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Axial baryonic charge and the spin content of the nucleon: A lattice investigation

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The axial baryonic charge of the nucleon, $\Delta\Sigma$, which measures the quark fraction of the nucleon spin, is computed in lattice QCD with dynamical staggered fermions. We obtain the value $\Delta\Sigma=0.18\pm0.02$. This suggests that the quark spin is responsible for very little of the nucleon spin.

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The European Muon Collaboration (EMC) measurement [1] of the spin-dependent structure function of the proton, $g_1^p(x,Q^2)$, has provoked many speculations [2] about the internal spin structure of the nucleon. The first moment of $g_1^p(x,Q^2)$, which through the operator product expansion is given by the proton matrix element of the axial vector current weighted by the square of the quark charges (modulo radiative corrections), was found to be

$$\int_0^1 dx\, g_1^p(x,Q^2)=\tfrac{1}{2}(\tfrac{4}{9}\Delta u+\tfrac{1}{9}\Delta d+\tfrac{1}{9}\Delta s)$$
$$=0.126\pm0.010\pm0.015 \quad (1)$$

with

$$\Delta q s_\mu=\langle \mathbf{p},s|\bar{q}\gamma_\mu\gamma_5 q|\mathbf{p},s\rangle\ , \quad q=u,d,s\ , \quad (2)$$

where s_μ is the covariant spin vector of the proton. If we combine the result (1) with information from hyperon decays, neutron β decay, and the assumption of flavor SU(3), we obtain

$$\Delta\Sigma=\Delta u+\Delta d+\Delta s=0.04\pm0.16\ . \quad (3)$$

The quantity $\Delta\Sigma$ is the axial baryonic charge of the nucleon:

$$\Delta\Sigma s_\mu=\langle \mathbf{p},s|[\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d+\bar{s}\gamma_\mu\gamma_5 s]|\mathbf{p},s\rangle$$
$$=\langle \mathbf{p},s|j_\mu^{;05}|\mathbf{p},s\rangle\ , \quad (4)$$

which in a naive wave-function picture can be interpreted as the fraction of the nucleon spin that is carried by the quarks. For example, in an SU(6)-type model of the nucleon we would obtain $\Delta\Sigma=1$. The vanishingly small experimental value of $\Delta\Sigma$ is referred to as the spin crisis of the nucleon.

The nucleon matrix element of the axial baryonic current can be written [3]

$$\langle \mathbf{p},s|j_\mu^{;05}|\mathbf{p}',s'\rangle$$
$$=\bar{u}(\mathbf{p},s)[G_1(k^2)\gamma_\mu\gamma_5-G_2(k^2)k_\mu\gamma_5]u(\mathbf{p}',s')\ , \quad (5)$$

where $k=p-p'$ and

$$G_1(0)=\Delta\Sigma\ . \quad (6)$$

Unlike the octet current, $j_\mu^{;05}$ is not conserved due to the anomaly. In the chiral limit,

$$\partial_\mu j_\mu^{;05}=N_f\frac{1}{8\pi^2}\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}\ , \quad \tilde{F}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}\ , \quad (7)$$

where N_f is the number of flavors. As a result, the form factor $G_2(k^2)$ does not develop a (Goldstone boson) pole at $k^2=0$. Writing

$$\left\langle \mathbf{p},s\left|N_f\frac{1}{8\pi^2}\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}\right|\mathbf{p}',s'\right\rangle$$
$$=2m_N A(k^2)\bar{u}(\mathbf{p},s)i\gamma_5 u(\mathbf{p}',s')\ , \quad (8)$$

where m_N is the nucleon mass, one then finds

$$A(0)=G_1(0)\equiv\Delta\Sigma\ . \quad (9)$$

Thus, the quark fraction of the nucleon spin is given by the matrix element of the anomalous divergence of the axial baryonic current.

A meaningful lattice calculation of $\Delta\Sigma$ should (i) include dynamical fermions and (ii) employ a proper definition of $\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}$ in order to account for the topological origin of the anomalous divergence. (The "naive" definition of $\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}$ would lead to ultraviolet divergent contributions which we do not know how to renormalize.) These requirements were not satisfied in earlier calculations [4,5]. For a preliminary calculation along these lines see, however, Ref. [6].

The calculations in this paper are done on a $16^3\times24$ lattice at $\beta=5.35$ and $m=0.01$ (in lattice units) with four flavors of dynamical staggered fermions using the hybrid Monte Carlo algorithm. Our data sample consists of 85 configurations separated by five trajectories. These configurations were used in Ref. [7] to compute the hadron mass spectrum. For further details of the lattice simulation the reader is also referred to this reference. The lattice parameters correspond to a renormalization group invariant quark mass of $m^{\mathrm{RGI}}=35$ MeV in the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme for $\Lambda_{\overline{\mathrm{MS}}}=200$ MeV. Taking the physical scale from the ρ mass, the lattice spacing is approximately 0.14 fm.

For $\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}$ we use the "geometric" definition of Lüscher [8]:

$$\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}(x)=\frac{2}{3}\sum_{\mu\nu\rho\sigma}\epsilon_{\mu\nu\rho\sigma}\left[3\int_{p(x+\hat{\mu}+\hat{\nu},\mu,\nu)}d^2z\,\mathrm{Tr}[P^x_{x+\hat{\mu}+\hat{\nu},\mu\nu}\partial_\rho(P^x_{x+\hat{\mu}+\hat{\nu},\mu\nu})^{-1}(R^x_{x+\hat{\mu},\mu;\nu})^{-1}\partial_\sigma R^x_{x+\hat{\mu},\mu;\nu}]\right.$$
$$-3\int_{p(x+\hat{\nu},\mu,\nu)}d^2z\,\mathrm{Tr}[P^x_{x+\hat{\nu},\mu\nu}\partial_\rho(P^x_{x+\hat{\nu},\mu\nu})^{-1}(R^x_{x,\mu;\nu})^{-1}\partial_\sigma R^x_{x,\mu;\nu}]$$
$$-\int_{f(x+\hat{\mu},\mu)}d^3z\,\mathrm{Tr}[S^x_{x+\hat{\mu},\mu}\partial_\nu(S^x_{x+\hat{\mu},\mu})^{-1}S^x_{x+\hat{\mu},\mu}\partial_\rho(S^x_{x+\hat{\mu},\mu})^{-1}S^x_{x+\hat{\mu},\mu}\partial_\sigma(S^x_{x+\hat{\mu},\mu})^{-1}]$$
$$\left.+\int_{f(x,\mu)}d^3z\,\mathrm{Tr}[S^x_{x,\mu}\partial_\nu(S^x_{x,\mu})^{-1}S^x_{x,\mu}\partial_\rho(S^x_{x,\mu})^{-1}S^x_{x,\mu}\partial_\sigma(S^x_{x,\mu})^{-1}]\right], \tag{10}$$

where P, R, and S are certain parallel transporters extrapolated to the interior of the plaquettes p and faces f. This expression proceeds from the principal bundle which is reconstructed from the lattice gauge field. The resulting topological charge, which is given by

$$Q=-\frac{1}{16\pi^2}\sum_x \mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}, \tag{11}$$

assumes integer values as in the continuum. The major drawback of Eq. (10) is that it involves a three-dimensional integral over the faces of the hypercubes.

According to Eqs. (8) and (9), $\Delta\Sigma$ is given by

$$\Delta\Sigma=\lim_{\mathbf{p}\to 0} i\frac{|\mathbf{s}|}{\mathbf{p}\cdot\mathbf{s}}\left\langle \mathbf{p},s\left|\frac{1}{2\pi^2}\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}\right|0,s\right\rangle . \tag{12}$$

On a finite lattice we cannot reach $\mathbf{p}=0$ continuously. We therefore shall evaluate $\Delta\Sigma$ at the smallest (nonzero) momentum transfer which in our case is $|\mathbf{p}|=2\pi/16$ (≈ 500 MeV). In the present calculation the latter is of the order of the pion mass. Choosing $\mathbf{s}/|\mathbf{s}|=\pm\mathbf{p}/|\mathbf{p}|$, we thus have to compute

$$C(t)=\pm\frac{i}{|\mathbf{p}|}\left\langle B_{\mathbf{p}}(t)P_{\pm}\frac{1}{2\pi^2}\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}(x_4)\bar{B}_0(0)\right\rangle , \tag{13}$$

where $\bar{B}$, B are the baryon creation and annihilation operators, $P_{\pm}$ is the spin projection operator and

$$\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}(x_4)=\sum_{\mathbf{x}}e^{i\mathbf{p}\cdot\mathbf{x}}\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}(x) . \tag{14}$$

Equation (13) leads to twelve independent correlation functions corresponding to the six possible directions of the momentum and the two spin orientations. For $0\le x_4\le t\ll 12$ the average of these correlation functions has the form

$$C(t)=\Delta\Sigma A_{+}e^{-m_N x_4-E_N(t-x_4)}+\Delta\Sigma_\Lambda A_{-}(-1)^t e^{-m_\Lambda x_4-E_\Lambda(t-x_4)}+\cdots \tag{15}$$

(and similarly for $12\ll t\le x_4\le 24$), where $\Delta\Sigma_\Lambda(m_\Lambda)$ is the axial baryonic charge (mass) of the Λ, the opposite parity partner of the nucleon [7], and

$$E_{N,\Lambda}=2\operatorname{arcsinh}\frac{1}{2}\left[\left[2\sinh\frac{m_{N,\Lambda}}{2}\right]^2+\sum_i 2(1-\cos p_i)\right]^{1/2} \tag{16}$$

are the lattice energies. Note that the matrix elements between N and Λ vanish for the weighted average. The amplitudes A_+, A_- are those of the correlation function $\langle B_0(t)\bar{B}_0(0)\rangle$. The dots in Eq. (15) stand for contributions from higher excitations.

A nontrivial problem is the computation of $\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}$, Eq. (10), for a given lattice gauge field configuration. For the gauge group SU(2) we could do one integration $\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}$ analytically [9], which makes its computation just feasible. In the present case of the gauge group SU(3) we shall make use of the fact that the calculation can be reduced to the case of SU(2) by means of the so-called reduction of the structure group [10,11]. This is known to be true for the topological charge. The reason is that $\pi_3[SU(3)/SU(2)]=0$. It is also true for $\Delta\Sigma$. To see this, we can write [12]

$$-\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}(x)=8\pi\sum_\mu(-1)^\mu(k_{x,\mu}-k_{x+\hat{\mu},\mu})+16\pi^2 n(x) , \tag{17}$$

where $k_{x,\mu}$ is the Chern-Simons density given by Seiberg [13] and $n(x)$ is the local winding number of a section of the bundle. The latter is topological and hence is invariant under reduction to SU(2). This leaves us with the lattice divergence. For the sake of argument we choose the momentum along the three-axis. The matrix element of the derivative in four-direction is proportional to $E_{N,\Lambda}-m_{N,\Lambda}$ and thus vanishes like p_3^2. The contributions from the derivatives in the one- and two-directions give zero when summed over the lattice points in the time slice. The contribution from the derivative in the three-direction can be written as

$$-8\pi\sum_{\mathbf{x}}e^{ip_3x_3}(1-e^{-ip_3})k_{x,3} . \tag{18}$$

Making use of translation invariance in the one- and two-directions, we can restrict ourselves to the sum over x_3. Exploiting the transformation properties of $k_{x,3}$ under gauge variations [13], we can gauge $K_{x,3}$ to zero. This shows that $\Delta\Sigma$ is determined by the topological properties of the gauge fields only.

In the actual calculation the reduction of the SU(3) link matrices to SU(2) link matrices is done by a coset decomposition [11,14] $U(x,\mu)=\omega(x,\mu)\tilde{U}(x,\mu)$, where $U(x,\mu)\in SU(3)$, $\tilde{U}(x,\mu)\in SU(2)$. In order to be able to use the "geometric" definition of $\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}$ on the SU(2) matrices, the latter must be sufficiently smooth, which is not necessarily the case. This is achieved by fixing to a maximal SU(2) gauge before the reduction is done. It amounts to minimizing $\sum_{x,\mu}[3-\mathrm{Re\,Tr}\,\omega(x,\mu)]$. Note that this gauge is manifestly renormalizable and therefore reduces ultra-violet fluctuations in the SU(2) variables. For the minimizing procedure we apply a combination of Metropolis and overrelaxation steps. We have checked for most of our configurations that this results in a gauge invariant $\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}$.

The computation of the baryon propagators follows Ref. [7]. For $\bar{B}$ we take the wall source, while for B we take the ordinary local baryon operator (i.e., operator No. 1). We have found that the lattice dispersion relation is rather well satisfied for the meson states at the present value of β, so that it is justified to assume the validity of Eq. (16). The current is placed at two different times: $x_4=4$ and $x_4=20$. The correlation function (13), averaged over all possible momentum and spin combinations, is shown in Fig. 1 for the choice $x_4=4$. (For $x_4=20$ and backtracking baryon the correlation function oscillates between positive and negative values.) In contrast with earlier calculations [4] we obtain a clear signal over several lattice spacings. The data are fitted by the function (15) with A_+, A_-, m_N $(=0.77)$, and $m_\Lambda(=0.90)$ taken from a fit [7] of $\langle B_0(t)\bar{B}_0(0)\rangle$ in the interval $4\leq t\leq 20$. The result of the fit of the $x_4=4$ and $x_4=20$ data combined is indicated by the solid line. It leads to

$$\Delta\Sigma=0.18\pm0.02\ , \qquad (19)$$

while the axial baryonic charge of the Λ comes out to be $\Delta\Sigma_\Lambda=0.22\pm0.04$.

The error analysis for the correlation function was done by a jackknife method. The error quoted for $\Delta\Sigma$ is the standard MINUIT error which neglects correlations between the data points. In order to check our results for systematic errors, we have also done a fit with the data points at $t=4$ and $t=20$ discarded. This gave $\Delta\Sigma=0.17\pm0.02$. Furthermore, we have repeated the calculation for $x_4=3$, where we found $\Delta\Sigma=0.16\pm0.05$. Thus, we can assume that $x_4=4$ and $x_4=20$ are far enough away from the source, so that the excited states have died out, and that the current does not excite the nucleon noticeably.

Our result (19) shows that a QCD calculation based on first principles can reproduce a value for $\Delta\Sigma$ which is substantially below one, in qualitative agreement with experiment. One might object that our calculations are not entirely realistic, as we work with four light quarks instead of two plus a heavier strange quark. Furthermore,

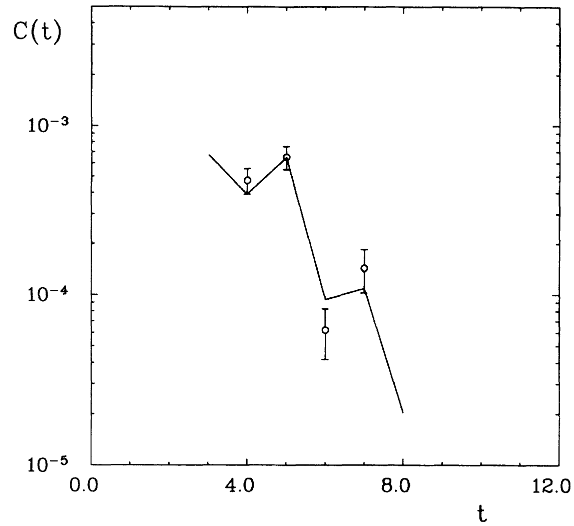

FIG. 1. The averaged correlation function (13) as a function of t for $x_4=4$. Also shown is a two-parameter fit of Eq. (15) to the $x_4=4$ and $x_4=20$ data combined, the fit interval being from $t=4$ to $t=7$ and from $t=20$ to $t=17$, respectively. For $8\leq t\leq 16$ the data are too noisy to be of much use.

we neglect contributions proportional to the quark masses. But there are reasons to believe that none of these approximations will alter our conclusions. As far as the flavor dependence is concerned, one expects $\Delta\Sigma\propto\sqrt{N_f}$ [15], which leads to a small correction only. The quark mass and momentum dependence, on the other hand, are controlled by the η' mass which is large compared to the pion mass. A larger correction might be expected from the strange quark. By the same analysis which led to (3) we obtain $\Delta s=-0.19\pm0.07$. Given that the anomalous contribution to Δs is small, the effect could be that altogether $\Delta\Sigma\approx 0$.

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