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Sphaleron transitions in the symmetric phase of the standard model

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Using a massive pure gauge theory with a dynamically generated magnetic mass $\sim g^2T$ as an effective theory of the standard model at high temperatures we estimate the rate of $B+L$ violating processes in the symmetric phase. Treating the magnetic mass as a parameter of the theory we find a small range of mass values for which it is possible to apply steepest descent methods and to obtain transition rates larger than the expansion rate of the universe. However, the largest rate that can possibly occur in this model lies in a range where the saddle-point approximation breaks down and is by at least one order of magnitude smaller than those obtained by lattice simulations.

In the past few years there has been much interest in baryon and lepton number violating processes of the standard electroweak model at high temperatures. The reason is that for sufficiently large rates such processes wash out any preexisting $B+L$ asymmetry of the early universe created in the framework of some unified theory. There are several calculations by now which suggest that in a narrow temperature range below the electroweak phase transition sphaleron [1] mediated $B+L$ violating processes are indeed in thermal equilibrium [2–6]. These calculations employ the fact that at high temperatures it is possible for thermally excited gauge and Higgs fields to cross over the potential barrier separating topologically inequivalent vacua. In the broken phase the height of this barrier is given by the sphaleron energy

$$E_{\mathrm{Sp}}=\frac{2m_W(T)}{\alpha_{\mathrm{w}}}B(\lambda/\alpha_{\mathrm{w}}), \tag{1}$$

where $B(\lambda/\alpha_{\mathrm{w}})$ is a slowly varying function of the Higgs coupling λ and the weak coupling $\alpha_{\mathrm{w}}=g^2/4\pi$. It is then possible to compute the rate of such crossings using the Langer formula [7,8]

$$\Gamma=\frac{|\kappa|}{\pi T}\operatorname{Im}F\approx\frac{|\kappa|}{\pi}\frac{\operatorname{Im}Z_{\mathrm{Sp}}}{Z_0}, \tag{2}$$

where F is the free energy of the sphaleron and κ is a dynamical factor to coincide with the frequency ω_- of an unstable mode in the underdamped case. In refs. [2–6] F has been calculated semiclassically by considering quadratic fluctuations around the sphaleron configuration and evaluating the corresponding partition function in a gaussian approximation. The results of these calculations are supported by numerical real time simulations on the lattice [9]. However, analytical methods meet severe difficulties as T approaches the critical temperature T_c of the phase transition and above. Since the high temperature effective coupling of the three dimensional theory is given by $\alpha_3=\alpha_{\mathrm{w}}T/2m_W(T)$ and the mass of the W-boson goes to zero as $T\to T_c$, the one loop approximation breaks down and the potential barrier vanishes for high temperatures [3]. Moreover, in the symmetric phase the high temperature loop expansion is plagued by infrared singularities. From dimensional considerations it can be inferred, however, that the rate for transitions between different vacuum states in the symmetric phase has to behave like [3,10]

$$\frac{\Gamma}{V}=\kappa(\alpha_w T)^4\,, \tag{3}$$

where the only quantitative estimate of the prefactor so far comes from lattice simulations which yield $\kappa\sim O(1)$ [9,11]. To circumvent the afore mentioned difficulties and still try to gain some insight by analytical methods we would like to elaborate on the idea to describe the electroweak model in the symmetric phase by a massive pure gauge theory where the gauge boson mass is dynamically generated [12]. The corresponding effective lagrangian is the gauged nonlinear σ model

$$\mathscr{L}_{\rm eff}=-\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}-\frac{m_W^2}{g^2}\,\mathrm{Tr}[(U^{-1}\,\mathrm{D}_\mu U)(U^{-1}\,\mathrm{D}^\mu U)]\,, \tag{4}$$

where $\mathrm{D}_\mu=\partial_\mu-igW_\mu$ and U is a unitary matrix transforming as $U'=VU$ under gauge transformations $V(x)=\exp[-(i/2)\Lambda^a(x)\tau^a]$, while $\mathscr{L}_{\rm eff}$ and $U^{-1}\,\mathrm{D}_\mu U$ remain unchanged. Concerning the choice of this model some comments are in order. It is well known [13] that higher order contributions to the self-energy of the gauge boson propagator in high temperature Yang–Mills theories develop directional singularities which potentially generate a "magnetic" mass $\sim g^2T$ for the spatial components of the gauge fields. The value of this mass cannot be calculated perturbatively and is as yet unknown. However there are arguments [13] and analytical [14] as well as lattice calculations [15] which indicate that it is nonzero. While these results are not entirely conclusive we assume the magnetic mass to be nonzero and give it a coefficient m_0 which we treat as a parameter of the theory, i.e. $m_W=m_0g^2T$ henceforth. Now the theory has a sphaleron solution again around which we can expand semiclassically. Since we will be working in the background gauge $W_0=0$ we can in a first approach neglect the fact that the time components of the gauge fields develop a different "electric" mass. The only effect of the Higgs fields here is to give a dynamical contribution to the magnetic mass. For the known classical solutions of the field equations the Higgs fields remain at their vacuum expectation value zero and hence do not contribute to the saddle-point. in fact there are indications that they partly decouple from the gauge fields in the symmetric phase [16]. The high temperature effective coupling of the theory (4) is now given by $\alpha_3=4\pi/m_0$ where all references to the coupling strength of the theory and the temperature has disappeared.

To obtain the classical solutions of the theory we choose the temporal gauge $W_0^a=0$ and parametrize the remaining fields by the spherically symmetric ansatz

$$W_i^a(\boldsymbol{x})=\frac{1}{g}\left(\frac{f_A(r)-1}{r^2}\,\epsilon_{iam}x_m+\frac{f_B(r)}{r^3}\,(r^2\delta_{ia}-x_ix_a)+\frac{f_C(r)}{r^2}\,x_ix_a\right),\quad U(x)=\exp\!\left(\frac{i}{2}\,\theta(r)\,\frac{x\cdot\tau}{r}\right). \tag{5}$$

We still have time independent gauge degrees of freedom left which are fixed by taking $f_B(r)=0$. This ansatz is then inserted into the field equations and the resulting differential equations for the radial functions are solved numerically. For the parametrization chosen here these equations are

$$f''_A=\frac{f_A}{r^2}\,(f_A^2-1)+m_W^2(f_A-\cos\theta)+f_Af_C^2,\quad f_C=\frac{r^2m_W^2\theta'}{r^2m_W^2+2f_A^2}\,,$$

$$r^2\theta''+2r\theta'-r^2f'_C-2rf_C-2f_A\sin\theta=0\,, \tag{6}$$

with boundary conditions

$$f_A(r)\to 1,\quad \theta(r)\to 0\quad \text{for}\quad r\to 0\,,$$

$$f_A(r)\to 1,\quad \theta(r)\to 2\pi\quad \text{for}\quad r\to\infty\,.$$

Our theory is mathematically equivalent to the one obtained from the SU(2)–Higgs theory by taking the limit of infinite Higgs mass and coupling [17,18]. We therefore know that it has a whole series of classical solutions. There is one solution with constant $\theta(r)=\pi$, energy $E_{\rm Sp}=5.41m_W/\alpha_w$ and winding number $q=0.5$, commonly referred to as the sphaleron solution which was considered in ref. [12] as the relevant configuration for $B+L$ violation. However, this solution is not the lowest energy solution of the field equations and furthermore has an infinite number of instabilities [18]. There are further solutions termed deformed sphalerons because they have winding numbers different from 0.5. In contrast to the sphaleron these deformed sphalerons are not CP invariant, so they always come in pairs related by CP conjugation. For the lowest energy deformed sphalerons with only one direction of instability we find $E_{\mathrm{Sp}}=5.07 m_W/\alpha_{\mathrm{w}}$ and winding number $q=\pm 0.375$ in agreement with refs. [17,18]. Note that these energies now rise linearly with $m_{\mathrm{W}}\sim T$, so one does not lower the Boltzmann suppression by raising the temperature. In ref. [19] it was shown for the case of quantum mechanical tunneling that only saddle-points with a single unstable mode are related to the decay of a metastable state. It is not entirely clear to us if this statement generalizes to the case of thermal transitions. But even if it does not we expect the lowest energy saddle to be the dominating one, so for the rest of this paper we will restrict ourselves to consider transitions over the lowest energy deformed sphalerons.

In ref. [6] it was shown that a calculation of the full fluctuation determinant is possible by exploiting the symmetries of the sphaleron solution and performing a partial wave decomposition of the fluctuations with respect to the total angular momentum operator $\boldsymbol{j}$ characterizing these symmetries. In terms of this decomposition the physical zero modes belong to the $j=1$ part of the determinant [4,6]. Apart from these modes we will in a first step only consider the spherically symmetric, i.e. $j=0$, part of the fluctuations, since the unstable mode is to be found in this channel [18]. We then determine the eigenfunctions of the unstable mode and, as a by-product of our numerical procedure, obtain a rough estimate for the $j=0$ part of the determinant. The calculation of the transition rate now essentially follows those in refs. [4,5]. Consider fluctuations around the sphaleron configuration,

$$W_i^a=W_{\mathrm{Sp}\,i}^a+\chi_i^a, \quad U=U_{\mathrm{Sp}}+\eta\,. \tag{7}$$

Spherically symmetric fluctuations of U are restricted to fluctuations of the radial function $\theta=\theta_{\mathrm{Sp}}+\beta$. The gauge zero modes obtained by transformations $V(x)=\exp[-(\mathrm{i}/2)\Lambda^a(x)\tau^a]$ from the sphaleron are given by

$$\chi_i^a=-\frac{1}{g}(\mathrm{D}_i\Lambda)^a, \quad \eta=-\frac{\mathrm{i}}{2}\tau^a\Lambda^a U_{\mathrm{Sp}}\,. \tag{8}$$

Here and in the following the covariant derivative is built with the sphaleron background field. To avoid counting these unphysical zero modes in the path integral one has to impose a gauge fixing condition

$$(\mathrm{D}_i\chi_i)^a-\mathrm{i}\frac{m^2}{2g}\mathrm{Tr}(\eta^\dagger\tau^a U_{\mathrm{Sp}}-U_{\mathrm{Sp}}^\dagger\tau^a\eta)=0\,. \tag{9}$$

The $(R_{\xi=1})$ gauge fixed static fluctuation hamiltonian then takes the form

$$\begin{aligned}H_{\mathrm{fl}}=\int \mathrm{d}^3x\bigg\{&\tfrac{1}{2}(\mathrm{D}_i\chi_j)^a(\mathrm{D}_i\chi_j)^a+g\epsilon_{abc}F_{ij}^a\chi_i^b\chi_j^c-\frac{m^2}{gr}\beta(\mathrm{D}_i\chi_i)^a x_a+\frac{m^4}{2g^2}\beta^2\\&+m_W^2\bigg[\tfrac{1}{2}\chi_i^a\chi_i^a-\frac{1}{gr^2}\beta'\chi_i^a x_i x_a+\frac{1}{g^2}\frac{\beta'^2}{2}-\cos\theta\,\beta\left(\frac{1}{gr}\chi_i^i-\frac{1}{gr^3}\chi_i^a x_i x_a\right)+\tfrac{1}{2}\sin\theta\,\beta^2\left(\frac{1}{gr}W_i^i-\frac{1}{gr^3}W_i^a x_i x_a\right)\\&+\sin\theta\,\beta\frac{1}{gr^2}\epsilon_{iam}\chi_i^a x_m+\cos\theta\,\beta^2\left(\frac{2}{g^2r^2}+\frac{1}{gr^2}\epsilon_{iam}W_i^a x_m\right)\bigg]-c^{\dagger a}[-(\mathrm{D}_i\mathrm{D}_i)_{ab}+m_W\delta_{ab}]c^b\bigg\}\,,\end{aligned} \tag{10}$$

where c^a, $c^{\dagger a}$ are Fadeev–Popov ghosts and we have dropped the index "Sp" from the sphaleron solution W_i^a, θ. Besides the gauge zero modes there are also physical zero modes corresponding to spatial translations and rotations of the sphaleron solution, which have to be integrated separately using collective coordinates. The naive spatial transformations have to be supplemented by appropriate gauge transformations such that they fulfill the gauge fixing condition (9). Carrying out the procedure described in refs. [4,5] we find the normalized volume factor

$$(\mathcal{N}V)=8\pi^2 V\frac{(\xi_{\rm tr}\xi_{\rm rot})^{3/2}}{(2\pi\beta\alpha_{\rm w})^3}, \tag{11}$$

with

$$\xi_{\rm tr}=\tfrac{1}{3}\int {\rm d}x\left(4f_{\rm A}'^2+2\frac{(f_{\rm A}^2-1)^2}{x^2}+4f_{\rm C}^2f_{\rm A}^2+x^2(f_{\rm C}'^2+\theta'^2-2\theta' f_{\rm C})+2f_{\rm A}^2+2-4f_{\rm A}\cos\theta\right)=5.07\,,$$

$$\xi_{\rm rot}=\tfrac{1}{3}\int {\rm d}x\,x^2\Big(2f_{\rm A}^2f_{\rm C}^2+2\frac{(f_{\rm A}^2-1)^2}{x^2}+2f_{\rm A}'^2+8\frac{1-f_{\rm A}^2}{x^2}Q-16\frac{f_{\rm A}'}{x}P-16\frac{f_{\rm A}f_{\rm C}}{x}R$$

$$+2(f_{\rm A}-1)^2+32(P^2+R^2)+16Q^2+(1-\cos\theta)(16P+4f_{\rm A})+16\sin\theta\, R\Big)=1.53\,, \tag{12}$$

where the auxiliary functions P, Q and R satisfy the differential equations

$$P''=-\frac{2}{x}P'+\left(\frac{f_{\rm A}^2+1}{x^2}+f_{\rm C}^2+1\right)P-\frac{2f_{\rm A}}{x^2}Q+2R'f_{\rm C}+f_{\rm C}'R-\frac{f_{\rm A}'}{2x},$$

$$Q''=-\frac{2}{x}Q'+\left(2\frac{f_{\rm A}^2+1}{x^2}+1\right)Q-\frac{4f_{\rm A}}{x^2}P+\frac{1-f_{\rm A}^2}{2x^2},$$

$$R''=-\frac{2}{x}R'+\left(\frac{(f_{\rm A}-1)^2}{x^2}+\frac{2}{x^2}+f_{\rm C}^2+1\right)R-2f_{\rm C}P'-Pf_{\rm C}'-\frac{2f_{\rm C}}{x}P-\frac{f_{\rm C}f_{\rm A}}{2x}, \tag{13}$$

with $x=m_W r$.

Next the spherically symmetric fluctuations are parametrized by

$$\chi_i^a(x)=\frac{1}{g}\left(\frac{\phi_{\rm A}(r)}{r}\epsilon_{iam}x_m+\frac{\phi_{\rm B}(r)}{r^2}(r^2\delta_{ia}-x_ix_a)+\sqrt{2}\frac{\phi_{\rm C}(r)}{r^2}x_ix_a\right),$$

$$\beta(r)=\frac{\sqrt{2}}{m_W}\phi_{\rm D}(r),\quad c^a(x)=f(r)\frac{x^a}{r}, \tag{14}$$

and yield the following set of coupling differential equations for the fluctuation frequencies ω:

$$\left(-\frac{{\rm d}^2}{{\rm d}x^2}-\frac{2}{x}\frac{{\rm d}}{{\rm d}x}+\frac{3f_{\rm A}^2-1}{x^2}+f_{\rm C}^2+1\right)\phi_{\rm A}-2f_{\rm C}\phi_{\rm B}'-\left(\frac{2f_{\rm C}}{x}+f_{\rm C}'\right)\phi_{\rm B}+\sqrt{2}\frac{\sin\theta}{x}\phi_{\rm D}+2\sqrt{2}\frac{f_{\rm A}f_{\rm C}}{x}\phi_{\rm C}=\bar\omega^2\phi_{\rm A}\,,$$

$$\left(-\frac{{\rm d}^2}{{\rm d}x^2}-\frac{2}{x}\frac{{\rm d}}{{\rm d}x}+\frac{3f_{\rm A}^2-1}{x^2}+f_{\rm C}^2+1\right)\phi_{\rm B}+2f_{\rm C}\phi_{\rm A}'+\left(\frac{2f_{\rm C}}{x}+f_{\rm C}'\right)\phi_{\rm A}+2\sqrt{2}\left(\frac{f_{\rm A}'}{x}-\frac{f_{\rm A}}{x^2}\right)\phi_{\rm C}+\sqrt{2}\frac{f_{\rm A}-\cos\theta}{x}\phi_{\rm D}$$

$$=\bar\omega^2\phi_{\rm B}\,,$$

$$\left(-\frac{{\rm d}^2}{{\rm d}x^2}-\frac{2}{x}\frac{{\rm d}}{{\rm d}x}+2\frac{f_{\rm A}^2+1}{x^2}+1\right)\phi_{\rm C}+2\sqrt{2}\left(\frac{f_{\rm A}'}{x}-\frac{f_{\rm A}}{x^2}\right)\phi_{\rm B}+2\sqrt{2}\frac{f_{\rm A}f_{\rm C}}{x}\phi_{\rm A}=\bar\omega^2\phi_{\rm C}\,,$$

$$\left(-\frac{{\rm d}^2}{{\rm d}x^2}-\frac{2}{x}\frac{{\rm d}}{{\rm d}x}+\cos\theta\frac{2f_{\rm A}}{x^2}+1\right)\phi_{\rm D}+\sqrt{2}\frac{\sin\theta}{x}\phi_{\rm A}+\sqrt{2}\frac{f_{\rm A}-\cos\theta}{x}\phi_{\rm B}=\bar\omega^2\phi_{\rm D}\,,$$

$$\left(-\frac{{\rm d}^2}{{\rm d}x^2}-\frac{2}{x}\frac{{\rm d}}{{\rm d}x}+\frac{4-2f_{\rm A}^2}{x^2}+1\right)f=\bar\omega_F^2 f, \tag{15}$$

where we have changed to the dimensionless radial variable $x=m_W r$ and frequency $\bar{\omega}=\omega/m_W$. Note that there is no decoupling of channels with definite parity because the deformed sphaleron is no eigenstate of the parity operator.

We estimate the frequency spectrum by putting the system in a sphere of radius R much larger than the sphaleron radius m_W^{-1} and imposing boundary conditions $r\phi_i(r)\to 0$ as $r\to 0$ and $\phi_i(r)\to 0$ as $r\to R$. Then the eigenvalue equations (15) are discretized and diagonalized numerically. We find one negative eigenvalue $\omega_-^2=-3.95m_W^2$. This value is confirmed by using it and the corresponding eigenfunctions as initial estimate for the solution of (15) in a relaxation routine. The eigenfunctions of the unstable mode are displayed in fig. 1. Fluctuations around the vacuum are obtained from (15) by letting $f_A(x)\to 1$, $\theta(x)\to 2\pi$. The difference between the free energies of the sphaleron and the vacuum is [4]

$$\Delta F=F_S-F_0=E_{Sp}+\frac{1}{\beta}\sum_{i=1}^{\infty}\left(\ln\frac{1-\exp(-m_0 g^2\bar{\omega}_i)}{1-\exp(-m_0 g^2\bar{\omega}_i^0)}+\ln\frac{1-\exp(-m_0 g^2\bar{\omega}_{Fi}^0)}{1-\exp(-m_0 g^2\bar{\omega}_{Fi})}\right), \tag{16}$$

where we have assumed that the divergent zero-point oscillations can be absorbed by a counterterm whose finite effects are small. A crude numerical estimate (accuracy $\sim$ (10–20)%) of the entropy factor is given in table 1. We have not pushed for higher numerical accuracy because the uncertainty due to our lack of knowledge about higher angular momentum parts of the entropy factor is larger than the numerical uncertainty. For the case of the sphaleron in the phase with broken symmetry it was shown that the higher partial waves give important contributions to the determinant [6]. The point here is merely to see if there is any entropy suppression already in the spherically symmetric channel.

Collecting the results the formula for the transition rate over the lowest energy deformed sphalerons reads

$$\frac{\Gamma}{V}=\gamma(m_0)T^4\exp[-\beta(\Delta F-E_{Sp})]\tilde{\kappa}, \tag{17}$$

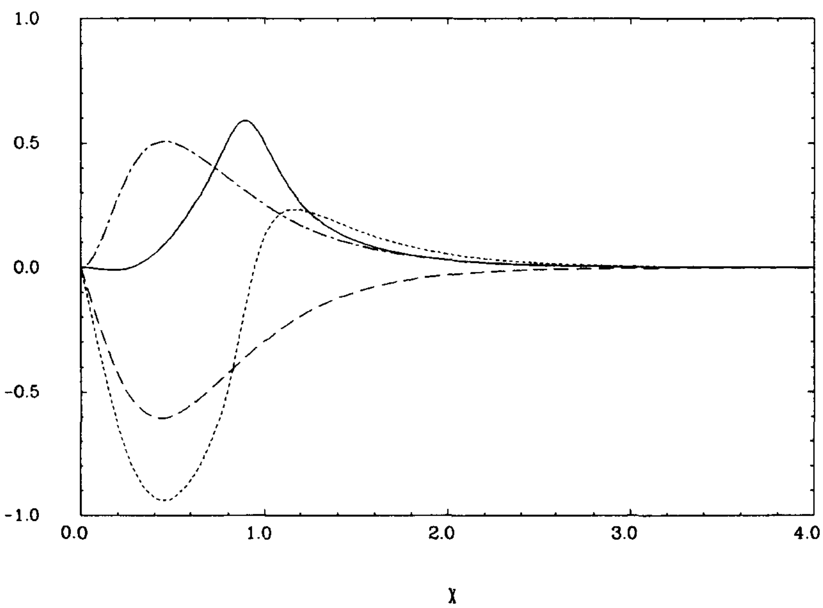

Fig. 1. Eigenfunctions of the unstable mode. Solid line: $x\phi_A(x)$; dotted line: $x\phi_B(x)$, dashed line: $x\phi_C(x)$, dash-dotted line: $x\phi_D(x)$. The eigenfunctions are normalized to $\int dx\,x^2(\phi_A^2+\phi_B^2+\phi_C^2+\phi_D^2)=1$.

Table 1
Entropy factor of the spherically symmetric fluctuations.

m_0	$\exp[-\beta(\Delta F-E_{\rm Sp})]$
0.1	42.3
0.2	27.1
0.3	19.5
0.4	14.8
0.5	11.8
0.6	9.5
0.7	8.1
0.8	6.9

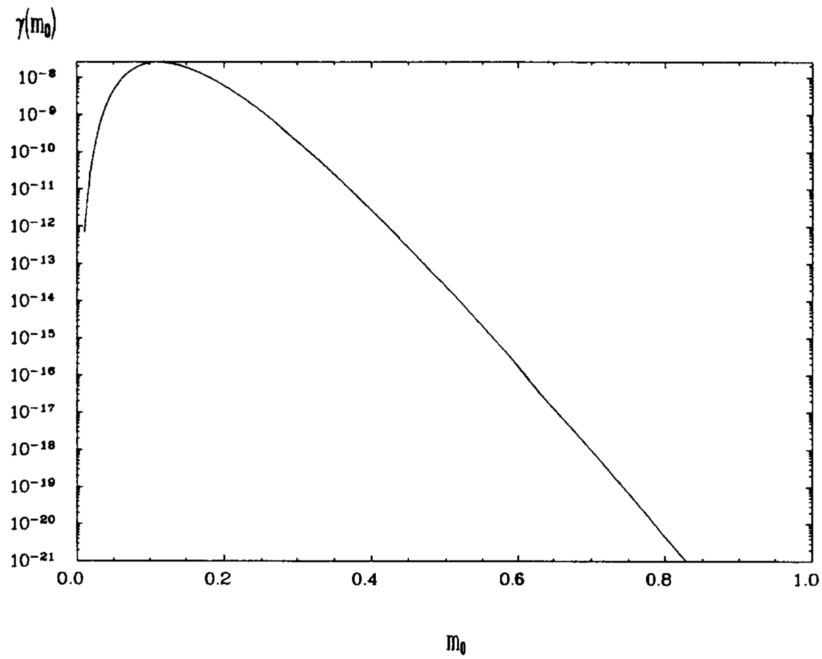

Fig. 2. The contributions of the single negative mode and the zero modes to the prefactor in the transition rate.

where

$$\gamma(m_0)=m_0\,|\bar{\omega}_-|\,\frac{4}{\pi}\frac{(\xi_{\rm tr}\xi_{\rm rot})^{3/2}}{\alpha_{\rm w}^3}\frac{\sinh(\frac{1}{2}m_0g^2\bar{\omega}_1^0)}{\sin(\frac{1}{2}m_0g^2\bar{\omega}_-)}\,[\sinh(\tfrac{1}{2}m_0g^2\bar{\omega}_{j=1}^0)]^6\exp(-\beta E_{\rm Sp})\,, \tag{18}$$

and $\tilde{\kappa}$ contains the fluctuation determinant of all channels with $j>0$. We have extracted a factor $[\sinh(\frac{1}{2}m_0g^2\bar{\omega}_{j=1}^0)]^6$ from the $j=1$ channel putting $\bar{\omega}_{j=1}^0\approx 1$ to compensate for the zero modes which have been integrated separately. The factor $\gamma(m_0)$ is shown in fig. 2. To see the relevance of this result one has to compare the rate of $B+L$ violating processes induced by the sphaleron transitions [3],

$$\Gamma_{\rm B}=\frac{{\rm d}N_{\rm B}}{N_{\rm B}\,{\rm d}t}\approx-\tfrac{13}{2}n_{\rm f}\beta^3\frac{\Gamma}{V}, \tag{19}$$

(here N_B denotes the baryon excess) with the Hubble expansion rate $H=1.66\sqrt{g^*}/\beta^2 M_{Pl}$. Plugging in our results and taking $g^*\sim 100$ one finds

$$\frac{|\Gamma_B|}{H}\approx 1.17\gamma(m_0)\frac{M_{Pl}}{T}\exp[-\beta(\Delta F-E_{Sp})]\tilde{\kappa}\,. \tag{20}$$

Without much playing around with different parameter values for m_0, $\tilde{\kappa}$ and T it is easy to see that for $m_0<0.5$, say, there will be a substantial effect even for some moderate entropy suppression coming from $\tilde{\kappa}$.

The effective coupling α_3 does not tell us anything about the validity of the one loop approximation since it does not contain any scale. Instead we have to compare the contributions of the saddle point and the prefactor to the final result. The quantity which was evaluated in saddle-point approximation is Im Z_{Sp}/Z_0. From this we divide out the volume factor VT^3 and thus compare $\ln\{\gamma(m_0)\exp[-\beta(\Delta F-E_{Sp})]/\exp(-\beta E_{Sp})\}$ with βE_{Sp}. We find that for $m_0>0.3$ the logarithm of the prefactor is always less than 10% of the saddle-point contribution such that one can trust a one loop calculation. Unfortunately, for the most interesting range with $m_0<0.2$ the prefactor is already $\sim 30\%$ of the saddle contribution and the approximation is not a good one anymore. Of course matters change again for values of $\tilde{\kappa}$ much different from one. Another remark concerns the validity of the dilute sphaleron gas approximation. In ref. [3] it was shown that one has to demand $(N/V)^{1/3}R\ll 1$ for the sphalerons to be sufficiently diluted, where $N\approx \mathrm{Im}\,Z_{Sp}/Z_0$ is the number of sphaleron transitions and $R\approx m_W^{-1}$ the sphaleron radius. Because of $m_W\sim g^2T$ in the model considered here the temperature cancels out of the condition such that it is always fulfilled.

Restating our result in the form of (3) we find that even in the best case $\kappa\sim O(10^{-1}-10^{-3})$ for the model (4) we considered. In conclusion we have a qualitative picture indicating that there is a parameter range for the magnetic mass of the W-boson in the symmetric phase for which we find a substantial rate of sphaleron transitions leading to $B+L$ violation. However, this rate is falling short of the lattice results by at least one order of magnitude.

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