SUMMARY REPORT - QUANTUM FLUCTUATIONS IN BEAM DYNAMICS

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The main topics and questions dicussed in working group A, "Quantum Fluctuations in Beam Dynamics", are summarized. Instead of going into details, only the underlying ideas and concepts are illustrated.

1 Introduction

The optimal performance of colliders and synchrotron light sources requires a good understanding of beam dynamics. A beam constitutes an ultrarelativistic ($v = c$) ensemble of charged particles with spin distributed in m bunches under the influence of

- external electromagnetic fields such as dipoles, multipoles and rf fields
- space charge, wakefields, and fields of the counter-rotating beam in colliders
- radiation
- restgas, and clouds of ions, electrons or positrons
- noise due to rf, power supplies, ground motion, intra-beam scattering etc.

Altogether it is a complicated nonlinear, explicitly stochastic many-particle system.

Questions of interest are

- what is the long-term behaviour ($\sim 10^9$ turns)?
- what is the kinetics of (statistically averaged) macroscopic quantities such as particle density or polarization?
- is there relaxation to an equilibrium?
- what is the beam response to perturbations?

The classical treatment of these problems is in the framework of statistical physics (dynamics) based on the Liouville-Vlasov-Fokker-Planck equation.[1,2]

The aim and main question of working group A was: where is $\hbar$? Sources of quantum manifestations on a macroscale are well known in electron storage rings namely radiation and spin. For example, a correct description of the beam emittances has to take into account recoil effects due to the quantum-like emission of the synchrotron light, and the building up of transverse beam polarization in electron storage rings via spin-flip synchrotron radiation is a purely quantum mechanical effect.[3] Further important questions are

- when is it necessary to perform quantum calculations (*i.e.* to formulate conditions for a quantum treatment of beam dynamics)?
- are there new quantum phenomena (suppression of orbit echoes, spin echoes)?
- what are the quantum foundations of the classical treatment of radiation in electron storage rings?
- can quantum tools such as supersymmetric quantum mechanics, path integrals and Feynman diagrams also be helpful for classical beam dynamics calculations (stochastic dynamics)?

The following topics were treated in working group A - partly in joint sessions with other working groups.

1. laser cooling and optical stochastic cooling[4,5,6]
2. quantum corrections to the electron equation of motion[7]
3. quantum effects in beam echoes[8]
4. stochastic theory of moving atoms-quantum field interaction[9]
5. path integral derivation of the Langevin-Abraham-Lorentz-Dirac equation for relativistic particle motion in quantum fields[10,11]
6. quantum fluctuations in free-electron lasers[12]
7. quantum limits (beam spot size, emittance)[13,14]
8. stochastic beam dynamics and Vlasov-Fokker-Planck description of particle motion in accelerators[15,16,17]

In the following I will describe four topics in more detail. I will only illustrate the main ideas and concepts. For the -sometimes- very sophisticated technical details the reader is referred to the original contributions to these proceedings. Thus, this summary should be considered merely as an appetizer for the quantum problems related to beam dynamics in accelerators.

2 Quantum foundations of the classical treatment of radiation in electron storage rings

The problem of radiative backreaction on the dynamics of charged particles and its selfconsistent treatment has a long history in classical electrodynamics - starting with the work of Abraham, Lorentz, Poincaré, Sommerfeld, von Laue, and culminating in Dirac's classical theory of the electron, which in relativistic covariant notation takes the form

$$ma^{\mu} = F^{\mu}_{ext} + \Gamma^{\mu} \tag{1}$$

with

$$\Gamma^{\mu} = \frac{2}{3}\frac{e^2}{c^3}(\dot{a}^{\mu} - \frac{1}{c^2}a^{\lambda}a_{\lambda}v^{\mu}) \tag{2}$$

(F^{μ}_{ext} external field term). For more information on the historical background and for a detailed discussion of the implications of Γ^{μ} and the problems related with $\dot{a}^{\mu}$ see for example Jackson [18], Erber [19], Kim [7] and Rohrlich. [20] As mentioned already, the classical theory is not sufficient to calculate the beam emittances of electrons in storage rings. Besides the damping effects due to radiation, recoil effects due to the stochastic emission of photons have to be taken into account, thus making the motion explicitly stochastic similar to a Brownian particle subject to a noisy environment. [21] [22] The calculation of the beam parameters such as emittances, beam sizes (*i.e.* average fluctuations of the particle around the closed orbit) and lifetimes usually start from the stochastically and dissipatively perturbed Hamiltonian of the coupled synchro-betatron motion [23] [24] or, similarly, from the stochastically perturbed Lorentz-Dirac equation [25] [26]

$$ma^{\mu} = F^{\mu}_{ext} + \Gamma^{\mu} + \eta^{\mu} \tag{3}$$

where η^{μ} is a noise term simulating the stochastic recoil effects.

An interesting and important question is: what are the limits of validity of this classical approach and what are the quantum foundations?

Using the concept of an open quantum system and techniques developed by Schwinger [27], Feynman and Vernon [28] a stochastic theory of relativistic particles moving in a quantum field was presented. The results of these investigations can be summarized as follows: it is a first principle derivation of the Abraham-Lorentz-Dirac equation as the consistent semiclassical limit for nonlinear particle-field systems. It offers a consistent resolution of the paradoxes of the Abraham-Lorentz-Dirac equation including the problems of runaway and acausal solutions and it shows the -generally- non-Markovian nature of the quantum particle open system. The technical and very subtle details can be found in the contributions of Hu and Johnson elsewhere in these proceedings.

Let me only illustrate in more detail the underlying ideas, namely the open quantum system concept. The main problem and question is: how can noise and dissipation arise in a quantum framework? Consider for example a quantum harmonic oscillator coupled linearly to a heat bath of infinitely many harmonic oscillators. The total system is described by the Hamiltonian (zero point energies neglected)

$$H = \omega a^\dagger a + \sum_j \omega_j a_j^\dagger a_j + \sum_j g_j a_j^\dagger (a^\dagger + a) + h.c. \qquad (4)$$

with the usual commutation relations for the operators $a, a^\dagger, a_j, a_j^\dagger$

$$[a, a^\dagger] = 1 \qquad [a_i, a_j^\dagger] = \delta_{ij}. \qquad (5)$$

We are only interested in the oscillator subsystem dynamics and not in the bath . The density matrix describing this subsystem is given by

$$\rho_s(t) = Tr_R(\rho_{R+s}(t)) \qquad (6)$$

where the summation (trace) is over the heat bath degrees of freedom of the density matrix of the total system. It can be shown [29], that the corresponding Wigner function (*i.e.* the quantum analogue of the classical phase space distribution), which describes the quantum subsystem, obeys a Fokker-Planck-type equation equivalent to a classical damped harmonic oscillator driven by white noise $f(t)$

$$m\ddot{q} + \alpha\dot{q} + \omega^2 q = f(t). \qquad (7)$$

These heat bath models have been studied extensively in quantum optics and statistical physics. [30] [31] [32] For example, (two-level) spin systems coupled to a heat bath have been used to explain and to study complicated relaxation phenomena in NMR. [33]

The results presented in the working group were generalizations of the above mentioned simple systems to relativistic particles moving in quantum fields. Coarse graining and averaging over the quantum fields then led to the Abraham-Lorentz-Dirac-Langevin-like equation for the particle motion.

3 Quantum calculations

During the working group sessions quantum calculations were presented for free-electron lasers, for optical stochastic cooling and beam echoes.

The free-electron laser mechanism, *i.e.* the generation of coherent radiation, is based on a subtle interaction and synchronization of an electron beam with a co-propagating radiation field in an undulator, thus transfering energy from the beam to the radiation. [34] Although the first investigations of the free-electron laser principle were done in a quantum mechanical framework [35], it is well known, that the gain (*i.e.* the energy transfer), the beam properties (microbunching) and the radiation characteristics can be calculated classically within the Vlasov-Maxwell theory. [36] Quantum mechanical effects in free-electron lasers can be neglected when the spreading of the electron wave packet is less than one wave period of the radiation over the length of the wiggler (see for example Freund and Antonsen [37]) *i.e.*

$$\frac{\lambda_c L}{\gamma_0 \lambda_w} \ll \lambda \tag{8}$$

with λ_c Compton wavelength , L wiggler length, λ_w wiggler period, γ_0 relativistic factor of the electrons and λ radiation wavelength. Stated in a different way, quantum effects can be neglected if the recoil due to radiation is unimportant, a condition which is usually fulfilled in most of the existing and planned devices.

However, from first principle considerations and for a better understanding of the start-up process of a free-electron laser, especially in the self amplified spontaneous emission (SASE) mode, and for an understanding of the photon statistics it is also desirable to have a general quantum theory of these devices.

Using anN_e-electron, $M-$mode quantized Hamilton operator for the free-electron laser process in a frame moving with the mean velocity of the electron beam (for details see Schroeder [12])

$$\begin{aligned} H = & \sum_{\lambda=1}^{M} \hbar\omega_\lambda (a_\lambda^\dagger a_\lambda + \frac{1}{2}) + \sum_{j=1}^{N_e} \hbar\Omega \frac{1}{2} \bar{p}_j^2 + \\ & + \sum_{\lambda=1}^{M} \hbar g_\lambda (a_\lambda^\dagger a_u \sum_{j=1}^{N_e} \exp(-i\Theta_{\lambda j}) + h.c.) \end{aligned} \tag{9}$$

(where the undulator is treated classically) the SASE mode, the start-up process due to electron beam fluctuations and the photon statistics has been discussed in detail.

For further discussions of quantum effects in free-electron lasers see also the review article by Dattoli and Renieri [38] and the contributions of Pellegrini and Kim in the first Conference on Quantum Aspects of Beam Physics [39].

In another contribution Chao [8] has described a project about possible quantum mechanical effects on beam echoes. Echoes in particle beams are based on the sensitive link between macroscopically measurable quantities and the microscopic phase space dynamics of the particles.[40] Measurements have been performed at various machines [41 42 43] and because of the above mentioned sensitive connection between macroscopic quantities and microscopic dynamics these measurements can provide valuable insight into various diffusion mechanisms in a beam. The question under study is: can quantum mechanics for example suppress the echo effect?

Optical stochastic cooling was another topic discussed extensively. This kind of cooling could be very important for future muon collider projects. In optical stochastic cooling the spontaneous radiation from pick-up undulators is amplified and fed back into the same bunch inside the kicker undulator with an appropriate phase delay. Heifets [6] and Charman [5] have presented fully quantum mechanical treatments of the beam-undulator and optical amplifier system.

In further contributions and discussions it was shown how concepts and tools from condensed matter physics such as Fermi liquid theory, quasiparticles, Wigner crystals and Coulomb chains can be used to get a better understanding of the quantum limits of beam emittances and crystallization phenomena in accelerators. [13]

4 Quantum tools

Quantum physics tools such as supersymmetric quantum mechanics, path integrals, Feynman diagrams etc. can also be used in classical beam dynamics calculations. As an example one can consider the influence of noise on the particle motion. Mathematically, noisy systems can be described by stochastic differential equations of the form ($\mu = 1, ...n$ and $\nu = 1, ...m$)

$$\dot{x}_\mu = f_\mu(x_1, ..x_n, t) + \sum_{\nu=1}^{m} g_{\mu,\nu}(x_1, ...x_n, t)\dot{W}_\nu \tag{10}$$

where $\dot{W}_\nu$ designates a Gaussian white noise vector process. For these Markovian diffusion processes there exists a fully developed mathematical theory.[44 45 46] An equivalent probabilistic description is via the Fokker-Planck equation for $p(x_1, ...x_n, t)$ or the transition (or conditional) probability density

$p(x_1, ...x_n, t|x_{1_0}, ...x_{n_0}, t_0)$. The corresponding Fokker-Planck equation is given by (Ito interpretation)

$$\frac{\partial p}{\partial t} = \hat{L}_{FP} p \tag{11}$$

with the Fokker-Planck-operator ($\underline{g} = (g_{\mu\nu}), \underline{g}^T$ transpose of $\underline{g}$)

$$\hat{L}_{FP} = -\sum_{\mu=1}^{n} \partial_\mu f_\mu + \frac{1}{2} \sum_{\mu=1,\nu=1}^{n} \partial_\mu \partial_\nu (\underline{g}\underline{g}^T)_{\mu,\nu}. \tag{12}$$

The Fokker-Planck equation for $p(\vec{x}, t)(\vec{x} = (x_1, ..x_n))$ is a linear partial differential equation, which describes the probability to find the stochastic system at time t between the phase space points $\vec{x}$ and $\vec{x} + d\vec{x}$.

A typical physical example is the horizontal motion of an electron in a collider under the influence of the nonlinear field of a counter-rotating beam (positron or proton beam) [17]

$$\ddot{x} + \alpha \dot{x} + V(x, t) = h(t) + \sigma \dot{W} \tag{13}$$

(α radiation damping, $V(x, t)$ focusing and nonlinear beam-beam force, $h(t)$ external (perturbing) field, $\sigma \dot{W}$ noise simulating quantum fluctuations). Questions of interest are: $p(x, \dot{x}, t)$, response of the dipole moment $e < x(t) >= e \int p(x, \dot{x}, t) x dx d\dot{x}$ to the external perturbation $h(t)$, lifetime τ, higher order moments, correlations etc.

From the theory of stochastic processes it is well known, that the propagator (or transition probability density) $p(\vec{x}, t|\vec{x_0}, t_0)$ with

$$p(\vec{x}, t) = \int p(\vec{x}, t|\vec{x_0}, t_0) p(\vec{x_0}, t_0) d\vec{x_0} \tag{14}$$

can be represented as a path integral (using the Chapman-Kolmogorov equation). For example, for a (scalar) stochastic differential equation of the form

$$\dot{x} = a(x) + \sqrt{D}\dot{W} \tag{15}$$

one obtains [47]

$$p(x', t'|x, t) = \int \mathcal{D}x(t) \exp\{-S[x]\} \tag{16}$$

with

$$S[x] = \frac{1}{2D} \int_t^{t'} d\hat{t} (\dot{x} - a(x))^2 \tag{17}$$

and

$$\mathcal{D}x(t) = \lim_{N\to\infty} \prod_{n=1}^{N-1} dx(t_n)(\frac{1}{2\pi\Delta t})^{\frac{N}{2}}. \tag{18}$$

Evaluation of these path integrals with the use of field theoretical tools [47] can help to get a better understanding of the complicated classical stochastic beam dynamics problems.

Analytical solutions of the Fokker-Planck equation are very rare. Usually one has to apply perturbative methods or numerical tools. For numerical checks of the underlying integration scheme and code it is however very helpful to have non-trivial exact solutions. For low-dimensional (1+1) problems supersymmetric quantum mechanics offers a way to construct exact solutions (see for example Risken [48] and the applications discussed by Bernstein and Brown [49] and Chen [50]).

5 Classical Vlasov-Fokker-Planck description of beam dynamics

An interesting new method for a stable long-term integration of the Vlasov-Fokker-Planck equation was presented by Warnock [15] and applied to the longitudinal motion of electrons in the SLAC damping ring. Within the classical approach one has to solve

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial q}\frac{\partial H}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial H}{\partial q} = 2\alpha\frac{\partial}{\partial p}(pf) + D\frac{\partial^2 f}{\partial p^2} \tag{19}$$

for the classical phase space distribution function $f(q,p,t)$. The left-hand side is the Vlasov-part and the right-hand side, the Fokker-Planck-part, is due to damping (α) and the quantum fluctuations. H is given by

$$H(q,p,t) = H_e(q,p,t) + H_{coll}(q,p,t,\{f\}) \tag{20}$$

where H_e describes the external fields and H_{coll} is the collective part depending on f. The time-dependent solution of

$$\frac{\partial f}{\partial t} = L_V f + L_{FP} f \tag{21}$$

is based on an operator splitting scheme

$$L_V \to L_{FP} \to L_V \to L_{FP} \to .... \tag{22}$$

For L_V (the Vlasov part) the method of local characteristics is used, *i.e.* one evaluates the Frobenius-Perron operator (time evolution operator of the Liouville equation) for small time steps Δt by treating the collective force as time-independent over Δt, and for L_{FP} (the Fokker-Planck part) elementary partial differential equation methods [16 51] can be used. This very efficient and stable scheme has been used to study the sawtooth instability and the beam-beam interaction in electron colliders.

Conclusions and acknowledgments

In this short summary I have tried to illustrate some of the questions, ideas and concepts which have been discussed during the workshop in working group A: "Quantum Fluctuations in Beam Dynamics". Because of lack of space I could not cover all presentations in full detail, and I want to apologize to those participants whose contributions were not appropriately mentioned in this summary. Like the first workshop in Monterey, this second meeting on "Quantum Aspects of Beam Physics" gave the opportunity to discuss interesting and challenging problems in beam physics of accelerators. The interdisciplinary character and the lively and clear presentations of the participants made this a very exciting conference. The author wants to thank all the colleagues, who have contributed to the work of our working group for their efforts. Special thanks, however, go to Pisin Chen and Stefania Petracca for their great work preparing this workshop, and the very efficient organization, which, altogether, made this a very enjoyable meeting.

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