

## A partial wave analysis of the decay $D^0 \rightarrow K_S^0 \pi^+ \pi^-$

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Received 28 April 1993

Editor: K. Winter

Using the ARGUS detector at the DORIS-II electron-positron storage ring at DESY, we have investigated the exclusive decay  $D^{*+} \rightarrow D^0\pi^+$ ,  $D^0 \rightarrow K_S^0\pi^+\pi^-$ . From a partial wave analysis of the  $K_S^0\pi^+\pi^-$  system we find that  $(71.8 \pm 4.2 \pm 3.0)\%$  are  $D^0 \rightarrow K^{*-}\pi^+$  and  $(22.7 \pm 3.2 \pm 0.9)\%$  are  $D^0 \rightarrow \bar{K}^0\rho^0$  with a relative phase of  $(-137 \pm 7 \pm 3)^\circ$  between the channels. The remaining fraction can be described by several channels involving excited resonances, but not by a three-body phase space decay, thus giving first evidence for  $D^0$  decays into  $K_0^*(1430)^-\pi^+$ ,  $\bar{K}^0 f_0(975)$ ,  $\bar{K}^0 f_2(1270)$ , and  $\bar{K}^0 f_0(1400)$ .

Weak hadronic decays of charmed mesons are expected [1,2] to proceed dominantly via resonant two-body decays in the spectator model, with a small nonresonant contribution. In this context an interesting final state<sup>#1</sup> is  $D^0 \rightarrow \bar{K}^0\pi^+\pi^-$ , which can proceed through a number of resonant two-body states. The substructure in this channel has been investigated previously [3–5], yielding the results given in table 1. All three experiments were limited by statistics to the two most prominent channels. The ARGUS data offer for the first time the opportunity to include all other resonant subchannels with known resonances, which are allowed to contribute due to their mass and branching ratio.

The analysis reported here is based on data taken with the ARGUS detector [6] in the energy region around  $\sqrt{s} = 10$  GeV on the  $\Upsilon$  resonances and in the nearby continuum, together comprising an integrated luminosity of 455/pb. In events with at least three charged tracks from the interaction region,  $D^0$  meson candidates are selected, which have been produced in the decay  $D^{*+} \rightarrow D^0\pi^+$ . Charged pions are required to have a relative likelihood greater than 1% for the pion mass hypothesis, as determined from time of flight and  $dE/dx$  information [6].  $K_S^0$  mesons are identified by a reconstructed secondary vertex formed by two oppositely-charged pions. Accidental vertices are suppressed by the additional requirement that the  $K_S^0$  momentum direction and the vector pointing

Table 1

Results from other experiments on the fractions and relative phases of the main contributions to  $D^0 \rightarrow \bar{K}^0\pi^+\pi^-$ .

	Mark II <sup>a)</sup> [3]	Mark III [4]	E687 [5]
$D^0 \rightarrow K^{*-}\pi^+$	$(70 \pm 1517 \pm 56)\%$	$(56 \pm 4 \pm 5)\%$	$(64 \pm 8 \pm 5)\%$
$\varphi$		$0^\circ$	$0^\circ$
$D^0 \rightarrow \bar{K}^0\rho^0$	$(2 \pm 142 \pm 2)\%$	$(12 \pm 1 \pm 7)\%$	$(20 \pm 6 \pm 3)\%$
$\varphi$		$(93 \pm 30)^\circ$	$(-143 \pm 29)^\circ$
$D^0 \rightarrow 3\text{-body}$	$(30 \pm 2321 \pm 5)\%$	$(33 \pm 5 \pm 10)\%$	$(26 \pm 8 \pm 5)\%$
$\varphi$		incoherent	$(-126 \pm 17)^\circ$

a) the Mark II data sample was too small to obtain results on the relative phase.

<sup>1</sup> DESY, IfH Zeuthen.

<sup>2</sup> Supported by the German Bundesministerium für Forschung und Technologie, under contract number 054DO51P.

<sup>3</sup> Supported by the German Bundesministerium für Forschung und Technologie, under contract number 054ER12P.

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<sup>8</sup> Supported by the Natural Sciences and Engineering Research Council, Canada.

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<sup>10</sup> Supported by the German Bundesministerium für Forschung und Technologie, under contract number 054KA17P.

<sup>11</sup> Supported by the Department of Science and Technology of the Republic of Slovenia and the Internationales Büro KfA, Jülich.

<sup>#1</sup> References in this paper to a specific charged state also imply the charged conjugate state.  $K_S^0\pi^+\pi^-$  is identified as  $\bar{K}^0\pi_1^+\pi_2^-$  or  $K^0\pi_1^-\pi_2^+$  depending on the charge of the additional pion from  $D^{*+}$  or  $D^{*-}$  decay, respectively.

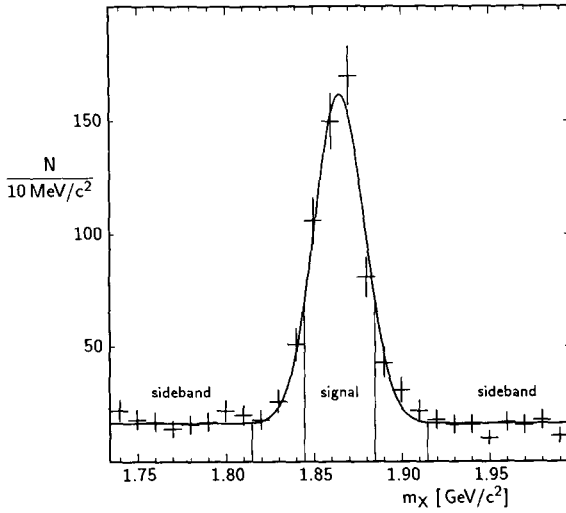


Fig. 1. Invariant mass distribution of  $D^0$  candidates in  $K_S^0\pi^+\pi^-$ , where the mass difference adding another  $\pi^+$  is the nominal difference  $m(D^{*+}) - m(D^0)$  within  $\pm 2 \text{ MeV}/c^2$ . The curve represents a fit using a Gaussian above a linear background. The signal and sideband regions are indicated.

from the primary to the secondary vertex have an opening angle  $\theta$  with  $\cos\theta > 0.9$ . A  $K_S^0\pi^+\pi^-$  combination ( $X$ ) is accepted if an additional  $\pi^+$  can be found where

$$143.5 \text{ MeV}/c^2 \leq |m(X\pi^+) - m(X)| \leq 147.5 \text{ MeV}/c^2.$$

For these  $D^{*+}$  candidates, we require a scaled momentum  $x_p = p/\sqrt{s/4 - m_{X\pi}^2} > 0.5$  making use of the hard charm fragmentation in continuum  $e^+e^-$  annihilation.

Fig. 1 shows the  $m_X$  distribution for the remaining candidates. We define a signal region around the  $D^0$  mass peak with

$$1.8446 \text{ GeV}/c^2 \leq m_X \leq 1.8846 \text{ GeV}/c^2,$$

which contains 507 events, and a sample from the upper and lower sidebands with

$$1.7346 \text{ GeV}/c^2 \leq m_X \leq 1.8146 \text{ GeV}/c^2,$$

$$1.9146 \text{ GeV}/c^2 \leq m_X \leq 1.9946 \text{ GeV}/c^2,$$

comprising 268 events. Assuming a linear dependence for the combinatoric background, we estimate  $67 \pm 8$  background events in the signal region.

The charm quantum number of the  $D$  is not defined by the final state  $K_S^0\pi^+\pi^-$ , which can be reached both by the  $D^0$  and  $\bar{D}^0$ , but can be determined from the charge of the accompanying soft pion, i.e.  $D^{*+} \rightarrow D^0\pi^+$  or  $D^{*-} \rightarrow \bar{D}^0\pi^-$ . There is a small fraction in the  $D$  signal from accidental  $D\pi$  combinations, which half of the time leads to the opposite charm. The fraction of wrong charge  $D\pi$  combinations has been determined to be  $\epsilon = (2.3 \pm 0.7)\%$  by extrapolating from  $m(D\pi) - m(D) > 150 \text{ MeV}/c^2$  into the signal region.

The Dalitz plot distribution of the 507 signal events is shown in fig. 2a. For each entry, the momenta are recalculated using a  $D^0$  mass constraint fit. This implies that the kinematical boundaries are strictly respected. The dominating channel  $D^0 \rightarrow K^{*-}\pi^+$  can be clearly distinguished. Fig. 2b shows the result of the same procedure for the sidebands.

A maximum likelihood fit is applied to the signal events, using a coherent sum of amplitudes for up to 8 different resonant channels, and a background of known shape and magnitude. The decay amplitude through a  $K\pi$  resonance ( $X$ ) is described by

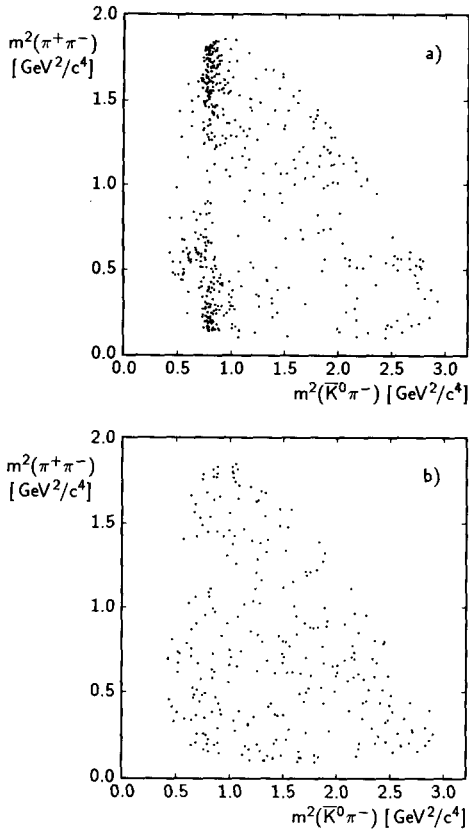


Fig. 2. Dalitz plot of  $K_S^0 \pi^+ \pi^-$ , where “ $\pi^+$ ” denotes the pion with the same charge as the additional pion from the  $D^*$ , in the  $D^0$  signal region (a) and the sidebands (b).

$$A(D \rightarrow X\pi, X \rightarrow K\pi) = g_{DX\pi} F_D a_{DX\pi} \frac{1}{(m_{K\pi}^2 - m_X^2) + im_X \Gamma(m_{K\pi})} g_{XK\pi} F_X a_{XK\pi},$$

where the coupling constants  $g_{DX\pi}$  and  $g_{XK\pi}$  contribute only to the normalization and are omitted. The form factors  $F_D$  and  $F_X$  are chosen according to Blatt and Weisskopf [7] as

$$L = 0: \quad F \sim 1,$$

$$L = 1: \quad F \sim \frac{1}{\sqrt{1 + R^2 p^{*2}}},$$

$$L = 2: \quad F \sim \frac{1}{\sqrt{9 + 3R^2 p^{*2} + (R^2 p^{*2})^2}},$$

where  $p^*$  is the momentum of the decay products in the rest frame of particle  $X$ , and  $R$  is a meson radius. Since we consider these formfactors as an effective description, we take the radii as free parameters, and find acceptable fit results with values of  $R_D = 3$  to  $7 \text{ GeV}^{-1}$  and  $R_X = 0$  to  $3 \text{ GeV}^{-1}$  for  $\pi\pi$  and  $K\pi$  resonances.

The widths are given by

$$\Gamma(m) = \Gamma_0 \cdot \left(\frac{p^*}{p_0^*}\right)^{2L+1} \frac{m_0 F_X^2(p^{*2})}{m F_X^2(p_0^{*2})},$$

where relevant values for  $m_0$  and  $\Gamma_0$  are taken from ref. [8].

The angular momentum parts,  $a_{D\pi\pi}$  and  $a_{KK\pi}$ , of the amplitude are given in the Zemach formalism [9] as functions of the four momentum transfer in the first ( $Q = p_X - p_\pi$ ) and second steps ( $q = p_K - p_\pi$ ) for  $L = 1$  by

$$a = -Q_\mu q^\mu + \frac{(Q_\mu p_X^\mu)(q_\nu p_X^\nu)}{m_X^2},$$

and for  $L = 2$  by

$$a = \left[ -Q_\mu q^\mu + \frac{(Q_\mu p_X^\mu)(q_\nu p_X^\nu)}{m_X^2} \right]^2 - \frac{1}{3} \left[ Q^2 q^2 - Q^2 \frac{q_\mu p_X^\mu}{m_X^2} - q^2 \frac{Q_\mu p_X^\mu}{m_X^2} + \frac{(Q_\mu p_X^\mu)(q_\nu p_X^\nu)}{m_X^4} \right].$$

All invariants in these factors are uniquely determined by the position in the Dalitz plot. The channels with resonances in the  $\pi\pi$  system are described accordingly.

Thus, the complete distribution fitted to the Dalitz plot is  $f(q, Q) = \eta(q, Q) \tilde{f}(q, Q)$  with

$$\tilde{f}(q, Q) = \beta \frac{b}{\int \eta b \, d\text{PS}} + (1 - \beta) \left[ \frac{(1 - \epsilon) \sum_{i,j} c_i c_j^* A_i A_j^* + \epsilon \sum_{i,j} c_i c_j^* \bar{A}_i \bar{A}_j^*}{\sum_{i,j} c_i c_j^* \int \eta A_i A_j^* \, d\text{PS}} \right], \quad (1)$$

where the acceptance factor  $\eta(Q, q)$  does not depend on the parameters and is omitted in the likelihood function, i.e. we maximize

$$\log L = \sum_{n=1}^{507} \log \tilde{f}(q_n, Q_n).$$

The constants  $\beta$  and  $\epsilon$  are the background and wrong charge contributions in the signal region, and  $A_i, \bar{A}_i$  are the individual amplitudes as given above for  $D^0 \rightarrow K_S^0 \pi_1^+ \pi_2^-$  and the charge conjugate decay  $\bar{D}^0 \rightarrow K_S^0 \pi_1^- \pi_2^+$ , respectively. The complex coefficients  $c_i$  are defined by eq. (1) only up to a common factor. Therefore we chose the most prominent channel,  $K^{*-} \pi^+$ , to have  $c_1 = 1$ . The relative phases to this channel are the phases of the coefficients  $c_i$ . The fractions can be obtained from

$$f_i = \frac{|c_i|^2 \int |A_i|^2 \, d\text{PS}}{\sum_{j,k} c_j c_k^* \int A_j A_k^* \, d\text{PS}}, \quad (2)$$

and do not, in general, add up to 100% due to the interference contributions.

The shape of the distribution of background events,  $b$  in eq. (1), has been determined by fitting an incoherent sum of resonances (those used in fit B of table 1, with two charge combinations for  $K\pi$  resonances), in addition to a flat distribution, to the sideband Dalitz plot, fig. 2b. This fit yields the fractions

$$f(3\text{-body}) = (83.8 \pm 5.4)\%,$$

$$f(K^{*-} \pi^+) = (8.5 \pm 2.7)\%,$$

$$f(K^{*+} \pi^-) = (5.7 \pm 2.7)\%,$$

$$f(K_S^0 \rho^0) = (2.0 \pm 3.9)\%.$$

All other contributions are zero to better than 0.2% and can safely be neglected. The total background contribution is

$$\beta = \frac{67 \pm 8}{507} = 0.132 \pm 0.017.$$

Table 2

Results of this analysis for fractions and phases using different sets of parameters for the resonance decomposition of  $D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-$ .

	Fit A	Fit B	Fit C	Fit D
$D^0 \rightarrow K^{*-} \pi^+$	$(70.2 \pm 4.3)\%$	$(71.8 \pm 4.2)\%$	$(68.9 \pm 3.4)\%$	$(69.5 \pm 3.5)\%$
$\varphi$	$0^\circ$	$0^\circ$	$0^\circ$	$0^\circ$
$D^0 \rightarrow K_0^*(1430)^- \pi^+$	$(9.3 \pm 3.2)\%$	$(12.9 \pm 3.4)\%$	$(9.0 \pm 3.4)\%$	0
$\varphi$	$(-152 \pm 14)^\circ$	$(-157 \pm 12)^\circ$	$(-171 \pm 19)^\circ$	
$D^0 \rightarrow K_2^*(1430)^- \pi^+$	$(1.2 \pm 1.0)\%$	0	0	0
$D^0 \rightarrow \bar{K}^0 \rho^0$	$(21.1 \pm 3.5)\%$	$(22.7 \pm 3.2)\%$	$(23.5 \pm 3.4)\%$	$(21.2 \pm 3.0)\%$
$\varphi$	$(-141 \pm 8)^\circ$	$(-137 \pm 7)^\circ$	$(-141 \pm 7)^\circ$	$(-143 \pm 8)^\circ$
$D^0 \rightarrow \bar{K}^0 \omega$	$(0.7 \pm 0.7)\%$	0	0	0
$D^0 \rightarrow \bar{K}^0 f_0(975)$	$(4.7 \pm 2.1)\%$	$(4.6 \pm 1.8)\%$	$(4.1 \pm 1.8)\%$	0
$\varphi$	$(68 \pm 17)^\circ$	$(68 \pm 15)^\circ$	$(71 \pm 19)^\circ$	
$D^0 \rightarrow \bar{K}^0 f_2(1270)$	$(4.1 \pm 2.0)\%$	$(5.0 \pm 2.1)\%$	$(4.3 \pm 2.0)\%$	0
$\varphi$	$(-166 \pm 15)^\circ$	$(-166 \pm 12)^\circ$	$(-175 \pm 15)^\circ$	
$D^0 \rightarrow \bar{K}^0 f_0(1400)$	$(7.2 \pm 2.8)\%$	$(8.2 \pm 2.8)\%$	0	0
$\varphi$	$(-37 \pm 16)^\circ$	$(-31 \pm 15)^\circ$		
$D^0 \rightarrow 3\text{-body}$	0	0	$(8.1 \pm 3.5)\%$	$(20.0 \pm 3.1)\%$
$\varphi$			$(152 \pm 16)^\circ$	$(107 \pm 9)^\circ$
#parameters $n$	14	10	10	4
$x^2 = -2 \max \log L$	5.0	13.7	24.1	85.7
$\langle x^2 \rangle$	1.4	60.9	28.9	-19.1
$\sigma(x^2)$	61.7	60.0	61.8	63.4
SL	48%	78%	53%	4.9%

In the fit function (1) the acceptance  $\eta$  enters only in the normalization constants. It has been determined with a full event and detector Monte Carlo simulation. The corresponding integrals of the matrix elements  $\int \eta A_i A_j^* d\text{PS}$  have been calculated from accepted Monte Carlo events, generated according to a phase space distribution using

$$\int \eta(q, Q) A_i(q, Q) A_j^*(q, Q) d\text{PS} = \frac{\sum_{n=1}^{N_{\text{acc}}} A_i(q_n, Q_n) A_j^*(q_n, Q_n)}{N_{\text{gen}}}.$$

The fit results for various combinations of channels are shown in table 2. Two channels ( $K_2^*(1430)^- \pi^+$  and  $\bar{K}^0 \omega$ ,  $\omega \rightarrow \pi^+ \pi^-$ ), which are found by fit A to make a negligible contribution with an indeterminate phase, are removed from consideration in the remainder of the analysis. The  $\bar{K}^0 \omega$  channel is known to have a fraction of  $(1.0 \pm 0.2)\%$  from measurements using more prominent  $\omega$  decay channels [8], in good agreement with our result in fit A.

The result of fit B is shown as a density plot in fig. 3, and compared to the projections of the Dalitz plot of our data.

The goodness-of-fit cannot be read directly from the value of  $\max \log L$ , as would be the case for a minimum  $\chi^2$  fit. However, since the number of degrees of freedom is large, the distribution of  $x^2 = -2 \max \log L$  is approximately Gaussian, although – in contrast to a  $\chi^2$  value – the expectation of  $x^2$  depends not only on the number  $n$ , but also on the values of the parameters. It is calculated as

$$\langle x^2 \rangle = 507 \int f(q, Q) [-2 \log \tilde{f}(q, Q)] d\text{PS} - n,$$

and the standard deviation is

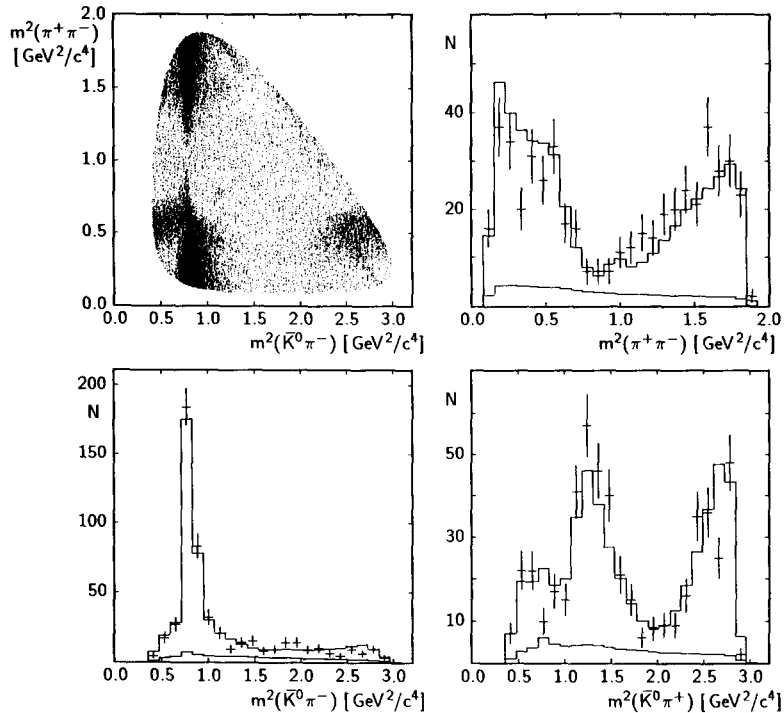


Fig. 3. Dalitz plot of the density function resulting from fit B. The three projections are compared to the data (crosses). The contribution from background is also indicated.

$$\sigma(x^2) = \sqrt{\langle x^4 \rangle - \langle x^2 \rangle^2} = \sqrt{507 \int f(q, Q) [-2 \log \tilde{f}(q, Q)]^2 dPS - (\langle x^2 \rangle + n)^2}.$$

From these values, which are listed in table 2 for all fits, a significance level (SL) can be calculated as the probability that  $x^2$  exceeds that of the fit. The validity of this procedure has been checked empirically using a Monte Carlo technique, by generating 250 samples of 507 random events each, distributed according to the density function (1), using the results of fit B and D as input parameters. The same function (1) with free parameters has been fitted to these events, and the resulting values of  $x^2 = -2 \max \log L$  have been recorded. Their distribution is shown in fig. 4, where also the values obtained from the fit to the real data are indicated. From this simulation, we obtain  $\langle x^2 \rangle = 56 \pm 4$ ,  $\sigma(x^2) = 63 \pm 3$ , SL = 75% for fit B and  $\langle x^2 \rangle = -23 \pm 4$ ,  $\sigma(x^2) = 66 \pm 3$ , SL = 3.2% for fit D, which agrees well within errors with the result from the Gaussian approximation in table 2. Thus, we accept fits A, B and C as good fits, whereas the minimum configuration of fit D can be rejected, since it has a significance level of less than 5%. From this we conclude, that a description using resonant subchannels is needed to describe the data.

In fit C, we have replaced the widest resonance  $f_0(1400)$  by three-body phase space. This gives also a good fit, but the increase in  $x^2$  of 10.4, while the expectation value for this set of parameters is even smaller, shows again that the data prefer resonant two-body decays over a direct three-body decay. Adding the three-body decay incoherently gives an even worse fit. This supports the claim by Cheng [2], that at most a small contribution, of the order of 2%, is nonresonant.

Systematic errors arise mainly from the parametrization of the amplitudes, and have been estimated by varying the masses and widths of the resonances, the radii in the form factor as given above, and the composition of the other channels. The dominant contribution is that of the  $f_0(1400)$ , where the mass has been varied from 1300

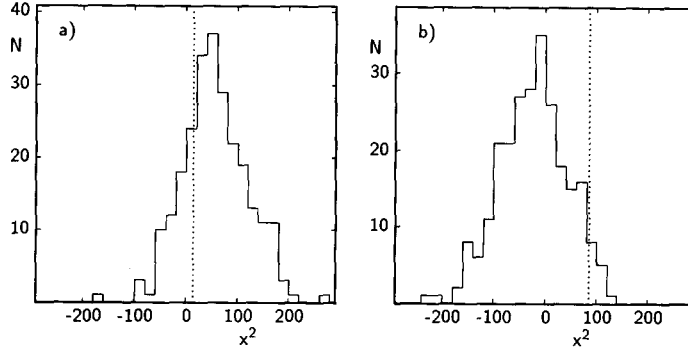


Fig. 4. Distribution of the test statistic  $x^2 = -2 \max \log L$  for best fits to 250 Monte Carlo samples with the same number of events as the data. Histogram (a) is for events generated according to the results of fit B and fitted with the 10 parameters of fit B, and (b) is for events generated according to the results of fit D and fitted with the 4 parameters of fit D. The dotted lines indicate  $x^2$  from the fits to the data.

to 1500 MeV, and the width from 150 to 400 MeV. Further contributions come from the errors on  $\beta$  and  $\epsilon$ , and the statistical error of the Monte Carlo normalization. Systematic errors from the distribution of the background events have been determined from a comparison of fits to the signal events using either a flat background  $b \equiv 1$  or a background with the shape  $b$  determined from the sidebands as given above.

Considering fit B to be our best estimate, we obtain for the dominant channels the fractions

$$f(D^0 \rightarrow K^{*-} \pi^+) = 0.718 \pm 0.042 \pm 0.030,$$

$$f(D^0 \rightarrow \bar{K}^0 \rho^0) = 0.227 \pm 0.032 \pm 0.009,$$

with a relative phase of  $(-137 \pm 7 \pm 3)^\circ$ . Using the world average branching ratio  $\text{BR}(D^0 \rightarrow \bar{K}^0 \pi^+ \pi^-) = (5.4 \pm 0.5)\%$  [8], we can convert these fractions to

$$\text{BR}(D^0 \rightarrow K^{*-} \pi^+) = (5.8 \pm 0.7)\%,$$

$$\text{BR}(D^0 \rightarrow \bar{K}^0 \rho^0) = (1.2 \pm 0.2)\%.$$

These values are both larger than the present world averages of  $(4.5 \pm 0.6)\%$  and  $(0.6 \pm 0.3)\%$ .

In the model of Bauer, Stech and Wirbel [1] the ratio of these two contributions can be used to estimate the ratio of the effective QCD parameters  $a_1$  and  $a_2$  via

$$\frac{\text{BR}(D^0 \rightarrow K^{*-} \pi^+)}{\text{BR}(D^0 \rightarrow \bar{K}^0 \rho^0)} = \frac{5.12 a_1^2}{3.14 a_2^2}.$$

From our results for these branching ratios, we find

$$\left| \frac{a_1}{a_2} \right| = 1.71 \pm 0.14,$$

which is considerably smaller than the value 2.4 used in ref. [1].

The results for the rarer decay channels have systematic uncertainties which are dominated by the poorly known resonance parameters. We obtain the fractions

$$f(D^0 \rightarrow K_0^*(1430)^- \pi^+) = 0.129 \pm 0.034 \pm 0.021,$$



$$f(D^0 \rightarrow \bar{K}^0 f_0(975)) = 0.046 \pm 0.018 \pm 0.006,$$

$$f(D^0 \rightarrow \bar{K}^0 f_2(1270)) = 0.050 \pm 0.021 \pm 0.008,$$

$$f(D^0 \rightarrow \bar{K}^0 f_0(1400)) = 0.082 \pm 0.028 \pm 0.013.$$

Although these results are highly correlated, and the errors for each are still very large, these resonances cannot be replaced by a single three-body phase space contribution, as discussed above, nor can we omit any one of them. If we leave one out of fit B, we obtain a corresponding increase in  $x^2$  of 46.1 ( $K_0^*$ ), 9.1 ( $f_0(975)$ ), 16.5 ( $f_2$ ) or 27.4 ( $f_0(1400)$ ), corresponding to more than three standard deviations in each branching fraction.

Using the known branching ratios [8] and Clebsch–Gordan coefficients, these fractions can be converted to the branching ratios

$$\text{BR}(D^0 \rightarrow K_0^*(1430)^- \pi^+) = (1.1 \pm 0.4)\%,$$

$$\text{BR}(D^0 \rightarrow K_2^*(1430)^- \pi^+) < 0.8\% \text{ (90 CL)},$$

$$\text{BR}(D^0 \rightarrow \bar{K}^0 f_0(975)) = (0.48 \pm 0.20)\%,$$

$$\text{BR}(D^0 \rightarrow \bar{K}^0 f_2(1270)) = (0.48 \pm 0.22)\%,$$

$$\text{BR}(D^0 \rightarrow \bar{K}^0 f_0(1400)) = (0.71 \pm 0.28)\%.$$

In summary, we have analysed the resonant substructure in the decay  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ . A satisfactory description can be obtained with resonant channels alone, without direct three-body decay. On the other hand, a description with three-body decay plus  $K^{*-} \pi^+$  and  $\bar{K}^0 \rho^0$  can be rejected at 5% significance level. For the dominant channels,  $K^{*-} \pi^+$  and  $\bar{K}^0 \rho^0$ , we obtain larger branching ratios than the present world average with comparable errors.

## Acknowledgement

It is a pleasure to thank U. Djuanda, E. Konrad, E. Michel, and W. Reinsch for their competent technical help in running the experiment and processing the data. We thank Dr. H. Neseemann, B. Sarau, and the DORIS group for the excellent operation of the storage ring. The visiting groups wish to thank the DESY directorate for the support and kind hospitality extended to them.

## References

- [1] M. Bauer, B. Stech and M. Wirbel, Z. Phys. C 34 (1987) 103;  
M. Bauer, Dissertation, Heidelberg, 1987.
- [2] H.-Y. Cheng, Z. Phys. C 32 (1986) 243;  
L.-L. Chau and H.-Y. Cheng, Phys. Rev. D 41 (1990) 1510.
- [3] Mark II Collab., R.H. Schindler et al., Phys. Rev. D 24 (1981) 78.
- [4] Mark III Collab., J. Adler et al., Phys. Lett. B 196 (1987) 107.
- [5] E687 Collab., P.L. Frabetti et al., Phys. Lett. B 286 (1992) 195.
- [6] H. Albrecht et al., Nucl. Instrum. Methods A 275 (1989) 1.
- [7] J. M. Blatt and V.F. Weisskopf, Theoretical Nuclear Physics, John Wiley & Sons, New York, 1952.
- [8] Particle Data Group, Phys. Rev. D 45, no. 11 part II (1992).
- [9] C. Zemach, Phys. Rev. B 140 (1965) 97, 109.