# Calculation of jet shapes in $\gamma p$ collisions at HERA 

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#### Abstract

We have calculated jet shapes in resolved $\gamma p$ collisions in perturbation theory at order $\alpha_{3}^{3}$ for the hard parton-parton processes. Various predictions concerning the dependence on transverse energy, rapidity and the inner cone extension are presented.


## 1. Introduction

The production of high transverse momentum ( $p_{\mathrm{T}}$ ) jets by quasi-real photons on protons is one of the major processes to gain further insight into the interaction of photons with quarks and gluons. First experimental results by the H1 Collaboration [1] and ZEUS Collaboration [2] at HERA have been published recently and more will be presented soon.

One distinguishes two mechanisms which contribute to the photoproduction of jets at large transverse momentum. The photon can either interact directly or via its quark and gluon content, the so-called resolved part, with the partons coming from the proton and producing high $-p_{\mathrm{T}}$ jets. The lowest order or Born term of the direct production is $\mathrm{O}\left(\alpha \alpha_{\mathrm{s}}\right)$, whereas the lowest order parton-parton scattering contribution is $\mathrm{O}\left(\alpha_{\mathrm{s}}^{2}\right)$. Since this term is multiplied by the photon structure function which has a term of $O\left(\alpha / \alpha_{s}\right)$, both contributions, i.e. the direct and the resolved part, are of the same order in the perturbative expansion. Actually, the resolved photoproduction dominates at HERA for low $p_{\mathrm{T}}$ and negative rapidities (we assign the rapidity as positive in the direction of the incoming electron). Near the maximum of the rapidity distribution ( $\eta=-2$ ) the resolved and the direct part become comparable at $p_{\mathrm{T}} \simeq 35 \mathrm{GeV}$. This result is not

[^0]changed when higher order QCD corrections are included in the resolved ( $O\left(\alpha_{\mathrm{s}}^{3}\right)$ ) and the direct ( $\mathrm{O}\left(\alpha \alpha_{\mathrm{s}}^{2}\right)$ ) contribution. This is in accordance with recent data from H 1 and ZEUS which indicate that the resolved production is dominant for the lower $p_{\mathrm{T}}$ 's of the high $p_{\mathrm{T}}$ range [1,2]. So as long as the jet $p_{\mathrm{T}}$ is not too large (below 25 GeV or so) one can safely neglect the contribution from direct photoproduction.
Some months ago we presented results for inclusive single jet production in next-to-leading order (NLO) QCD where we restricted ourselves to resolved photoproduction only [3]. The inclusive jet production was studied as a function of the jet cone size $R$. Two partons are combined to one jet when their momenta lie inside the cone with size $R$. We found that for $R \simeq 0.7$ the NLO cross section was approximately equal to the Born cross section over the whole ( $p_{\mathrm{T}}, \eta$ ) range considered. Our Born cross section was calculated with the same NLO set of parton distributions for the photon and the proton (only $\alpha_{\mathrm{s}}$ was calculated from the one-loop formula). The $R$ dependence of the cross section was approximately given by the formula
$\sigma=a+b \ln R+c R^{2}$,
where the first two terms were sufficient up to $R \simeq 0.3$, slightly increasing with $p_{\mathrm{T}}$. We also investigated the scale dependence of the cross section and found that the scale dependence is strongly reduced when adding the NLO corrections. This reduction depends on the cone size $R$. In the maximum of the rapidity distribution we observed a rather mild dependence on
the scale (the renormalization scale $\mu$ is set equal to both factorization scales $Q$ ) near $R \simeq 0.7$ over the whole range of $p_{\mathrm{T}}$ examined. These results for $\gamma p$ collisions are qualitatively very similar to the results obtained for inclusive jet production in $p \bar{p}$ collisions at $\sqrt{s}=1.8 \mathrm{TeV}$ [4,5]. They are also in qualitative agreement with the results obtained by other authors for resolved photoproduction [6]. The results of refs. [4,5] for $\bar{p} p$ agree quite well with data from CDF at the TEVATRON [7].

Although there are no experimental data yet which test the cone dependence of the inclusive one-jet photoproduction cross section, both experiments at HERA used only $R=1$ as cone size, it is worthwhile to study the cone dependence of the cross section in more detail. For this purpose we have analysed the internal structure of jets with cone size $R=1$. Following similar studies for $p \bar{p}$ collisions [8] we consider the fractional $E_{\mathrm{T}}$ profile $\rho\left(r, R, E_{\mathrm{T}}, \eta\right)$ as a function of $r, E_{\mathrm{T}}$ and $\eta$. This function is a good measure of the internal jet structure.

In section 2 we shall explain the basic input for our calculation and the precise definition of the jet profile $\rho$. Some of our final results are presented in section 3.

## 2. Theoretical input

As in our previous paper [3] we consider only the resolved photon. The initial electron energy is $p_{e}=30$ GeV and the proton energy $p_{p}=820 \mathrm{GeV}$ giving $\sqrt{s}=314 \mathrm{GeV}$. As usual, the photon emission from the electron is described by the Weizsäcker-Williams equivalent photon approximation with a fixed angle cut for the final electron with a formula as in ref. [3]. We consider only one combination of structure functions. For the proton structure functions we have chosen the set B1 of Morfin and Tung [9] in the $\overline{\text { MS }}$ factorization scheme. The parton distributions of the photon are taken from the work of Glück, Reya and Vogt [10] also in the $\overline{\text { MS }}$ scheme. Both are NLO parametrizations. The details are not essential for the calculation of $\rho$ since it is a ratio of cross sections. For $\alpha_{\mathrm{s}}(\mu)$ we employ the one-loop formula with $N_{\mathrm{f}}=4$ and with the QCD scale parameter $A$ taken from the proton structure function. We decided on the one-loop
expression for $\alpha_{\mathrm{s}}$ since we are doing essentially a treegraph calculation as we shall see later.
As jet definition we adopt the convention of the Snowmass Meeting [11]. According to this definition a jet is defined as transverse energy $E_{T}$ deposited in a cone of radius $R$ in the rapidity-azimuthal angle ( $\eta, \phi$ )-plane. The jet axis is given by
$\eta_{J}=\sum_{i \in \mathrm{jet}} E_{\mathrm{T}, i} \eta_{i} / E_{\mathrm{T}}$,
$\phi_{J}=\sum_{i \in \mathrm{jet}} E_{\mathrm{T}, i} \phi_{i} / E_{\mathrm{T}}$.
with the jet transverse energy $E_{\mathrm{T}}$ obtained from
$E_{\mathrm{T}}=\sum_{i \in \mathrm{jet}} E_{\mathrm{T}, i}$.
A parton with kinematic variables ( $\eta_{i}, \phi_{i}$ ) is included in the jet if the condition
$\left(\eta_{i}-\eta_{J}\right)^{2}+\left(\phi_{i}-\phi_{J}\right)^{2} \leqslant R^{2}$
is satisfied. Since we include only up to three partons in the final state not more than two partons can build up a jet. One should note that this jet definition is not fully defined since jets can overlap. For the $2 \rightarrow 3$ process it is possible for a one parton jet to lie within the cone of a jet combined out of two partons. In ref. [4] this particular configuration was considered as a twojet event only. In our previous work [3] we did not correct for this. However by subtracting the relevant three-jet events, the cross section is lowered by a few percent only. The quantity $\rho$ we present here is unaffected by these details of the jet recombination except at $r=0$.

The fractional $E_{\mathrm{T}}$ profile $\rho\left(r, R, E_{\mathrm{T}}, \eta\right)$ is defined in the same way as in the case for $p \bar{p}$ reactions [8]. For a sample of jets of transverse energy $E_{\mathrm{T}}$ defined with a cone radius $R$, the quantity $\rho$ is the average fraction of the jet's transverse energy that lies inside an inner cone with radius $r<R$ which is concentric with the jet defining cone. Then the quantity $1-\rho$ stands for the fraction of $E_{\mathrm{T}}$ that lies in the cone segment between $r$ and $R$, i.e.

$$
\begin{align*}
& 1-\rho\left(r, R, E_{\mathrm{T}, \mathrm{~J}}, \eta\right) \\
& =\frac{\int \mathrm{d} E_{\mathrm{T}} E_{\mathrm{T}}\left[\mathrm{~d} \sigma(\gamma p \rightarrow 3 \text { partons }+X) / \mathrm{d} E_{\mathrm{T}}\right]}{E_{\mathrm{T}, \mathrm{~J}} \sigma\left(E_{\mathrm{T}, \mathrm{~J}}\right)_{\mathrm{Bora}}}, \tag{6}
\end{align*}
$$

where the integral in the numerator is performed over the cone segment between $r$ and $R$. This quantity is
easily calculated from the contributing $2 \rightarrow 3$ parton subprocesses. For $r>0$ the integration does not include the collinear singularities which are at $r=0$. Therefore it is straightforwardly computed from the $E_{\mathrm{T}}$ weighted integral of the $\gamma+p \rightarrow 3$ partons $+X$ cross section over the cone segment between $r$ and $R$ normalized to $E_{\mathrm{T}, \mathrm{J}}$ times the Born cross section. 1- $\rho$ in (5) is $\mathrm{O}\left(\alpha_{s}\right)$ since the numerator is proportional to $\alpha_{\mathrm{s}}^{3}$ and the denominator is $\mathrm{O}\left(\alpha_{\mathrm{s}}^{2}\right)$. In the following we denote the total transverse energy of the jet as $E_{\mathrm{T}}$.

## 3. Results

We have calculated $\rho\left(r, R, E_{\mathrm{T}}, \eta\right)$ as a function of $r$ for a fixed $R=1$ for several $E_{\mathrm{T}}=5,15$ and 30 GeV and for $\eta=-2$. The results are plotted in fig. 1 . This plot shows that the jets become narrower with increasing $E_{\mathrm{T}}$. The Born result corresponds to a constant value of $\rho=1$ for all $r, E_{\mathrm{T}}$ and $\eta$, while the higher order calculation yields a characteristic distribution varying with $r, E_{\mathrm{T}}$ and $\eta$. In the limit $r \rightarrow 0$ the jet pro-


Fig. 1. Jet profile $\rho\left(r, R, E_{\mathrm{T}}, \eta\right)$ versus $r$ for $R=1, \sqrt{s}=314 \mathrm{GeV}$, $\eta=-2$ and $E_{\mathrm{T}}=5 \mathrm{GeV}$ (full), $E_{\mathrm{T}}=15 \mathrm{GeV}$ (dotted) and $E_{\mathrm{T}}=30$ GeV (dashed).
file $\rho$ becomes logarithmically singular. This behaviour signals the breakdown of perturbation theory. Therefore, in general, the predictions are more reliable for larger $r$ values in the vicinity of $r=1$, say above $r=0.3$ or so. In fig. 2 we show a different plot of $\rho$. Here we kept $r$ fixed at $r=0.3,0.5$ and 0.7 and considered $\rho\left(r, R . E_{\mathrm{T}}, \eta\right)$ as a function of $E_{\mathrm{T}}$ for fixed $R=1$ and $\eta=-2$. So these curves give the fraction of $E_{\mathrm{T}}$ inside fixed smaller cones with radii $r=0.3,0.5$, 0.7 versus $E_{\mathrm{T}}$. These plots show again that the jets become narrower with increasing $E_{\mathrm{T}}$. The $E_{\mathrm{T}}$ dependence of these three curves is very well accounted for through the $E_{\mathrm{T}}$ variation of $\alpha_{\mathrm{s}}$, i.e. $(1-\rho)=$ $A \alpha_{\mathrm{s}}\left(\mu=E_{\mathrm{T}}\right)$ and $A$ is almost $E_{\mathrm{T}}$ independent. All these results are qualitatively similar to results obtained for $p \bar{p}$ collisions at CM energy $\sqrt{s}=1.8 \mathrm{TeV}$ [8]. The $\eta$ dependence of $\rho$ for fixed $r=0.3$ and for three choices of $E_{\mathrm{T}}=5,15$ and 30 GeV is exhibited in fig. 3. The jets are narrower near the phase space limits in $\eta$. For fixed $\eta$ the narrowness increases with increasing $E_{\mathrm{T}}$ as we saw already for $\eta=-2$ in figs. 1 and 2.
Since $\rho$ is calculated in lowest nontrivial order


Fig. 2. Jet profile $\rho\left(r, R, E_{\mathrm{T}}, \eta\right)$ versus $E_{\mathrm{T}}$ for $R=1, \sqrt{s}=314$ $\mathrm{GeV}, \eta=-2$ and $r=0.3$ (full), $r=0.5$ (dotted) and $r=0.7$ (dashed).


Fig. 3. Jet profile $\rho\left(r, R, E_{T}, \eta\right)$ versus $\eta$ for $R=1, \sqrt{s}=314 \mathrm{GeV}$, $r=0.3$ and $E_{\mathrm{T}}=5 \mathrm{GeV}$ (full), $E_{\mathrm{T}}=15 \mathrm{GeV}$ (dotted) and $E_{\mathrm{T}}=30$ GeV (dashed).
( $\mathrm{O}\left(\alpha_{\mathrm{s}}\right)$ ) of perturbation theory we expect an appreciable scale dependence of the results. In all the results shown so far we have chosen as scale $\mu=Q=E_{\mathrm{T}}$. When $\mu$ is lowered the coupling $\alpha_{\mathrm{s}}$ increases from which we expect that $\rho$ will be reduced. This is indeed the case, $\rho$ decreases monotonically with increasing scale. Of course this scale dependence can be compensated only by including higher orders in $\alpha_{s}$ which at present are not known. Due to the scale dependence we can not expect perfect agreement with experimental data [12]. Another uncertainty are fragmentation effects. Experimentally jets are a spray of hadrons which cover a finite angular range. Only for jets with sufficient large $E_{\mathrm{T}}$ we expect the angular spread from fragmentation effects to be small compared to the jet cone size used above. Therefore we anticipate better agreement of the theoretical $\rho(r, R$, $E_{\mathrm{T}}, \eta$ ) with experimental data measured for the larger $r$ and/or $E_{\mathrm{T}}$. A second problem is that the implementation of the jet algorithm used in analysing the experimental data need not coincide in detail with the jet defining procedure in the theoretical evaluation.

According to Ellis et al. [8] this mismatch might be particularly important for the limiting configuration when two equal transverse energy partons (each with $\frac{1}{2} E_{\mathrm{T}}$ ) are just $2 R$ apart. In our calculation this is counted as a single jet of transverse energy $E_{\mathrm{T}}$ with its cone centered between the two partons. Due to the angular spread caused by the hadronization this configuration is likely to be treated differently in the experimental analysis. To account for this Ellis et al. [8] added an extra constraint to the theoretical jet algorithm. Namely, two partons $a$ and $b$ are no longer merged into a single jet when they are separated by more than $R_{\text {sep }}(\leqslant 2 R)$, i.e. when $R_{a b}^{2}=\left[\left(\eta_{a}-\eta_{b}\right)^{2}\right.$ $\left.+\left(\phi_{a}-\phi_{b}\right)^{2}\right] \geqslant R_{\text {sep }}^{2}$. The results shown in figs. 1,2 and 3 correspond to $R_{\text {sep }}=2 R$. We investigated the influence of a $R_{\text {sep }}<2 R$ on $\rho\left(r, R, E_{\mathrm{T}}, \eta\right)$ and found similar results as in ref. [8] for $p \bar{p}$ collisions. The fraction of $E_{\mathrm{T}}$ increases near the edge of the cone at $r \simeq R$. Further details on this will be reported in connection with the forthcoming presentation of the experimental measurements from ZEUS [13]. In addition we considered also different choices for $R<1$. The results are quite similar as for $R=1$. Details will be reported elsewhere [14].

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