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Miscellaneous results on the electroweak phase transition

E.-M. Ilgenfritz [a]*and A. Schiller [b]

[a]Institut für Physik, Humboldt-Universität zu Berlin, Invalidenstr. 110, D–10099 Berlin, Germany

[b]Fakultät f. Physik und Geowiss., Universität Leipzig, Augustusplatz 10–11, D–04109 Leipzig, Germany

We present new $4-D$ Monte Carlo results characterizing the strength of the finite temperature phase transition for Higgs/W mass ratios 1.0 and 0.6, obtained on isotropic lattices mainly with $N_s = 16, N_t = 2$. We discuss the distribution of an gauge invariant block spin order parameter, estimating the Higgs condensate ϕ_c at T_c. We use the Potvin/Rebbi method in order to find the interface tension α/T_c^3. We demonstrate how the multi-histogram method (giving free energy differences) can be used to avoid the limiting procedure $\delta\kappa \to 0$. From pure-phase histograms at κ_c, extrapolated with the help of this method, we estimate the latent heat $\Delta\epsilon/T_c^4$. Actual time series at lower Higgs mass require blocking in order to determine the jump of the lattice observables.

1. INTRODUCTION

Here is no need to dwell on the phenomenological importance to be able to calculate the physical quantities which characterize the electroweak phase transition (see K. Kajantie's review at this conference [1]). It is mainly due to the possible generation of the baryon asymmetry when our universe underwent this transition. Studying its nature and strength on the lattice, albeit restricted to a purely bosonic, $SU(2)$ gauge-Higgs model at unphysically small Higgs mass, may serve to state its intrinsically non-perturbative features. While perturbation theory describes well the broken phase up to T_c (in particular $\phi(T)$), it breaks down both at small $\phi << T_c$ and in the symmetric phase $T > T_c$. Before the symmetric phase is qualitatively understood, lattice Monte Carlo calculations are indispensable to quantify the strength of the transition.

We have studied the pure $SU(2)$ gauge-Higgs model with the action

$$S = \beta \sum_{plaq} (1 - \frac{1}{2} Tr U_p) - \kappa \sum_{links} Tr(\Phi_x^+ U_{x,\mu} \Phi_{x+\mu})$$
$$+ \sum_{sites} (\rho_x^2 + \lambda(\rho_x^2 - 1)^2) \quad (1)$$

($\rho_x^2 = \frac{1}{2} Tr(\Phi_x^+ \Phi_x)$) at $\beta = 8.0$ for $m_H \le m_W$ (medium $\lambda = 0.00172$[2,3] and small $\lambda = 0.0005$). The algorithm combined a $3-D$ Gaussian heat bath for $U_{x,\mu}$ and a $4-D$ Gaussian heat bath (improved for acceptance) for $\Phi_x = \rho_x V, V \in SU(2)$. The autocorrelation was optimized by one heat bath step followed by 8 reflections for the Higgs and 1 reflection for the gauge field (see B. Bunk[4]). The lattice scale was determined at κ_c for medium λ on a 24^4 lattice, giving $m_H/m_W = 1.0(1)$ corresponding to $m_H = 49 GeV$ and $T_c/m_W = 1.74(5)$. For small λ the most precise calibration at κ_c was obtained on an anisotropic ($\gamma_G = \gamma_H = 2$) $16^3 \times 32$ lattice, giving $m_H a_s/m_W a_s = 0.62(2)$ and $m_H a_t/m_W a_t = 0.60(1)$ corresponding to $T_c/m_W = 1.13(1)$. Large statistics Quadrics Q16 results were presented by the DESY group for $m_H = 49 GeV$ and $m_H = 18 GeV$ ($\lambda = 0.0001$) at this conference[5].

2. ORDER PARAMETER

In order to define a gauge invariant order parameter we employed the projective block spin construction[3]. Solving the covariant Laplace eigenvalue problem

$$-D_a^2[U] C_x^a = \lambda_0^a C_x^a \quad (2)$$

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on various blocks a (including the lattice as single block, with Neumann boundary conditions), the eigenvector C_x^a corresponding to the lowest eigenvalue (normalized as $\frac{1}{2}\sum_{x\in a} Tr(C_x^{a+}C_x^a) = |a|$, $|a|$ is the block volume) is used to define a block Higgs field

$$\Phi^a = \frac{1}{|a|}\sum_{x\in a} C_x^{a+}\Phi_x. \quad (3)$$

The lowest eigenvalue is obtained by using the conjugate gradient method to minimize the Ritz functional. Convergence is found to be much slower (several hundreds of iterations) in the symmetric than in the broken phase. The Higgs length $\phi^a = \sqrt{det(\Phi^a)}$ is the scalar order parameter.

It is instructive to compare the order parameter distributions for the whole lattice $16^3 \times N_t$ and for subblocks, at $N_t = 4$ and 2. On lattices of that size a two-state signal can be easily seen on the whole lattice (see Fig. 1), but it becomes generically weaker for $8^3 \times N_t$ subblocks as well as for $N_t = 2$ instead of 4. In no case it was possible to apply the equal-area criterion to determine κ_c (which is instead defined by the link susceptibility). The distribution for the symmetric phase is known from simulations well below κ_c, to shrink and move towards $\phi = 0$ with larger block size. For the broken phase well above κ_c the distribution becomes narrower with block size but moves only with rising κ. The same is true for the two-state histograms near to κ_c.

From the maximum of the peak describing the broken phase in phase equilibrium we estimate $\phi_c/T_c = \sqrt{2\kappa_c}\phi_{max}N_t$. We find 1.0 at medium λ and 1.15 at small λ. A detailed study of the distribution near $\phi = 0$ (and its scaling properties with N_s/ξ for lattice size comparable to correlation length) would require multicanonic updating *i.e.* the knowledge of C_x^a in every Higgs update.

3. INTERFACE TENSION

For the small λ case we have examined the method of Potvin and Rebbi[6] to determine the surface tension in a relatively small system. Two lattices Λ of size $16^2 \times 32 \times 2$ are kept at κ_1 and κ_2 and put into contact along the xyt hyperplanes.

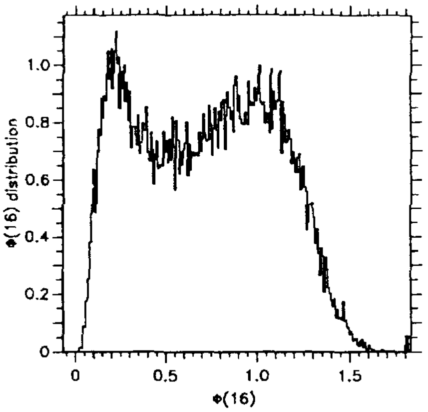

Figure 1. *Order parameter distribution for whole lattice* $16^3 \times 2$ *at* $\kappa = 0.12887$ *and small* λ *(30000 measurements, autocorrelation 9.3)*

Due to periodic boundary conditions there should be eventually two interfaces. Runs with many pairs (κ_1, κ_2) of couplings (grid size $\delta\kappa = 10^{-5}$) have been performed around $\kappa_c = 0.12887$ (with a number of measurements 1000 to 4000 per point). Data could then be grouped into "heat baths" according to paths in the $\kappa_1 - \kappa_2$-plane and processed by the multihistogram technique to give smooth interpolations, for example the average action per link

$$E^1_{\kappa_1,\kappa_2} = \langle \sum_{l\in 1} \frac{1}{2} Tr(\Phi_x U_{x,\mu}\Phi_{x+\mu})/4|\Lambda^1|\rangle(\kappa_1,\kappa_2) \quad (4)$$

refering to the subsystem 1 in the heat bath κ_2 as function of κ_1. According to Ref.[6] the main contribution to the interface tension should be given by the integral

$$\alpha/T_c^3 = 2N_z N_t^3 \int_{\kappa_1}^{\kappa_2} (E^1_{\kappa,\kappa_2} - E^1_{\kappa,\kappa_1})d\kappa, \quad (5)$$

in our case over the multihistogram interpolated curves. The delicate task, however, is to perform the limit $\kappa_c - \kappa_1, \kappa_2 - \kappa_c \to 0$, but keeping

$$\kappa_1 < \kappa_c(\kappa_2) < \kappa_c(\kappa_1) < \kappa_2, \quad (6)$$

away from the critical κ's of the subsystems in the presence of a heat bath at another κ. This condition could be fulfilled only for unsymmetric

κ-spacing, $|\kappa_c - \kappa_1|/|\kappa_2 - \kappa_c| = 2$, which takes the unsymmetric character of the phase transition into account (*i.e.* link susceptibility much higher in the broken phase). Actually our lattice was too small to allow for a reasonable limit not consistent with $\alpha = 0$, even for unsymmetric (κ_1, κ_2) with respect to κ_c.

Uncontrolled contributions to the actual α as a free energy difference are hidden in the paths approaching $\kappa_1 = \kappa_2 = \kappa_c$ along the homogeneous-phase and the mixed-phase paths, respectively. The multihistogram technique can implicitely evaluate integrals along arbitrary curves in coupling space by estimating free energy differences[7]. We have employed this idea along the piecewise straight paths $(\kappa_c, \kappa_c) - (\kappa_1, \kappa_1) - (\kappa_2, \kappa_1) - (\kappa_c + \epsilon, \kappa_c + \epsilon)$. For the preferable unsymmetric case $(\kappa_1 = 0.12881, \kappa_2 = 0.12890)$ this procedure gives an upper estimate

$$\alpha/T_c^3 = \frac{\Delta f}{2}(\frac{N_t}{N_x})^2 = 4.4 \times 10^{-3} \qquad (7)$$

(which is twice as large as for the symmetric case $(\kappa_1 = 0.12883, \kappa_2 = 0.12891)$). This result refers to a mixed-phase point at $(0.12888, 0.12885)$. It should be mentioned that the Monte Carlo configurations along the mixed-phase part of the integration contour must be monitored to make sure that both subsystems are in the appropriate phases. At medium λ huge lattices are necessary.

4. LATENT HEAT

The part of data for $\kappa_1 = \kappa_2$ has been analysed to give an estimate of the latent heat as well. We have looked for the discontinuity of the interaction strength $\delta = \frac{\epsilon}{3} - p$. Due to the continuity of pressure p the jump of this quatity gives access to the latent heat per unit volume $\Delta\epsilon$ [8],

$$\Delta\epsilon/T_c^4 = N_t^4(\frac{\partial\kappa}{\partial\tau}8\Delta\langle E_l\rangle - \frac{\partial\lambda}{\partial\tau}\Delta\langle(\rho^2-1)^2\rangle - \frac{\partial\beta}{\partial\tau}6\Delta\langle P\rangle) \qquad (8)$$

with $P = TrU_p/2$ and $\tau = -log(aM)$. Using the one-loop RG equations for the derivatives of bare couplings along lines of constant physics [8] and

$$-\frac{\partial\kappa}{\partial\tau} = \frac{1}{N_t}\frac{\partial\kappa_c}{\partial(1/N_t)}, \qquad (9)$$

which is 0.008(2) at medium λ[2] and 0.0011(2) at small λ[8]. We have used both theoretical histograms at $\kappa_c = 0.12887$ at small λ. obtained by multihistogram extrapolation from pure phase data away from κ_c and actual data from the Monte Carlo at κ_c for the discontinuities in eq. (8).

For this purpose, the actual data need blocking over 5..15 subsequent configurations. But then remains a systematic difference between the corresponding estimates for the latent heat at this λ: $\Delta\epsilon/T_c^4 = 0.103(10)$ for the extrapolated histograms, $\Delta\epsilon/T_c^4 = 0.087(8)$ for the blocked actual data at κ_c.

5. CONCLUSIONS

We have investigated two values of the Higgs mass $m_H \leq m_W$ on modest lattices.

Compared with the results of the DESY group our results show that lattices of correlation length size can characterize the strength of the transition in the right ballpark of parameters.

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