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# The decay $B \to D\pi\ell\nu$ using chiral and heavy quark symmetry

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We calculate the rate and angular correlation parameters for the decay $B \to D^*\ell\nu \to D\pi\ell\nu$ using the recently combined methods of chiral and heavy quark symmetry to evaluate the current matrix elements. An interesting interference effect between the direct $D^*$ exchange term and the $B^*$ exchange term yields constraints on the width of the $D^*$. If the $D^*$ width is small compared with the experimental resolution, the model fits recent ARGUS data with $|V_{bc}| = 0.043$.

## 1. Introduction

Heavy quark symmetry [1] has turned out to be a useful tool to obtain model-independent information on the weak decay matrix elements of heavy mesons. In the limit of infinitely large quark mass additional symmetries beyond the ones of QCD arise which reduce the number of independent weak current matrix elements. The approach is conveniently formulated in terms of an effective theory called heavy quark effective theory (HQET) [2]. In this framework corrections to the heavy quark limit may be discussed systematically.

In heavy to heavy transitions like $b \to c$ decays all heavy quark bilinear current matrix elements between heavy quarks are described in terms of only one independent form factor, the so-called Isgur–Wise function. In addition the symmetries of the heavy quark limit give the normalization of the Isgur–Wise function at the point of equal velocities for the initial and final heavy meson. This is phenomenologically very useful, since it allows a model-independent determination of $|V_{bc}|$ from semileptonic $B \to D$ and $B \to D^*$ decays [3].

When heavy quark decays also involve the emission of soft pions, the methods of chiral symmetry (CS) may be used alongside the heavy quark symmetry to describe matrix elements of currents involving heavy mesons and soft pions. Such is the case in the decay $B \to D^*\ell\nu$ where the $D^*$ subsequently decays to $D\pi$. The fundamental process is thus $B \to D\pi\ell\nu$ with an intermediate $D^*$ state in the $D\pi$ system and, to satisfy the CS, an intermediate $B^*$ in the $B\pi$ cross channel of the fundamental process. (See fig. 1.)

Recently several authors have studied the combined HQET/CS limit and written down the amplitude for the current matrix element $\langle D(v')\pi(p_3)|V_\mu - A_\mu|B(v)\rangle$ where $v$ and $v'$ are the four-velocities of the heavy mesons [4]. It is remarkable that this amplitude depends only on the normalized Isgur–Wise function and on an overall scale related to the $D^*$ width.

In the present paper we shall use this amplitude to calculate the partial width, the polarization parameter $\alpha$, and the forward–backward asymmetry parameter $A_{\rm fb}$ of $B \to D^*\ell\nu$. We shall compare and correlate our results with data on the branching ratio for $B \to D^*\ell\nu \to D\pi\ell\nu$, $B \to (D, D^*, D^{**})\ell\nu$, the total semileptonic branching ratio, and the parameters $\alpha$ and $A_{\rm fb}$.

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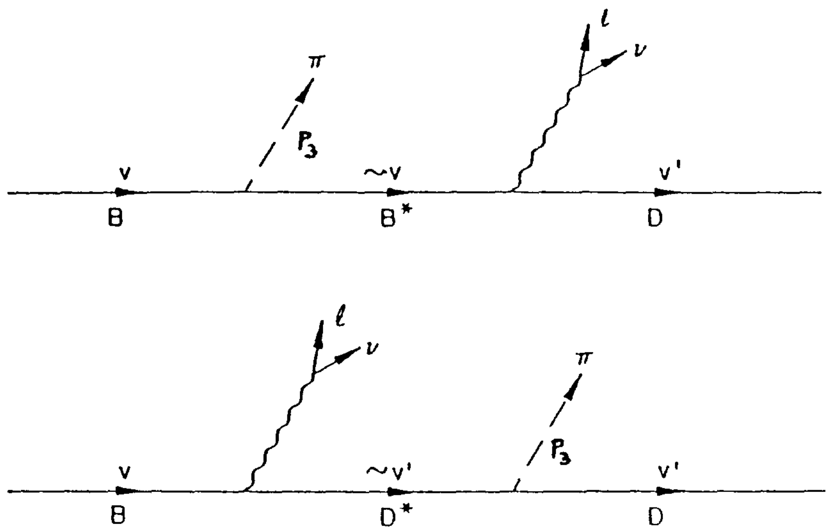


Fig. 1. $B^*$ and $D^*$ pole diagrams contributing to the decay $B \to D\pi\ell\nu$.

## 2. Kinematics and angular distribution of $B \to D\pi\ell\nu$

A complete treatment of the angular distribution and kinematics of the decay $B \to D^*\ell\nu \to D\pi\ell\nu$ can be found in our earlier work [5] in the context of $D \to K^*\ell\nu$. The only difference for $B$ decay is that signs of the vector current must be reversed [6]. Here we only review that the process is described by the momenta

$$p_1 \to p_2 + p_3 + k + k', \tag{1}$$

where $p_2$ and $p_3$ are the momenta of the $D$ and $\pi$, respectively, $k$ is the electron momentum, $k'$ is the neutrino momentum, and $q = k + k'$. The fully differential decay distribution integrated over the angle $\chi$ between the lepton and hadron decay planes is

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta\, ds_{23}\, d\cos\theta^*} = \sum_i l_i \frac{d^3\Gamma_i}{dq^2\, ds_{23}\, d\cos\theta^*}, \tag{2}$$

where the lepton coefficients are

$$l_U = \tfrac{3}{8}(1 + \cos^2\theta),$$
$$l_L = \tfrac{3}{4}\sin^2\theta,$$
$$l_P = \tfrac{3}{4}\cos\theta, \tag{3}$$

and the hadron tensors are given by

$$\frac{d^3\Gamma_i}{dq^2\, ds_{23}\, d\cos\theta} = \frac{G_F^2|V_{bc}|^2 q^2\sqrt{a_2}\, X}{96(2\pi)^5 m_1^3} H_i, \tag{4}$$

with $i = $ U, L, P.

In terms of the current decomposition

$$J_\mu \equiv \langle p_2, p_3|A_\mu + V_\mu|p_1\rangle = (1/m_1)[f(p_2+p_3)_\mu + g(p_2-p_3)_\mu + rq_\mu + (ih/m_1^2)\epsilon_{\mu\nu\alpha\beta}q^\nu(p_2+p_3)^\alpha(p_2-p_3)^\beta], \tag{5}$$

the hadronic structure functions are

$$H_U = C_1\left(|g|^2 + |h|^2X^2/m_1^4\right),$$
$$H_L = (1/q^2m_1^2)|Xf + C_2 g|^2,$$
$$H_P = (2X/m_1^2)C_1\,\mathrm{Re}(g^*h), \tag{6}$$

where we defined

$$C_1 = \left(a_2 s_{23}/m_1^2\right)\sin^2\theta^*,$$
$$C_2 = \kappa X + \sqrt{a_2}[(m_1^2 - s_{23} - q^2)/2]\cos\theta^*,$$
$$\kappa = (m_2^2 - m_3^2)/s_{23}. \tag{7}$$

The dynamics of the decay is contained in the structure functions $H_i$. They depend on $s_{23}$, $\cos\theta^*$, and $q^2$, the invariant mass of the $D\pi$ system, the polar angle of the $D$ in the $D\pi$ system, and the momentum transfer to the leptons, respectively. In (4) and (6) we defined

$$X = \sqrt{s_{23}}|\boldsymbol{q}| = \sqrt{s_{23}}|\boldsymbol{p}_1| = \lambda^{1/2}(m_1^2, s_{23}, q^2)/2$$

and

$$\sqrt{a_2} = 2|\boldsymbol{p}_2|\sqrt{s_{23}} = 2|\boldsymbol{p}_3|/\sqrt{s_{23}} = \lambda^{1/2}(s_{23}, m_2^2, m_3^2)/s_{23}.$$

They are related to the momenta of the $B$ and the $D$ meson in the $D\pi$ rest system $\boldsymbol{P} = \boldsymbol{p}_2 + \boldsymbol{p}_3 = 0$. The structure functions $H_i$ can be calculated from the decomposition of the hadronic matrix element.

Finally, the coefficients $\alpha$ and $A_{\rm fb}$ are given by

$$\alpha = 2H_{\rm L}/H_{\rm U} - 1,$$
$$A_{\rm fb} = \tfrac{3}{4}H_{\rm P}/(H_{\rm L} + H_{\rm U}). \tag{8}$$

### 3. Chiral and heavy quark symmetry results for $B \to D^*\ell\nu \to D\pi\ell\nu$

The HQET/CS result for the current matrix element is [4]

$$\langle D(v')\pi(p_3)|A_\mu + V_\mu|B(v)\rangle = \frac{\mathrm{i}\hat{f}}{\sqrt{2}f_\pi}\xi(vv')\sqrt{m_B m_D}$$
$$\times\Bigg(\frac{1}{2vp_3 + 2(m_B^* - m_B)}[-\mathrm{i}\epsilon_{\mu\nu\alpha\beta}p_3^\nu v'^\alpha v^\beta$$
$$+p_3(v+v')v_\mu - (1+vv')p_{3\mu}]$$
$$-\frac{1}{2v'p_3 + 2(m_D - m_D^*)}[-\mathrm{i}\epsilon_{\mu\nu\alpha\beta}p_3^\nu v'^\alpha v^\beta$$
$$+p_3(v+v')v'_\mu - (1+vv')p_{3\mu}]\Bigg). \tag{9}$$

This expression determines the relative sign and magnitudes of the two diagrams in fig. 1, up to the overall constant $\hat{f}$ (which can be directly related to the width for $D^* \to D\pi$) and the Isgur–Wise function. Recently, Lee, Lu and Wise have advocated the use of this amplitude above the $D^*$ resonance to determine the fundamental parameter $\hat{f}$ related to the width of the $D^*$. This region however is subject to large experimental backgrounds. We instead concentrate on the interference effects in the resonance region and the dependence of the rate on the $D^*$ width. This also has the advantage that the pion is still soft in this region. Our procedure is to calculate the diagrams of fig. 1 and then require that they match (9) in the heavy quark limit. This imposes the constraint

$$(g_{D^{*0}D^-\pi^+}/m_D)^2 = (g_{B^*B^-\pi^+}/m_B)^2$$
$$= \hat{f}^2/8f_\pi^2 \tag{10}$$

and also determines all of the form factors $f$, $g$, and $h$ in terms of a single Isgur–Wise function. (The important relative sign between the two terms of equations is the same as that used by us [5,6] many years ago in the context of $D \to K^*\ell\nu$.) For the Isgur–Wise function we used a dipole form (with the mass fixed by the $\bar{b}c$ vector and axial vector state, estimated to have a mass 6.34 GeV [7]) so that the large momentum transfer behavior of the first axial vector form factor is that of a monopole. This form factor also gives an excellent fit to the $q^2$ dependence of the rate as measured by ARGUS [8] and is quite similar to their "model A" from $\xi = 1$ to $\xi = 1.3$, slightly higher for large $\xi$. In doing the actual numerical evaluation we did not use the strict heavy quark limit incorporated in (9) but rather used (9) to relate the form factors of the $B^*$ and $D^*$ pole terms, employing the general kinematical framework of our earlier work which includes all terms with finite masses for the heavy mesons [5].

The two terms arising from the $D^*$ pole and $B^*$ pole interfere in the neighborhood of the narrow $D^*$, an effect that vanishes in the narrow-width approximation. In table 1 we show the results of our calculation for three cases: no $B^*$ pole, $B^*$ pole with sign according to the HQET/CS model ($D^* + B^*$), and $B^*$ pole with opposite sign ($D^* - B^*$). The only input to the calculation is the width of the $D^*$. We show results for $\Gamma_{D^*} = 0$ (narrow-width approximation) up to $\Gamma_{D^*} = 0.4$ MeV. The $B^*$ term interferes destructively with the $D^*$ pole if the HQET/CS sign is chosen.

The effect of the narrow-width approximation is clearly seen in the first model (no $B^*$ exchange). In this model the effect of a finite-width Breit–Wigner factor is to enhance the rate because of the rapid variation of the rest of the matrix elements under the Breit–

Table 1
$B \to D^* \ell \nu \to D\pi\ell\nu$ decay parameters for various values of $\Gamma_{D^*}$ (MeV). Units of $\Gamma$ are $|V_{bc}|^2 \times 10^{-11}$ GeV.

| Model | | $\Gamma_{D^*}$ | | | | |
|---|---|---|---|---|---|---|
| | | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 |
| no $B^*$ pole | $\Gamma$ | 1.398 | 1.528 | 1.658 | 1.789 | 1.919 |
| | $\alpha$ | 1.058 | 1.084 | 1.104 | 1.122 | 1.144 |
| | $A_{\rm fb}$ | 0.192 | 0.190 | 0.188 | 0.186 | 0.185 |
| $D^* + B^*$ exchange | $\Gamma$ | 1.398 | 1.458 | 1.517 | 1.576 | 1.637 |
| | $\alpha$ | 1.058 | 1.116 | 1.174 | 1.224 | 1.276 |
| | $A_{\rm fb}$ | 0.192 | 0.189 | 0.186 | 0.184 | 0.181 |
| $D^* - B^*$ exchange | $\Gamma$ | 1.398 | 1.885 | 2.376 | 2.865 | 3.358 |
| | $\alpha$ | 1.058 | 1.316 | 1.494 | 1.620 | 1.720 |
| | $A_{\rm fb}$ | 0.192 | 0.175 | 0.165 | 0.158 | 0.153 |

Wigner factor. That this occurs for $\Gamma_{D^*}$ values which are very small compared to $m_{D^*}$ is rather unexpected, in particular that such a tiny width as 0.4 MeV increases the $B \to D^*\ell\nu$ width by 29% compared to the narrow-width approximation. The reason for this is that the $D^*$ mass is only 6 MeV from the $D\pi$ threshold, where the phase space has a strong dependence on $s_{23}$, the invariant mass of the $D\pi$ system. Then the Breit–Wigner function is distorted and has a very asymmetric shape with a long tail on the high side. [We have run the calculation for a $D^*$ mass of 2.100 GeV (instead of the accepted value of 2.010 GeV) and find no difference between zero-width and $\Gamma_{D^*} = 0.4$ MeV.] In view of the sensitivity of the $B \to D\pi\ell\nu$ rate on the total width of the $D^*$ one might think that accurate measurement of $B \to D\pi\ell\nu$ is a practical albeit indirect way of gaining information on $\Gamma_{D^*}$. (Of course a direct measurement of $\Gamma_{D^*}$ seems very difficult if not impossible if it is less than 1 MeV.) Unfortunately up to now this has not been possible due to the large background above the $D^*$ resonance which must be subtracted in the experimental analysis.

The analysis of ARGUS [8] is based on the assumption that the $D^*$ width is small compared to that group's experimental resolution of 1 MeV. Thus the most direct comparison with this reference is in the narrow-width approximation, where the effect of $B^*$ exchange vanishes. Using the $B$ lifetime ($1.32 \times 10^{-12}$ s) and the branching ratio reported by ARGUS [8],

$$\mathrm{Br}(B \to D^*\ell\nu) = (5.2 \pm 0.5 \pm 0.6)\%, \tag{11}$$

we find from the narrow-width column of table 1 that

$$|V_{bc}| = 0.043. \tag{12}$$

This agrees with the value of ARGUS derived from HQET with their "model A" for the Isgur–Wise function, which is numerically very close to our dipole form. ARGUS also reports a new measurement of $\alpha$ and $A_{\rm fb}$,

$$\alpha = 1.1 \pm 0.4 \pm 0.2,$$
$$A_{\rm fb} = 0.20 \pm 0.08 \pm 0.06. \tag{13}$$

These results are independent of $V_{bc}$ and agree nicely with our results in the zero-width approximation [#1]. Although the angular correlation parameters change as a function of $\Gamma_{D^*}$, as we can see from table 1, the change is still inside the limited accuracy of the ARGUS data. In addition we must remember that their analysis is based on the assumption that $\Gamma_{D^*}$ is small compared to their experimental resolution of 1 MeV.

In their recent ARGUS paper the group also found evidence for $D^{**}$ production in semileptonic decays. In fact they report the total semileptonic branching ratio to $D$, $D^*$ and $D^{**}$ final states,

$$\mathrm{Br}(\overline{B} \to (D, D^*, D^{**})\ell^-\overline{\nu})$$
$$= [(9.4\text{–}9.8) \pm 1.0 \pm 0.9]\%, \tag{14}$$

where the numbers in the parentheses represent the spread due to theoretical models used in the analysis. Thus the states $D$, $D^*$, $D^{**}$ almost exhaust the semileptonic rate [10],

$$\mathrm{Br}(B \to X\ell^-\overline{\nu}) = 10.7 \pm 0.5\%, \tag{15}$$

leaving at most a few percent for states like the additional finite-width effects we found with or without $B^*$ pole contributions. This offers the possibility of ruling out some of the models in table 1. Averaging the numbers in (14) and combining it with (15) we get a limit for the extra $D\pi$ contribution of

[#1] Similar results are reported by the CLEO Collaboration [9], with a cut in the lepton momentum $p_\ell < 1$ GeV.

$(2.4 \pm 0.5)\%$. This means that the total $D\pi$ contribution including the $D^*$ should obey $\Gamma < 2.18$ in units of table 1. By examining the results of table 1 we find that $\Gamma_{D^*} < 0.16$ MeV for the admittedly unrealistic model of $D^* - B^*$. For the other two models the limits are less strict. By making runs with larger $\Gamma_{D^*}$ we obtain $\Gamma_{D^*} < 0.60$ MeV for the model with no $B^*$ pole and $\Gamma_{D^*} < 1.3$ MeV for the HQET/CS model $(D^* + B^*)$. If in the future the error on the branching ratios in (14) and (15) can be reduced, better limits on $\Gamma_{D^*}$ for the more interesting HQET/CS model may be obtained. If on the other hand the experimental accuracy is high enough that a deficit between the total semileptonic rate and the rate from the sum of resonances could be established this deficit could be explained by a finite-width effect of the $D^*$. So, for example, if the $D^*$ width is as large as 0.2 MeV the finite-width effect yields as much as 0.4% to the total semileptonic rate in the most realistic HQET/CS model.

The experimental upper bound on $\Gamma_{D^*}$ is $\Gamma_{D^*} < 1.1$ MeV [10] which is about the same as the bound we have found for the $D^* + B^*$ model. From the theoretical point of view, using SU(4) or potential models, the total width of the $D^*$ is expected to be much smaller, about 0.1 MeV. For a review see ref. [11]. (See also Yan et al. [4].)

We carried through this calculation assuming that the $D^*(2010)^+$ decays entirely to $D^0\pi^+$ and $D^+\pi^0$. The particle data group [10] reports that the branching ratio of these channels is $(82 \pm 4)\%$ leaving the rest to the $D^+\gamma$ channel. However it seems increasingly unlikely, on theoretical as well as experimental grounds, that this branching ratio is more than a few percent [11].

Finally, to improve the calculation we unitarized the angular momentum $J = 1$ partial wave by satisfying Watson's theorem using the method described in our earlier work [12]. The effect was insignificant.

In conclusion we have used HQET/CS to calculate $B \to D^*\ell\nu \to D\pi\ell\nu$. Because the $D^*$ is so close to threshold, finite-width effects are important and the rate depends on the $D^*$ width. The model is consistent with the data for $|V_{bc}| = 0.043$. Improved data on $\mathrm{Br}(B \to (D, D^*, D^{**})\ell\nu)$ and the total semileptonic rate may well provide the best constraint on the $D^*$ width.

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