymmetry [6].

Recent experimental values of  $R_h$  from the four LEP collaborations are given in table 2 and lead to an average value of

$$R_h \equiv \frac{\Gamma_{had}}{\Gamma_{lept}} = 20.86 \pm 0.08. \quad (10)$$

For the value of  $\sin^2\theta_w^{eff}$  given above and  $\alpha_s = 0.117 \pm 0.004$ , one then obtains assuming the validity of the MSM

Table 3  
Values of  $\sin^2\theta_w^{eff}$  from forward-backward asymmetry and  $\tau$  polarization measurements at LEP. The results are taken from ref. [18].

Observable	$\sin^2\theta_w^{eff}$
$A_{FB}^{lept}$	$0.2323 \pm 0.0017$
$A_{FB}^{q\bar{q}}$	$0.2314 \pm 0.0032$
$A_{FB}^{b\bar{b}}$	$0.2341 \pm 0.0024$
$A_{pol}^\tau$	$0.2323 \pm 0.0029$

$$\begin{aligned} \Delta_{bV} &= 0.010 \pm 0.016_{(R)} \pm 0.007_{(\sin^2 \theta_w^{\text{eff}})} \pm 0.006_{(\alpha_s)} \\ &= 0.010 \pm 0.019, \end{aligned} \quad (11)$$

which leads to the 95% confidence level limits of

$$\Delta_{bV} > -0.033 \Rightarrow m_t < 208 \text{ GeV}. \quad (12)$$

The corresponding values for  $\Delta_{bV}$  and  $m_t$  if one uses the more conservative choice  $\alpha_s = 0.117 \pm 0.007$  would be  $\Delta_{bV} = 0.010 \pm 0.021$ , leading to the 95% confidence level limits of  $\Delta_{bV} > -0.036$  and  $m_t < 221$  GeV. These results improve substantially over previous, similar determinations [4].

4. The constraints eq. (12) on the value of  $m_t$  that we have obtained in the previous analysis are remarkably close to those that one can obtain from the oblique corrections, i.e. from  $\Delta\rho$ . As a matter of fact, the top mass value which can be extracted, *in the MSM*, from a global fit to the electroweak data, from LEP, from neutrino–nucleon scattering, and from the W mass, is the following:

$$m_t = 160_{-17}^{+16} \pm 17_{-23}^{+17} \Rightarrow m_t < 204 \text{ GeV (95% CL)}, \quad (13)$$

where the first error is due to the experimental uncertainties and the second one to the uncertainty generated by varying the Higgs boson mass from its present experimental lower limit of 60 GeV to the theoretical upper bound of  $\sim 1$  TeV. Let us now make a few comments on these two different determinations of  $m_t$ .

We first note that the precision on the value of  $m_t$  that is determined, mostly from  $\Delta\rho$ , in eq. (13) is limited by the theoretical error,  $\sim \pm 20$  GeV, due to the unknown contribution of the Higgs boson. In contrast, the error on the  $m_t$  value that is extracted from  $\Delta_{bV}$  is only of experimental nature. Therefore, if more precise measurements of  $\alpha_s$  and  $\sin^2 \theta_w^{\text{eff}}$  are available in the future, the combined error in the latter quantities (which already now is approximately of the same size as the one due to the Higgs contribution in  $\Delta\rho$ ) will be smaller, leading to a smaller error on the value of  $m_t$ .

The two independent determinations of  $m_t$  that we have performed here become even more relevant as soon as the possibility of deviations from the MSM is considered. Indeed, for a wide class of models of New Physics there is a certain complementarity which

prevents contributions to both  $\Delta\rho$  and  $\Delta_{bV}$  from the same model as discussed in refs. [4–6]. Therefore for every such model, it would be possible to use one of the two radiative corrections as a real constraint on  $m_t$  and the other measurement as a constraint on the new parameters of the model itself. Furthermore, once the top quark is found, one can use both measurements to probe the effects of New Physics <sup>#6</sup>.

In conclusion, using recent LEP measurements of the ratio of partial Z widths  $\Gamma_{\text{had}}/\Gamma_{\text{lept}}$ , and those of the effective electroweak mixing angle  $\sin^2 \theta_w^{\text{eff}}$  and the strong coupling constant  $\alpha_s$ , we have determined the size of the vertex radiative correction to the  $Z \rightarrow b\bar{b}$  partial width,  $\Delta_{bV} = 0.010 \pm 0.019$ . In the context of the minimal standard model, this allows to set the upper bound on the top quark mass  $m_t < 208$  GeV at 95% CL, independently of the Higgs boson mass. This bound is remarkably close to the available bound which is extracted from the analysis of the oblique radiative correction  $\Delta\rho$ . Since the two corrections  $\Delta_{bV}$  and  $\Delta\rho$  are of completely different nature, the two bounds are largely independent and therefore provide us with a very important test of the model.

<sup>#6</sup> For instance, the value of  $\Delta_{bV}$  given in eq. (11) will impose strong constraints on the extension of the MSM in which an extra Higgs doublet is added [23]. In this extension, the charged Higgs boson gives rise to additional contributions to the  $Zb\bar{b}$  vertex that are proportional to  $m_t^2/M_Z^2 \cot^2 \beta$  [24], where  $\tan \beta$  is the ratio of the vacuum expectation values of the two Higgs fields. For a top quark heavier than 150 GeV and a charged Higgs lighter than 200 GeV, values of  $\tan \beta$  smaller than 0.5 can be ruled out. This improves over previous constraints [25].

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