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On the Quantum Equivalence Principle

Hermann Hessling

Deutsches Elektronen-Synchrotron (DESY), Notkestrasse 85, D-22603 Hamburg, Germany

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According to Galilei "all bodies fall with the same speed". How can the idea, upon which Einstein's explanation of this fact is founded, be transferred into quantum field theory? A formulation of a *Quantum Equivalence Principle* (QEP) is suggested here. It is applicable to linear quantum field theories but cannot be applied directly to interacting quantum fields. Equilibrium states of free quantum fields in the Rindler wedge are analyzed and it is shown that only the state with the Hawking–Bisognano–Wichmann temperature is admitted by the Quantum Equivalence Principle.

1. Introduction

Galilei was the first to realize that gravitational and inertial mass are equivalent. He discovered that gravity acts on all bodies in such way that they fall with the same speed. Other forces have been found but none of these show this behavior of equivalence. For example electrons and positrons in an electromagnetic field move in different directions.

Einstein gave a paradoxical explanation of the equivalence principle: "locally" there is no gravity.

Consider a classical pointlike testparticle in a gravitational field. In every point of the worldline it is possible to find a coordinate system ("local inertial system") where the equation of motion looks trivial, i.e. does not differ from the equation of motion in the absence of gravitation. To observe gravitational effects the worldline has to be considered in some neighborhood of a point and not only in a single point. The influence of gravitation on other classical objects, e.g. classical fields, can also be transformed away pointwise by choosing a local inertial system.

It is not so clear how to describe the influence of gravitation on quantum fields because on the one hand the Equivalence Principle is a consequence of the small-distance behavior of the gravitational force but on the other hand the small-distance behavior of quantum fields is singular.

The small-distance behavior of quantum field theories is related to the problem of characterizing "physically realizable states". In quantum field theories in Minkowski spacetime energy plays an important role in describing physically realizable states. Since in experiments only a finite amount of energy is available, only those states have to be considered as physically realizable whose energy differs by a finite amount from the energy of the vacuum state. This suggests that all physically realizable states are locally quasi-equivalent * to the vacuum state [14].

Because of its translation invariance and invariance under Lorentz transformations the vacuum state is a distinguished state from a geometrical point of view. Moreover the vacuum state is (in its GNS representation) a ground state of the generator of the time evolution along the time axis of an inertial coordinate system. In curved spacetimes there exists no global inertial system. For that reason one cannot in general define a "vacuum state" in a curved spacetime. Is it possible to characterize physically realizable states without knowledge of a reference state? The *Principle of Local Definiteness* (PLD) of Haag, Narnhofer and Stein [13] postulates: physically realizable states are locally quasi-equivalent. The physical content of this principle is that the expectation values $\langle A \rangle$, $\langle A \rangle'$ of a field observable A in the physically realizable states $\langle\ \rangle$, $\langle\ \rangle'$ become indistinguishable as the localization region of A is contracted to a point. There are criteria in the case of linear quantum fields in the Robertson–Walker spacetime from which the PLD follows [1,17].

A general characterization of physically realizable states in curved spacetimes has not been given till now, but the concept of the *scaling limit of states* introduced by Fredenhagen and Haag [11] is of great importance in this context. The scaling limit allows the formulation of a *Principle of Local Stability* (PLS): the scaling limit of a state in a curved spacetime does not differ from the scaling limit of the vacuum state in Minkowski spacetime [13,11]. But local stability is not sufficient to fix the quasi-equivalence class of a state. A counterexample in connection with the Robertson–Walker spacetime has already been given in ref. [13]. Another counterexample in connection with the problem of the Hawking temperature will be discussed in sect. 3.

The general concepts just mentioned are well explained in the case of Hadamard states. Hadamard states are definable in linear quantum field theories and are

* Let $\langle A \rangle$ be the expectation value of a field observable A localized in a finite spacetime region. It is possible (via the GNS construction; e.g. ref. [14]) to obtain a Hilbert space $\mathscr{H}$ and in this Hilbert space $\mathscr{H}$ a representation π of the field observables as linear operators $\pi(A)$. A state $\langle\ \rangle'$ is *locally quasie-equivalent* to the state $\langle\ \rangle$ if it is representable by a density matrix ρ in the Hilbert space $\mathscr{H}$: $\langle A \rangle' = \mathrm{Tr}\, \rho \pi(A)$.

quasifree states * with a specific singularity structure: the symmetric part of the two-point function is identical with Hadamard's fundamental solution of the wave equation [10]

$$\langle\{\phi(P'), \phi(P)\}\rangle = u/\sigma + v \ln \sigma + w. \tag{1}$$

The functions u, v, w are regular in P and P'. The information about the state is contained in w; u and v are state-independent. In the limit where the localization points P, P' of the quantum field ϕ coincide ("scaling limit") only the most singular term u/σ contributes. According to the Principle of Local Stability the u/σ term has to be compared with the most singular part of the vacuum state of the quantum field ϕ in the Minkowski spacetime. This gives the right "$i\varepsilon$ prescription", i.e. the interpretation of the $1/\sigma$ singularity in the sense of distributions. It was shown by Verch [21] that Hadamard states satisfy the PLD, i.e. Hadamard states are "physically realizable".

There is a conjecture by Haag [14] that quasifree states fulfill the PLD, if the symmetric part of their two-point functions allows an expansion

$$\langle\{\phi(P'), \phi(P)\}\rangle = -\frac{1}{2\pi^2}\frac{1}{\sigma(P', P)} + \Delta w(P', P) \tag{2}$$

in which the leading singularity is proportional to the square of the geodesic distance σ between P' and P, i.e. has a singularity of order two, and if Δw has a singularity of order less than one. Hadamard states fulfill the assumptions of Haag's conjecture.

The motivation of Haag, Narnhofer and Stein [13] to introduce the PLD was to understand the Hawking temperature of black holes [15] from a fundamental point of view. They consider KMS states ** with respect to the timelike Killing vector field of a black hole and showed that the only KMS state, which fulfills the PLS, is the one with the Hawking temperature. But their derivation of the Hawking temperature has some shortcomings. They used a very singular scaling procedure, not in accordance with the prescription of ref. [11], and put one point of the two-point function of the KMS state on the intersection of the past and future horizon of a static black hole. But a realistic black hole is the final state of a collapsed star and there is no past horizon. As we will show, the PLS is not sufficent, but the QEP is needed in the case of a realistic black hole in order to arrive at the Hawking temperature as the unique equilibrium temperature of a black hole.

In sect. 2 we attempt a formulation of the QEP. While this appears appropriate

* A state is called quasifree, if its truncated n-point functions vanish for $n \neq 2$.

** KMS = Kubo, Martin, Schwinger [16,18].

for linear fields it is not directly applicable in asymptotically free theories like QCD *.

In sect. 3 the QEP is applied to equilibrium states of Klein–Gordon and Dirac fields in the Rindler wedge to demonstrate that it leads to some interesting physical insights even for linear theories. Especially it is shown that only one equilibrium state is allowed by the QEP, the one with the Hawking–Bisognano–Wichmann temperature. The result for the Dirac field disproves a statement in ref. [19].

2. Quantum Equivalence Principle

In General Relativity the gravitational field is represented by the metric tensor. It is always possible to introduce coordinates $P^{\mu} = (t, x, y, z)$ around an arbitrary point P_* in spacetime in such a way that the components of the metric tensor $g_{\mu\nu}$ in P_* coincide with the components of the Minkowski metric $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ and the partial derivatives vanish,

$$g_{\mu\nu}(P_*) = \eta_{\mu\nu}, \quad \partial_{\rho} g_{\mu\nu}(P_*) = 0. \tag{3}$$

Such a coordinate system is called a *local inertial system* around P_*. The physical meaning of eqs. (3) is that the gravitational field can be transformed away in an infinitesimal neighborhood of P_*. There is no gravitation "locally". Gravitation is an effect of second order. This explains the experimentally very well tested Equivalence Principle: "All bodies fall with the same speed" (Galilei).

We want to transfer Einstein's idea, that the influence of gravitation is observable only beyond the first order, to other interactions. Are there interactions which vanish at small length scales, comparable to the gravitational force? Since the other fundamental classical interaction, electromagnetism, does not show this behavior we have to got to the quantum level. If there should be an interaction with this short-distance behavior this would show that there is a QEP.

We first concentrate on the problem how to define quantum field theories which are "free up to first order". This is a nontrivial task, because the short-distance behavior of quantum fields is singular even in the linear case.

To begin with, we remark that the content of eqs. (3) can be reformulated as follows: there is a coordinate system $P^{\mu} = (t, x, y, z)$ around P_* such that for all

* One of the referees of this paper mentioned that the term "Quantum Equivalence Principle" has already been introduced in the literature. This was not known to us. The definition presented in ref. [22] is interesting but differs from our notation. It is formulated as some differential conditions on a hypersurface. But a hypersurface is a global concept and exactly this global aspect does not fit into our understanding of the QEP as a principle which describes the behavior of nature at small distances like the Equivalence Principle.

points P in a small neighborhood around P_*

$$\lim_{\lambda \to 0} g_{\mu\nu}(\chi_\lambda P) = \eta_{\mu\nu}, \quad \lim_{\lambda \to 0} \frac{\mathrm{d}}{\mathrm{d}\lambda} g_{\mu\nu}(\chi_\lambda P) = 0, \tag{4}$$

where (with respect to this coordinate system)

$$(\chi_\lambda P)^\mu = P^\mu{}_* + \lambda(P^\mu - P_*{}^\mu) \tag{5}$$

is a one-parametric scaling diffeomorphism with $\chi_1 P = P$ and $\chi_0 P = P_*$. In the limit $\lambda \to 0$ the point P is scaled into the point P_* along the path defined by the diffeomorphism χ_λ. The first equation in (4) gives the "value" of the metric tensor in the scaling point P_* by a one-parametric scaling procedure. The second equation in (4) means that the metric tensor is "constant up to first order in λ".

A one-parametric scaling procedure is also customary to analyze the short-distance behavior of quantum fields. For simplicity we consider in this section the case of a scalar quantum field $\phi(P)$. We assume that the field ϕ has been renormalized relative to the state $\langle\ \rangle$.

The one-parametric diffeomorphisms χ_λ give rise to of an action α_λ on products of the quantum field *,

$$\alpha_\lambda(\phi(P_1)\dots\phi(P_n)) = N(\lambda)^n \phi(\chi_\lambda P_1)\dots\phi(\chi_\lambda P_n). \tag{6}$$

In the limit $\lambda \to 0$ the localization points $P_1, \dots, P_n$ of the product $\phi(P_1)\dots\phi(P_n)$ move under the action of α_λ along the path χ_λ into the point P_*. In addition each field $\phi(P_i)$ is scaled by a function $N(\lambda)$. The *scaling function* $N(\lambda)$ is important to define the short-distance behavior of products of the quantum field.

The state $\langle\ \rangle$ has a *scaling limit* [11] in the point P_* if there is a scaling function $N(\lambda)$, which is monotonous and nonnegative for $\lambda > 0$, such that for every n-point function $\langle\phi(P_1)\dots\phi(P_n)\rangle$ the limit

$$\lim_{\lambda \to 0} \langle \alpha_\lambda(\phi(P_1)\dots\phi(P_n))\rangle = \omega_{P_1,\dots,P_n}(P_*) \tag{7}$$

exists and is nonzero for some n.

The right hand side of (7) can be considered as the "value" of the n-point function in the scaling point P_*. It was shown by Fredenhagen and Haag [11] that the scaling limit $\omega_{P_1,\dots,P_n}(P_*)$ is independent of the choice of the coordinate system, i.e. the special diffeomorphism χ_λ may be replaced by a general contractive one which has the point P_* as a fixpoint.

* The existence of the products is to be understood in the sense of distributions, i.e. (6) has to be smeared out with test functions $f^{(n)}(P_1, \dots, P_n)$ with supports contained in $\mathcal{O} \times \dots \times \mathcal{O} = \mathcal{O}^n$, $\mathcal{O}$ being a region in spacetime.

To adapt Einstein's idea as directly as possible into quantum field theory we introduce the following definition, motivated by (4). We call the state $\langle\ \rangle$ *constant up to first order* in P_* if its scaling limit (7) exists and if with respect to a local inertial system around P_* the derivative condition

$$\lim_{\lambda\to 0}\frac{\mathrm{d}}{\mathrm{d}\lambda}\langle\alpha_\lambda(\phi(P_1)\dots\phi(P_n))\rangle=0 \tag{8}$$

is fulfilled for all n-point functions.

In the case of quasifree states the derivative condition (8) is comparable with the assumption of Haag's conjecture about Δw in (2) to be *less singular than of order one.*

To make the derivative condition (8) independent of the local intertial system, the behavior of the derivative condition (8) under a general coordinate transformation has to be analyzed. One finds that (8) has to be replaced by

$$\lim_{\lambda\to 0}\frac{D}{D\lambda}\langle\alpha_\lambda(\phi(P_1)\dots\phi(P_n))\rangle=0, \tag{9}$$

where

$$\frac{D}{D\lambda}=\frac{\mathrm{d}}{\mathrm{d}\lambda}-\frac{1}{2}\sum_{i=1}^{n}\Gamma^{\mu}_{\nu\rho}(P_*)u_i^\nu u_i^\rho\partial^i_\mu$$

is the corresponding covariant derivative. The symbols $\Gamma^{\mu}_{\nu\rho}$ are the Christoffel symbols,

$$u_i=\frac{\mathrm{d}}{\mathrm{d}\lambda}(\chi_\lambda P_i)$$

is the tangential vector of the scaling diffeomorphism $\chi_\lambda P_i$ and

$$\partial^i_\mu=\frac{\partial}{\partial P_i^\mu}$$

is the partial derivative relative to the ith localization point P_i of the n-point function.

We say the state $\langle\ \rangle$ is admitted by the QEP if it is constant up to first order in every point P_* of spacetime and if the scaling limit $\omega_{P_1,\dots,P_n}(P_*)$ of every n-point function depends continuously on P_*.

An important class of states which is admitted by the QEP are the Hadamard states. (A suitable scaling function is $N(\lambda)=\lambda$ and it is not difficult to check that Hadamard states fulfill the derivative condition (8).) In particular the vacuum state of the Klein–Gordon field in Minkowski spacetime is allowed by the QEP. This shows that at least for linear quantum field theories the QEP can be a criterion which selects theories *free up to first order* (*in* λ).

We now want to check whether the QEP is applicable to interactive quantum field theories. It seems natural to consider the QEP only for quantum field theories which are asymptotically free in the sense of renormalization group theory because in asymptotically free theories the interactions go to zero in the short-distance limit.

A simple example of an asymptotically free theory is the ϕ^3-theory in the six-dimensional Minkowski spacetime,

$$\mathscr{L} = \tfrac{1}{2}\partial^{\mu}\phi\ \partial_{\mu}\phi - \tfrac{1}{2}m^2\phi^2 - \frac{g}{3!}\phi^3.$$

The energy is not bounded from below. This is why this theory has no stable states. But perturbatively it is well defined. We want to analyze the scaling limit of the propagator

$$\langle T\phi(P')\phi(P)\rangle = i\int\frac{\mathrm{d}^6p}{(2\pi)^6}\frac{\exp[-ip\cdot(P'-P)]}{p^2-m^2-\Sigma(p)+i\varepsilon}.$$

Σ is the self-energy. With dimensional regularization and the method of minimal subtraction the renormalized self-energy becomes in one-loop approximation [9, p. 58]

$$\Sigma_{1\mathrm{R}} = -\frac{g^2}{128\pi^3}\Bigg((m^2-\tfrac{1}{6}p^2)(\gamma_E-1) + \int_0^1\mathrm{d}x\left[m^2-p^2x(1-x)\right]\,\ln\frac{m^2-p^2x(1-x)}{4\pi\mu^2}\Bigg). \tag{10}$$

$\gamma_E = 0.5772\ldots$ is the Euler number and μ the "unit of mass". A renormalization group analysis for

$$\Gamma = p^2 - m^2 - \Sigma_{\mathrm{R}} \tag{11}$$

yields the following scaling property under the scaling transformation $p \to \sigma p$ [20]:

$$\Gamma(\sigma p, g, m, \mu) = \sigma^2\exp\left(-2\int_g^{g(\sigma)}\mathrm{d}g'\frac{\gamma(g')}{\beta(g')}\right)\Gamma(p, g(\sigma), m(\sigma), \mu),$$

with $g(\sigma)$ the *running coupling constant*,

$$\sigma\frac{\mathrm{d}}{\mathrm{d}\sigma}g(\sigma) = \beta(g) \tag{12}$$

and $m(\sigma)$ the *running mass*, which we put equal to zero because we are interested in the short-distance limit $P \to P'$, i.e. high-energy limit, where the mass should be negligible and not influence our later arguments. In the ϕ^3-theory the β- resp. γ-function are perturbatively given by [9, pp. 181 and 184]

$$\beta = -A_1 g^3 + O(g^5), \quad \gamma = C_1 g^2 + O(g^4), \tag{13}$$

where $A_1 = 3/256\pi^3$ and $C_1 = 1/384\pi^3$. After integration of (12) the running coupling constant becomes in lowest order

$$g(\sigma)^2 = \frac{g^2}{1 + g^2 A_1 \ln \sigma^2}.$$

The right hand side goes to zero as $\sigma \to \infty$. Because of this property the ϕ^3-theory in six dimensions is asymptotically free. We note that the running coupling constant does not "smoothly" become zero,

$$\frac{\mathrm{d}}{\mathrm{d}\lambda} g\left(\frac{1}{\lambda}\right)^2 \sim \frac{1}{\lambda(\ln \lambda)^2} \to \infty, \quad \text{as} \quad \lambda \to 0. \tag{14}$$

In lowest order

$$\exp\left(-2\int_g^{g(\sigma)} \mathrm{d}g' \frac{\gamma(g')}{\beta(g')}\right) = \left(\frac{g(\sigma)^2}{g^2}\right)^{C_1/A_1} \sim (\ln \sigma^2)^{-C_1/A_1}, \quad \text{as} \quad \sigma \to \infty.$$

Thus for the scaling function we choose *

$$N(\lambda) = \lambda^2 \left(\frac{g(1/\lambda)}{g}\right)^{C_1/A_1}.$$

We obtain in one-loop approximation for the renormalized and scaled propagator

$$N(\lambda)^2 \langle T\phi(\lambda P')\phi(\lambda P)\rangle = i\frac{N(\lambda)^2}{\lambda^6}\int \frac{\mathrm{d}^6 p}{(2\pi)^6} \frac{\exp[-ip\cdot(P'-P)]}{\Gamma(p/\lambda, g, 0, \mu) + i\varepsilon}$$

$$= i\int \frac{\mathrm{d}^6 p}{(2\pi)^6} \frac{\exp[-ip\cdot(P'-P)]}{\Gamma(p, g(1/\lambda), 0, \mu) + i\varepsilon}.$$

* The fact that $N(\lambda)$ is not positive and monotonous for $\lambda > 0$, but only for $0 < \lambda < \exp(1/2g^2 A_1)$, does not matter because merely $\lambda \approx 0$ is analyzed.

If we differentiate with respect to λ, we see by using (10), (11) and (14), that the limit $\lambda \to 0$ diverges and the derivative condition (8) of the QEP is not fulfilled. This is due to the property of the running coupling constant $g(1/\lambda)$ that it does not smoothly become zero in the short-distance limit $\lambda \to 0$.

From the viewpoint of perturbation theory QCD is very similar to the ϕ^3-theory in six dimensions *. Therefore the QEP does not harmonize with QCD. This means that the derivative condition (8) has to be modified if one wants to formulate a QEP for interactive quantum field theories.

3. Equilibrium states in the Rindler wedge

In this section we calculate equilibrium states of a Klein–Gordon field $\phi(P)$ and a Dirac field $\psi(P)$ in the Rindler wedge and investigate with the QEP which temperatures are physically allowed.

In quantum mechanics of finite degrees of freedom equilibrium states are characterized by

$$\langle A \rangle = \frac{\mathrm{Tr}(\mathrm{e}^{-\beta H} A)}{\mathrm{Tr}\ \mathrm{e}^{-\beta H}}, \tag{15}$$

where H is the Hamilton operator and A an observable. They are equally well described by the *KMS boundary condition*

$$\langle A_\tau B \rangle = \langle B A_{\tau + i\beta} \rangle. \tag{16}$$

$A_\tau = \mathrm{e}^{iH\tau} A \mathrm{e}^{-iH\tau}$ is the time translated observable A.

In quantum field theory the right hand side of (15) does not exist. But the KMS boundary condition (16) remains valid [12]. Therefore equilibrium states in quantum field theory are considered as KMS states:

Let $\tau \mapsto A_\tau$ be the time evolution of a field observable A. A *KMS state* $\langle\ \rangle$ with the temperature $1/\beta$ relative to this time evolution is characterized by the conditions that $\langle B A_{\tau + i\varepsilon} \rangle$ considered as a function in $\tau + i\varepsilon$ is analytic in the strip $0 < \varepsilon < \beta$ and that $\langle B A_{\tau + i\varepsilon} \rangle$ fulfills in the limit $\varepsilon \to \beta$ the KMS boundary condition (16) [12,8,14].

We first consider the Klein–Gordon field and choose as the observable the field itself $A = \phi(P)$. The time evolution is defined via the one-parametric group of automorphisms

$$\tau \mapsto \phi_\tau(P) = \phi(\Lambda_\tau P), \tag{17}$$

* The β- and γ-functions formally have the same perturbative expansions as (13).

where $P_\tau = \Lambda_\tau P$ is the one-parametric group of diffeomorphisms defined by

$$P_\tau^\mu = \begin{pmatrix} t_\tau \\ x_\tau \\ y_\tau \\ z_\tau \end{pmatrix} = \begin{pmatrix} \cosh\tau & \sinh\tau & & \\ \sinh\tau & \cosh\tau & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}. \tag{18}$$

P_τ describes a velocity transformation in the x-direction which leaves the *Rindler wedge*

$$\mathscr{W} = \{P^\mu \mid |t| < x\}$$

invariant. The coordinates $(t, x, y, z,)$ are the standard inertial coordinates of the Minkowski spacetime ($\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2$). These coordinates are regular not only in the Rindler wedge $\mathscr{W}$ but also on the horizon $|t| = x$ of the Rindler wedge.

By Fourier transforming the KMS boundary condition and by using their analytical properties KMS states are representable in terms of the commutator function. Assuming a vanishing one-point function $\langle\phi(P)\rangle$ one has for $P', P \in \mathscr{W}$

$$\begin{aligned} &\langle\phi(P')\phi_{\tau+i\varepsilon}(P)\rangle \\ &= \frac{1}{2\pi}\int \mathrm{d}\omega\ \mathrm{d}\tau' \langle[\phi_{\tau'}(P), \phi(P')]\rangle \frac{\exp[-i\omega(\tau' - \tau - i\varepsilon)]}{\mathrm{e}^{-\beta\omega} - 1} \\ &= \frac{i}{2\beta}\int \mathrm{d}\tau' \langle[\phi_{\tau'}(P), \phi(P')]\rangle \coth\frac{\pi}{\beta}(\tau' - \tau - i\varepsilon). \end{aligned} \tag{19}$$

The crucial point is that in linear field theories the commutator function is state-independent. Thus equilibrium states of the Klein–Gordon field are computable via (19). For the nonequal time commutator of a massless Klein–Gordon field one finds

$$[\phi(P), \phi(P')] = \frac{1}{2\pi i}\ \mathrm{sign}(t - t')\delta(\sigma(P, P')), \tag{20}$$

where

$$\sigma(P, P') = (t - t')^2 - (x - x')^2 \tag{21}$$

is the square of the geodesic distance between P, P' and sign(t) the sign function

(= 1 if $t > 0$ and $= -1$ if $t < 0$) [7]. Inserting the commutator function (20) into (19) we obtain

$$\langle \phi(P')\phi_{\tau+i\varepsilon}(P)\rangle = \frac{1}{4\pi\beta}\frac{1}{\dot{\sigma}}\left(\coth\frac{\pi}{\beta}(\tau_+ - \tau - i\varepsilon) - \coth\frac{\pi}{\beta}(\tau_- - \tau - i\varepsilon)\right) \tag{22}$$

$$= \frac{1}{2\pi\beta\dot{\sigma}}\left(\cosh\frac{\pi}{\beta}(\tau_+ - \tau_-) - \cosh\frac{\pi}{\beta}[\tau_+ + \tau_- - 2(\tau + i\varepsilon)]\right)^{-1}$$

$$\times \sinh\frac{\pi}{\beta}(\tau_+ - \tau_-). \tag{23}$$

The two times $\tau_\pm = \tau_\pm(P', P)$ are implicitly given by

$$\sigma(P_{\tau\pm}, P') = 0 \tag{24}$$

and mark the intersection points of the light cone in P' with the curve P_τ. From

$$\sigma(P_\tau, P') = \sigma(P, P') + 2(xx' - tt')(\cosh\tau - 1) + 2(tx' - xt')\sinh\tau \tag{25}$$

follows

$$\tau_\pm = \ln\frac{C \pm \sqrt{C^2 - 4(x^2 - t^2)(x'^2 - t'^2)}}{2(x + t)(x' - t')}, \tag{26}$$

where

$$C = 2(xx' - tt') - \sigma(P, P').$$

The quantities

$$\sigma_\pm = \left|\frac{d\sigma(P_\tau, P')}{d\tau}\right|_{\tau=\tau_\pm} \tag{27}$$

are the velocities $\dot{\sigma}_\pm = \dot{\sigma}_\pm(P', P)$, with which the square of the geodesic distance σ between P_τ and P' varies at the two intersection times $\tau_\pm$. We obtain

$$\dot{\sigma}_+ = \dot{\sigma}_- = \sqrt{C^2 - 4(x^2 - t^2)(x'^2 - t'^2)} \equiv \dot{\sigma}. \tag{28}$$

From (23) we observe that the correlation between $\phi(P')$ and $\phi(P_\tau)$ decays exponentially ($\sim e^{-2\pi|\tau|/\beta}$). In the ground state ($\beta = \infty$) the correlation decays only like τ^{-2} (use in (22) that $\coth\varepsilon \sim 1/\varepsilon$ for $\varepsilon \approx 0$).

Substituting (26) and (28) into (23) we are able to express the two-point function by elementary functions

$$\langle\phi(P')\phi_\zeta(P)\rangle = \frac{\alpha}{4\pi^2}\frac{(C+\dot\sigma)^\alpha-(C-\dot\sigma)^\alpha}{\dot\sigma}\big[(C+\dot\sigma)^\alpha+(C-\dot\sigma)^\alpha$$
$$-2^\alpha(x+t)^\alpha(x'-t')^\alpha\,\mathrm{e}^{\alpha\zeta}-2^\alpha(x-t)^\alpha(x'+t')^\alpha\,\mathrm{e}^{-\alpha\zeta}\big]^{-1}, \tag{29}$$

with

$$\alpha\equiv\frac{2\pi}{\beta},\quad \zeta\equiv\tau+i\varepsilon,\quad 0<\varepsilon<\beta.$$

With the help of the QEP we want to analyze which temperatures $1/\beta>0$ of the KMS state (29) are "physically realizable". (It was shown by Beyer [2] that the $\beta=\infty$ case is not locally quasi-equivalent to the vacuum of the Minkowski space-time.)

Let $P^\mu_\lambda=P^\mu_*+\lambda(P^\mu-P^\mu_*)$ and $P'^\mu_\lambda=P^\mu_*+\lambda(P'^\mu-P^\mu_*)$ be one-parametric diffeomorphisms which scale P and P' from the inside of the Rindler wedge in the limit $\lambda\to0$ into the point P_* with the coordinates $(t_*,\ x_*,\ y_*,\ z_*)$. There are three cases singled out geometrically: P_* inside the Rindler wedge, P_* on the horizon and P_* in the intersection of the past and future horizon.

We introduce the abbreviations

$$\begin{aligned}
&\xi_+=(x-x_*)+(t-t_*),\quad \xi_-=(x-x_*)-(t-t_*),\\
&U'=(x_*-t_*)\xi_++(x_*+t_*)\xi'_-,\quad 'U=(x_*-t_*)\xi'_++(x_*+t_*)\xi_-,\\
&\xi'_+=(x'-x_*)+(t'-t_*),\quad \xi'_-=(x'-x_*)-(t'-t_*),\\
&X'=\xi_+\xi'_-,\quad 'X=\xi'_+\xi_-.
\end{aligned}$$

One has ($\sigma=\sigma(P',\ P)$)

$$\sigma_\lambda\equiv\sigma(P'_\lambda,\ P_\lambda)=\lambda^2\sigma,$$

$$C_\lambda\equiv2(x_\lambda x'_\lambda-t_\lambda t'_\lambda)-\sigma_\lambda=2(x_*^2-t_*^2)+('U+U')\lambda+('X+X'-\sigma)\lambda^2,$$

$$(x_\lambda+t_\lambda)(x'_\lambda-t'_\lambda)=x_*^2-t_*^2+U'\lambda+X'\lambda^2,$$

$$(x_\lambda-t_\lambda)(x'_\lambda+t'_\lambda)=x_*^2-t_*^2+'U\lambda+'X\lambda^2,$$

$$\dot\sigma_\lambda\equiv\sqrt{C_\lambda^2-4(x_\lambda^2-t_\lambda^2)(x'^2_\lambda-t'^2_\lambda)}=\lambda\sqrt{A^2+2B\lambda+D\lambda^2},$$

where

$$A^2 = ('U - U')^2 - 4(x_*^2 - t_*^2)\sigma,$$

$$B = ('U - U')('X - X') - ('U + U')\sigma,$$

$$D = ('X - X')^2 - 2('X + X')\sigma + \sigma^2.$$

Case 1: Scaling point inside the Rindler wedge ($x_* > |t_*|$). If in (29) P and P' are substituted by P_λ resp. P'_λ, we are able to write down the expansion of the KMS state (29). We restrict ourselves to the symmetric part of the state because we know already the antisymmetric part, the commutator. After an expansion of (29) we arrive at

$$\lambda^2 \langle \{\phi(P'_\lambda), \phi(P_\lambda)\} \rangle = -\frac{1}{2\pi^2}\frac{1}{\sigma} + O(\lambda^2). \tag{30}$$

Because of the temperature independence of the first term and the absence of the term of order λ we conclude that for scaling points inside the Rindler wedge the QEP gives no restriction to the temperature of the KMS state.

Case 2: Scaling point on the future horizon ($x_* = t_* > 0$). With

$$Z_\alpha = \frac{\alpha}{'U^\alpha - U'^\alpha}\left(\frac{X' + 'X - \sigma}{2}('U^{\alpha-1} - U'^{\alpha-1}) + \frac{B}{2A}(U'^{\alpha-1} + 'U^{\alpha-1})\right)$$

and

$$Y_\alpha = \frac{\alpha - 1}{2\sigma}\frac{'U - U'}{'U^\alpha - U'^\alpha}\left(U'^\alpha \frac{('X + X' - \sigma - B/A)^2 - 4X'^2}{(2U')^2}\right.$$

$$\left. + 'U^\alpha \frac{('X + X' - \sigma + B/A)^2 - 4'X^2}{(2'U)^2}\right) + \frac{'U^{\alpha-1} - U'^{\alpha-1}}{'U^\alpha - U'^\alpha}\frac{A^2 D - B^2}{2A^2\sigma}$$

we can evaluate the anticommutator in the KMS state (29),

$$\lambda^2 \langle \{\phi(x'_\lambda), \phi(x_\lambda)\} \rangle = -\frac{1}{2\pi^2}\frac{1}{\sigma}\left[1 + \left(Y_\alpha + Z_\alpha - \frac{B}{A^2}\right)\lambda\right] + O(\lambda^2).$$

The first term is again independent of the temperature, so that the scaling limit gives no restriction to the temperature of the KMS state. But if the derivative condition (8) of the QEP is taken into account, one sees that the KMS state is constant up to first order only if the coefficient of the term of order λ vanishes for all P', P in the Rindler wedge,

$$Y_\alpha + Z_\alpha - B/A^2 = 0. \tag{31}$$

This, in turn, is only possible if $\alpha \equiv 2\pi/\beta = 1$, as we shall see immediately, and consequently the KMS state has the temperature $1/\beta = 1/2\pi$. This is the Hawking–Bisognano–Wichmann temperature [15,3,4].

To solve eq. (31) is equivalent to determining α, which solves the equation $(E = (x + t - 2t_*)/2t_*,\ E' = (x' + t' - 2t_*)/2t_*)$

$$\frac{U'^\alpha}{'U^\alpha}\left[\frac{1-\alpha}{2}A\left(\sigma + 2AE'\frac{'U}{U'}\right) + \alpha A\left(\sigma + E'A\frac{'U}{U'}\right) + \frac{A^2D - B^2}{2\sigma U'} - B\right]$$

$$= \frac{\alpha - 1}{2}A\left(\sigma - 2AE\frac{U'}{'U}\right) - \alpha A\left(\sigma - EA\frac{U'}{'U}\right) + \frac{A^2D - B^2}{2'U\sigma} - B \tag{32}$$

for all P', P in the Rindler wedge. Since this equation cannot be solved in closed form, we use the following trick. If the points $P = (t,\ x_0 + \xi,\ y,\ z)$, $P' = (t',\ x_0 + \xi',\ y',\ z')$ are far away from the edge of the Rindler wedge, i.e. $x_0 \gg |\xi|$, $|\xi'|$, $|t|$, $|t'|$, we are able to bring down the exponent α in (32): $(U'/'U)^\alpha = 1 + \alpha\delta$ with $|\delta| = |\xi' - t' - \xi + t|/x_0 \ll 1$. Thus, in the far away limit, (32) is reduced to $(\alpha - 1)\delta + O(\delta^2) = 0$. From this follows $\alpha = 1$, as claimed.

Case 3: Scaling point in the intersection of future and past horizon $(x_* = t_* = 0)$. The scaled two-point function (29) is independent of the scaling parameter λ,

$$\lambda^2\langle\{\phi(P'_\lambda),\ \phi(P_\lambda)\}\rangle$$

$$= \frac{\alpha}{2\pi^2}\frac{1}{\sqrt{D}}\frac{\left(X' + 'X - \sigma + \sqrt{D}\right)^\alpha - \left(X' + 'X - \sigma - \sqrt{D}\right)^\alpha}{\left(X' + 'X - \sigma + \sqrt{D}\right)^\alpha + \left(X' + 'X - \sigma - \sqrt{D}\right)^\alpha - 2^\alpha X'^\alpha - 2^\alpha {'X}^\alpha}.$$

According to the continuity part of the QEP the scaling limit of the right hand side has to connect continuously with the scaling limit inside the Rindler wedge. The right hand side has to be $-1/2\pi^2\sigma$, the value given by (30). By repeating the trick of case 2 one can show that continuity is only possible if $\alpha = 1$ resp. $\beta = 2\pi$.

Let use summarize: in the Rindler wedge only the KMS state with the Hawking–Bisognano–Wichmann temperature is allowed by the QEP. If the scaling point is in the intersection of the future and past horizon it is sufficient to require that the scaling limit of the KMS state depends *continuously* on the scaling point P_* in order to fix the temperature. This shows that in this case the PLS can be replaced by the continuity requirement of the QEP. If the scaling point is on the horizon a unique temperature is singled out only by the QEP and not by the PLS.

Finally, we consider the scaling limit of KMS states of a Dirac field with the scaling point on the future horizon.

A Dirac field $\psi(P)$ is a solution of the equation

$$(i\gamma^\mu\partial_\mu - m)\psi(P) = 0.$$

The γ-matrices fulfill the anticommutation relations

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbf{1}.$$

A possible representation is

$$\gamma^0 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix},$$

with the Pauli matrices

$$\mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The anticommutator between the spinor ψ and the adjoint spinor $\bar{\psi} = \psi^\dagger\gamma^0$ is [6]

$$\{\psi(P'), \bar{\psi}(P)\} = i(i\gamma^\mu\partial_{\mu'} + m)\Delta(P', P). \tag{33}$$

For massless spinors, on which we restrict ourselves in the following, one has (cf. 20))

$$\Delta(P', P) = -\frac{1}{2\pi}\operatorname{sign}(t'-t)\delta(\sigma(P, P')).$$

The theory is invariant under the global gauge transformation $\gamma_\eta^{(1)}$, defined by

$$\psi \mapsto e^{i\eta}\psi, \quad \bar{\psi} \mapsto e^{-in}\bar{\psi}.$$

In the massless case one has moreover the chiral symmetry $\gamma_\theta^{(2)}$, defined by

$$\psi \mapsto \exp(i\theta\gamma^5)\psi, \quad \bar{\psi} \mapsto \bar{\psi}\exp(-i\theta\gamma^5),$$

with $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

Let $P_\tau = \Lambda_\tau P$ be the boost transformation defined in (18). Then α_τ, defined by

$$\psi(P) \mapsto S(\tau)\psi(\Lambda_\tau P), \quad \bar{\psi}(P) \mapsto \bar{\psi}(\Lambda_\tau P)S^{-1}(\tau),$$

where

$$S(\tau) = \exp(-\tfrac{1}{2}i\tau\sigma^{01}), \quad \sigma^{01} = \tfrac{1}{2}i[\gamma^0, \gamma^1],$$

acts as a velocity transformation in the x-direction [5].

The one-parametric symmetry transformation $\tau \mapsto \alpha^{\mu}_{\tau}$, defined by

$$\alpha^{\mu}_{\tau} = \alpha_{\tau} \circ \gamma^{(1)}_{-\mu_1 \tau} \circ \gamma^{(2)}_{-\mu_2 \tau},$$

where μ_1 and μ_2 are the chemical potentials belonging to the internal symmetries, has the following action on the adjoint spinor:

$$\alpha^{\mu}_{\tau}\bar{\psi}(P) = \bar{\psi}(P_{\tau}) \exp(i\mu_1 \tau)\left(\mathbf{1} \cos(\mu_2 \tau) + i\gamma^5 \sin(\mu_2 \tau)\right) \times \left(\mathbf{1} \cosh(\tfrac{1}{2}\tau) - \gamma^0\gamma^1 \sinh(\tfrac{1}{2}\tau)\right).$$

Let $\langle\, \rangle$ be a KMS state of the massless Dirac field $\psi(P)$ with temperature $1/\beta$ relative to the time evolution $\tau \mapsto \alpha^{\mu}_{\tau}$. Which temperatures and which chemical potentials of the KMS state are allowed by the QEP?

As before, KMS states can also be represented in terms of the anticommutator function,

$$\begin{aligned}\langle \psi(P')\alpha^{\mu}_{\tau+i\varepsilon}\bar{\psi}(P)\rangle &= \frac{1}{2\pi}\int \mathrm{d}\omega\, \mathrm{d}\tau' \langle \{\psi(P'),\, \alpha^{\mu}_{\tau'}\bar{\psi}(P)\}\rangle \frac{\exp[-i\omega(\tau'-\tau-i\varepsilon)]}{\mathrm{e}^{-\beta\omega}+1} \\ &= \frac{1}{2i\beta}\int \mathrm{d}\tau' \Big\langle \{\psi(P'),\, \alpha^{\mu}_{\tau'}\bar{\psi}(P)\}\Big\rangle \left(\sinh\frac{\pi}{\beta}(\tau'-\tau-i\varepsilon)\right)^{-1}. \end{aligned} \tag{34}$$

The symmetric part of the state (34) is given by the anticommutator (33). Therefore we consider in the following only the antisymmetric part of (34). From (34) we compute

$$\begin{aligned}\Big\langle \big[\psi(P'),\, \alpha^{\mu}_{0}\bar{\psi}(P)\big]\Big\rangle &= \frac{1}{2i\beta}\int \mathrm{d}\tau \{\psi(P'),\, \alpha^{\mu}_{\tau}\bar{\psi}(P)\} \sinh(\pi\tau/\beta) \\ &= \frac{1}{2i\beta}\int \frac{\mathrm{d}\tau}{\sinh(\pi\tau/\beta)} \\ &\quad \times \exp(i\mu_1\tau)\Big(\{\psi(P'),\, \bar{\psi}(P_{\tau})\} \cos(\mu_2\tau) \cosh(\tfrac{1}{2}\tau) \\ &\quad + i\{\psi(P'),\, \bar{\psi}(P_{\tau})\gamma^5\} \sin(\mu_2\tau) \cosh(\tfrac{1}{2}\tau) \\ &\quad - \{\psi(P'),\, \bar{\psi}(P_{\tau})\gamma^0\gamma^1\} \cos(\mu_2\tau) \sinh(\tfrac{1}{2}\tau) \\ &\quad - i\{\psi(P'),\, \bar{\psi}(P_{\tau})\gamma^5\gamma^0\gamma^1\} \sin(\mu_2\tau) \sinh(\tfrac{1}{2}\tau)\Big). \end{aligned} \tag{35}$$

By using (33) the anticommutator becomes

$$\{\psi(P), \bar{\psi}(P_\tau)\} = -\frac{1}{2\pi}\gamma^\mu \partial_{\mu'} \frac{1}{\dot{\sigma}}[\delta(\tau-\tau_+) - \delta(\tau-\tau_-)],$$

where $\dot{\sigma} = \dot{\sigma}(P', P)$ is given by (28) and $\tau_\pm = \tau_\pm(P', P)$ by (26).

The commutator function between the first spinor component and the first component of the adjoint spinor becomes

$$\begin{aligned}
&\langle[\psi_1(P'), \alpha_0^\mu \bar{\psi}_1(P)]\rangle \\
&= \frac{i}{2\pi\beta} \\
&\quad\times\left[\partial_{t'}\frac{1}{\dot{\sigma}}\left(\exp(i\mu_1\tau_+)\cos(\mu_2\tau_+)\frac{\cosh(\frac{1}{2}\tau_+)}{\sinh(\pi\tau_+/\beta)}\right.\right. \\
&\left. - \exp(i\mu_1\tau_-)\cos(\mu_2\tau) - \frac{\cosh(\frac{1}{2}\tau_-)}{\sinh(\pi\tau_-/\beta)}\right) \\
&- i\partial_{z'}\frac{1}{\dot{\sigma}}\left(\exp(i\mu_1\tau_+)\sin(\mu_2\tau_+)\frac{\cosh(\frac{1}{2}\tau_+)}{\sinh(\pi\tau_+/\beta)}\right. \\
&\left. - \exp(i\mu_1\tau_-)\sin(\mu_2\tau) - \frac{\cosh(\frac{1}{2}\tau_-)}{\sinh(\pi\tau_-/\beta)}\right) \\
&+ (\partial_{x'} + i\partial_{y'})\frac{1}{\dot{\sigma}}\left(\exp(i\mu_1\tau_+)\cos(\mu_2\tau_+)\frac{\sinh(\frac{1}{2}\tau_+)}{\sinh(\pi\tau_+/\beta)}\right. \\
&\left.\left. - \exp(i\mu_1\tau_-)\cos(\mu_2\tau) - \frac{\sinh(\frac{1}{2}\tau_-)}{\sinh(\pi\tau_-/\tau)}\right)\right].
\end{aligned} \tag{36}$$

We want to calculate the scaling limit of (36) onto a point $P_* = (t_*, x_*, y_*, z_*)$, $t_* > 0$, on the future horizon of the Rindler wedge. If we replace $P \to P_\lambda = P_* + \lambda(P - P_*)$ and $P^* \to P'_\lambda = P_* + \lambda(P' - P_*)$, we get from (26) and by using the abbreviations introduced above

$$\tau_{+,\lambda} \equiv \ln\frac{C_\lambda + \dot{\sigma}_\lambda}{2(x_\lambda + t_\lambda)(x'_\lambda - t'_\lambda)} = \ln\frac{'U}{U'} + O(\lambda),$$

$$\tau_{-,\lambda} \equiv \ln\frac{C_\lambda - \dot{\sigma}_\lambda}{2(x_\lambda + t_\lambda)(x'_\lambda - t'_\lambda)} = Q\lambda + R\lambda^2 + O(\lambda^3), \tag{37}$$

where

$$Q = \frac{'X - X' - \sigma - B/A}{2U'},$$

$$R = -\frac{A^2D - B^2}{4A^3U'} - \frac{('X + X' - \sigma - B/A)^2 - 4X'^2}{8U'^2}.$$

$\tau_{-,\lambda}$ tends to zero as $\lambda \to 0$. Substituting these expansions into (36) gives

$$\lambda^3 \langle [\psi_1(P'_\lambda), \alpha_0^\mu \bar{\psi}_1(P_\lambda)] \rangle = \frac{i}{2\pi^2} \left(\partial_{t'} \frac{1}{AQ} + \frac{\pi}{\beta} S\lambda \right) + O(\lambda^2). \tag{38}$$

with

$$S = \partial_{t'} \frac{1}{A} \left[\exp(i\mu_1 \tau_{+,0}) \cos(\mu_2 \tau_{+,0}) \frac{\cosh(\frac{1}{2}\tau_{+,0})}{\sinh(\pi\tau_{+,0}/\beta)} + \frac{\beta}{\pi} \left(\frac{B}{A^2 Q} + \frac{R}{Q^2} - i\mu_1 \right) \right]$$

$$- i\partial_{z'} \frac{1}{A} \left(\exp(i\mu_1 \tau_{+,0}) \sin(\mu_2 \tau_{+,0}) \frac{\cosh(\frac{1}{2}\tau_{+,0})}{\sinh(\pi\tau_{+,0}/\beta)} - \frac{\beta\mu_2}{\pi} \right)$$

$$+ (\partial_{x'} + i\partial_{y'}) \frac{1}{A} \left(\exp(i\mu_1 \tau_{+,0}) \cos(\mu_2 \tau_{+,0}) \frac{\sinh(\frac{1}{2}\tau_{+,0})}{\sinh(\pi\tau_{+,0}/\beta)} - \frac{\beta}{2\pi} \right).$$

The first term in (38) is independent of the temperature and the chemical potentials, so the PLS is not sufficient to fix any of the state parameters. The coefficient of the term of order λ has to vanish according to the QEP. With the same trick which made it possible to solve (31), one finds that S is zero only if the chemical potentials μ_1, μ_2 vanish and the temperature $1/\beta$ is the Hawking–Bisognano–Wichmann temperature $1/2\pi$.

The QEP fixes not only the temperature but also the chemical potentials.

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