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Study of the frequency multiplication process in a multistage HGHG FEL

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Abstract

A new design for a multistage High-Gain Harmonic Generation (HGHG) scheme is proposed. The main difference with previous HGHG schemes is that in our scheme the HGHG technique can be applied more than once in a HGHG chain (single bunch scheme). This is consequence of the fact that the growth of the energy spread due to the HGHG process in our case is much less than initial energy spread, and exponential growth rate in the main undulator is practically the same as without stage sequence. Problems relating to X-ray HGHG FEL are discussed. Our studies have shown that the frequency multiplication process produces a noise degradation proportional to at least the square of the multiplication ratio. This prevents operation of HGHG FEL at a very short wavelength range. The results presented in this paper have demonstrated that the HGHG FEL approach is quite adequate for the VUV coherent source, but not scalable to X-ray devices. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The improvement of the longitudinal coherence of the X-ray SASE FEL is of great practical importance. At present two ways to overcome this problem are under development. One of them is based on an idea to use a self-seeding scheme [1,2]. Another approach to produce completely coherent radiation consists in utilizing a high-gain harmonic generation (HGHG) FEL scheme. In the HGHG FEL the radiation output is derived from a coherent subharmonic seed pulse. Consequently, the optical properties of the HGHG FEL are expected to be a map of the characteristics of the high-quality seed laser. This has the benefit of providing radiation with a high degree of stability and control of the central wavelength, bandwidth energy and pulse duration which is absent from the SASE FEL source [3–7]. An idea of using two undulators, with the second undulator resonant to

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one of the harmonics of the first one, has been proposed in Ref. [3]. The next step in this direction was taken in Ref. [4], where dispersion section is introduced between the two undulators. Recently, an approach utilizing a HGHG scheme, which is capable of producing longitudinally coherent pulses, was demonstrated experimentally [8,9].

To generate short wavelengths we have to go to a high harmonic number. In order to obtain an efficient coherent harmonic generation, the energy modulation δE, introduced by the modulator, needs to be larger than the initial energy spread by the factor of harmonic number N. Therefore, for very high harmonics, the energy modulation becomes very large and this makes the exponential growth gain length too large. This problem can be solved with a multistage HGHG scheme [10]. A simple solution to solve the energy spread problem, which has been proposed in Ref. [10], is to use fresh electron bunches in each stage. Once the coherent harmonic generation process in the first stage is over, the resulting radiation at frequency 4ω is guided to the next amplifier/radiator unit for interaction with a fresh electron bunch. Following the second stage the radiation at frequency 16ω enters the third stage. Like the second stage, the third stage makes use of fresh electron bunches, etc. The results of these studies are considered very promising because they indicate that the HGHG FEL technique could allow the production of fully coherent X-rays [10].

In this paper, we propose a single-bunch, multistage HGHG FEL scheme (see Fig. 1). In contrast to usual the HGHG approach [4], in our scheme the density modulation at the fundamental frequency at the exit of the dispersion section is about 10% only. This is achieved at a relatively small energy modulation which is much less than the natural (local) energy spread in the electron beam. This trick reveals an opportunity to use single bunch in all stages of the HGHG scheme. Attractive feature of multistage HGHG technique can be an essential pulse shortening mechanism. Successive multiplication to the Nth harmonic resulting in $\sqrt{N}$-fold compression of the Nth harmonic pulse duration. It is shown in this paper that at $N = 32$ we can obtain 80 fs pulse at the wavelength of 8 nm.

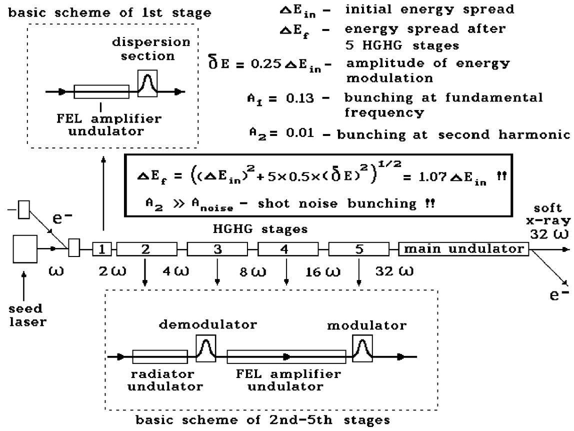

Fig. 1. Single bunch HGHG FEL scheme proposed in this paper.

Up to now operation of HGHG schemes has been analyzed within the framework of idealized models which do not take into account shot noise effects [4–7]. The results of these studies were very promising and allowed the authors to make the conclusion that HGHG FEL technique would allow to reach the X-ray wavelength range starting from visible light. In this paper we take into account shot noise in the electron beam. It has been found that a general disadvantage of HGHG FEL schemes (as well as any frequency multiplication scheme) is due to strong noise degradation of the properties of output radiation with increasing harmonic number N. In the case of HGHG FEL this means that the effect of frequency multiplication by a factor of N results in multiplication of the ratio of noise power to carrier by a factor of N^2. This prevents successful operation of HGHG FEL at very short wavelengths. On the basis of our study we can make the definite conclusion that the applicability region of the HGHG scheme is significantly narrower than claimed before. The results presented in this paper demonstrate that the HGHG scheme is quite adequate for the 10–100 nm coherent light source, but cannot be used to produce hard X-rays. It is explicitly shown that noise degradation prevents operation HGHG multistage scheme at Angstrom wavelength range.

2. Operation of a single-bunch subharmonically seeded multistage HGHG FEL

General scheme of a single-bunch, multistage HGHG FEL scheme is shown in Fig. 1. Each of HGHG stages (except of the first) consists of a radiator undulator, first dispersion section (demodulator), FEL amplifier undulator and end-stage dispersion section (modulator) as it is illustrated in Fig. 2 (Table 1). The first stage is a conventional FEL amplifier seeded by an external laser. The undulator is followed by a dispersion section to increase spatial bunching. Optimal parameters of the dispersion section can be calculated in the following way. The phase space distribution of the particles in the first FEL amplifier is described in terms of the distribution function $f(P,\psi)$ written in "energy-phase" variables $P = E - E_0$ and $\psi = k_w z + \omega_0(z/c - t)$, where E_0 is the nominal energy of the particle, $k_w = 2\pi/\lambda_w$ is the undulator wave number, and ω_0 is the frequency of the seed radiation. Before entering the first undulator, the electron distribution is assumed to be Gaussian in energy and uniform in phase ψ. The present study assumes the density modulation at the end of first undulator to be very small compared to the desired value (10%) and there is an energy modulation $P_0 \sin\psi$ only. After passing through the dispersion section with dispersion strength $\mathrm{d}\psi/\mathrm{d}P$, the electrons of phase ψ and energy deviation P will come to a new phase $\psi + P\,\mathrm{d}\psi/\mathrm{d}P$. The integration of "energy-phase" distribution over energy provides the beam density distribution, and the Fourier expansion of this function gives the harmonic components of the density modulation converted from the energy modulation [4]

$$\int_{-\infty}^{\infty} f(P,\psi)\,\mathrm{d}P = 1 + 2\sum_{n=1}^{\infty} \exp\left[-\frac{1}{2}n^2\langle(\Delta E)^2\rangle\left(\frac{\mathrm{d}\psi}{\mathrm{d}P}\right)^2\right] \times J_n\left(nP_0\frac{\mathrm{d}\psi}{\mathrm{d}P}\right)\cos(n\psi). \quad (1)$$

The Bessel function factor represents the microbunching. If its argument is much smaller than unity, the microbunching would be reduced proportionally to its nth power. Hence, $P_0\,\mathrm{d}\psi/\mathrm{d}P$ must be comparable to a_1, where a_1 is the desirable value of first harmonic bunching factor. The first exponential factor shows that the energy spread suppresses significantly the microbunching when $\Delta E\,\mathrm{d}\psi/\mathrm{d}P \simeq a_1\sqrt{\langle(\Delta E)^2\rangle}/P_0$ is larger or equal to unity. Hence, the energy modulation can be smaller than the energy spread in order to have a small first harmonic components ($a_1 \ll 1$). Parameters in our case are: $\sqrt{\langle(\Delta E)^2\rangle} \simeq 1$ MeV, $P_0 \simeq 0.25$ MeV, $P_0\,\mathrm{d}\psi/\mathrm{d}P \simeq 0.16$. Put these parameters in (1) we find the amplitudes of the first and the second harmonic of density modulation: $a_1 \simeq 0.13$, $a_2 \simeq 0.01$.

In contrast to usual the HGHG approach [4], in our scheme the density modulation at the fundamental frequency at the exit of the dispersion section is about 10% only. Nevertheless, at the chosen parameters, the amplitude of the second harmonic of the density modulation is high enough, about 1%, and dominates significantly over the amplitude of shot noise harmonics (about 0.01%). This modulation density serves as an input signal for the second stage which is resonant to the second harmonic. An important feature of our design is that a very small energy modulation is sufficient to produce 10% microbunching in the dispersion section. In particular, the amplitude of the energy modulation δE can be much smaller than the natural (local) energy spread in the

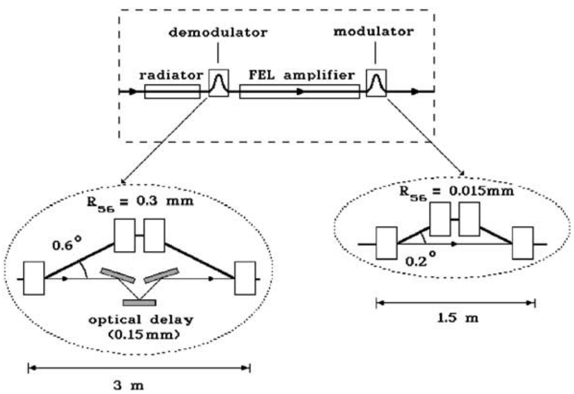

Fig. 2. Schematic illustration of design configuration for the second stage of subharmonically seeded soft X-ray FEL at TTF.

Table 1
Parameters of the magnetic system of HGHG FEL scheme

	Stage 1 256 nm	Stage 2 128 nm	Stage 3 64 nm	Stage 4 32 nm	Stage 5 16 nm	Main 8 nm
Radiator undulator						
Length of undulator, (m)		1.4	1.7	1.9	2	
Period, (cm)		5	4.5	4	3.3	
Peak field, (T)		1.1	0.94	0.75	0.65	
K-value		5.5	4	2.8	2	
Demodulator chicane						
Net momentum compaction, (mm)		0.3	0.3	0.3	0.03	
Total chicane length, (m)		3	3	3	1.5	
Length of each dipole magnet, (m)		0.25	0.25	0.25	0.25	
Bend angle of each dipole, (deg.)		0.6	0.6	0.6	0.27	
Magnetic field for each dipole, (T)		0.15	0.15	0.15	0.067	
FEL amplifier undulator						
Length of undulator, (m)	1.95	1.6	2.4	2.8	4.6	13.1
Period, (cm)	6.5	5	4.5	4	3.3	2.73
Peak field, (T)	1.1	1.1	0.94	0.75	0.65	0.5
K-value	6.8	5.5	4	2.8	2	1.26
Modulator chicane						
Net momentum compaction, (μm)	18	7.5	3.5	1.5		
Total chicane length, (m)	1.5	1.5	1.5	1.5		
Length of each dipole magnet, (m)	0.25	0.25	0.25	0.25		
Bend angle of each dipole, (deg.)	0.2	0.13	0.09	0.06		
Magnetic field for each dipole, (T)	0.05	0.033	0.023	0.015		

electron beam ΔE_{in}. The analysis of the parameters of the subharmonically seeded FEL has shown that it will operate reliably even for an energy modulation amplitude equal to $\delta E = 0.25\Delta E_{\mathrm{in}}$.

Fig. 1 illustrates how the soft X-ray wavelength range may be reached by successive multiplication ($\omega \rightarrow 2\omega \rightarrow 4\omega \rightarrow 8\omega \rightarrow 16\omega \rightarrow 32\omega$) in a stage sequence. Following the first stage (FEL amplifier undulator and dispersion section) the electron beam and the seed radiation enter a short undulator (radiator) which is resonant with the second harmonic of the seed radiation. In the radiator the seed radiation plays no role and is diffracted out of the electron beam, while the bunched beam generates radiation with frequency 2ω. At the exit of the radiator undulator the radiation power exceeds significantly the effective power of shot noise. After leaving the radiator the electron beam is guided through a dispersion section (demodulator). The function of this dispersion section consists in suppressing the density and energy modulation of the electron bunch produced in the first stage. The problem of suppressing the beam modulation can be solved quite naturally due to the presence of the local energy spread in the electron beam. After passing the dispersion section, the demodulated electron beam enters the FEL amplifier and amplifies in exponential regime the radiation with frequency 2ω produced by the radiator undulator. The length of the FEL amplifier undulator is chosen in such a way that the energy modulation at the undulator exit has the same value of $\delta E = 0.25\Delta E_{\mathrm{in}}$ as at the exit of the first stage. This energy modulation is then converted to spatial bunching while the electron beam traverses the end-stage dispersion section (modulator). The values of the 2nd and the 4th harmonics of density modulation at the second stage exit are about 10% and 1%, respectively. These values are approximately the same as the amplitudes of the first and second harmonics at first stage exit. Following the second stage the beam enters the third stage which is resonant with 4th harmonic of the seed radiation, etc. Finally, after the 5th stage the electron beam enters main undulator which is resonant to the 32nd harmonic of the seed radiation. The process of amplification in the main undulator starts from the modulation of the beam density. By the time the beam is overbunched in the main undulator, the 32ω radiation reaches saturation. In order to reach saturation, the main undulator should be sufficient long. An important feature of the proposed scheme is that the energy modulation (i.e. correlated energy spread) induced in the nth stage, transforms to local (i.e. uncorrelated) energy spread in the $(n+1)$th stage. As a result, the dispersion of the electron energy distribution at the exit of the multistage scheme is calculated as the sum of induced dispersions. The small energy perturbation of the electron beam is one of the advantages of the adopted subharmonically seeded FEL design. For instance, the total energy spread generated to the end of the 5th stage, can be estimated as $\sqrt{\langle(\Delta E)^2\rangle} \simeq \sqrt{(\Delta E_{\mathrm{in}})^2 + 5\times 0.5\times(\delta E)^2} \simeq 1.07\Delta E_{\mathrm{in}}$ for $\delta E = 0.25\Delta E_{\mathrm{in}}$ which does not differ much from the number obtained from numerical simulations (see Section 4 for more details). Such a small degradation of the energy spread allows effective generation of powerful radiation in the main undulator.

3. Estimation of essential shot noise effects

As for the HGHG FEL operating in the short wavelength range, its noise properties are defined only by the shot noise. To describe the noise output quantitatively, we should define the quality criterion of the HGHG FEL. One possible definition can be made as follows. A HGHG FEL can be characterized by a noise factor F that related the input to output signal to noise ratio

$$F = \frac{(P_{\mathrm{s}}/P_{\mathrm{n}})_{\mathrm{in}}}{(P_{\mathrm{s}}/P_{\mathrm{n}})_{\mathrm{out}}}$$

where P_{s} and P_{n} are the power of signal and noise, respectively. It is natural to describe the input signal and noise power by the radiation power of signal and noise at the first FEL amplifier exit. As a rule, the first FEL amplifier has gain of about 10–20 dB only which is insufficient for transverse mode selection. Nevertheless, for a long last (main) undulator the only fundamental TEM_{00} mode, which has maximal gain, should survive. That is why the input noise power $(P_{\mathrm{n}})_{\mathrm{in}}$ should be treated as a contribution to fundamental radiation mode only.

An intrinsic disadvantage in the short wavelength HGHG FEL is the enormous value of noise factor. This is the direct effect of the frequency multiplication. The dependence of a noise factor can be given as a function of a frequency multiplication factor N. Successive multiplication to the Nth harmonic resulting in at least N^2-fold increasing of the noise factor

$$F > N^2 \quad \text{at } \omega_0 \to n_1\omega_0 \to n_1 n_2\omega_0 \to \cdots \to N\omega_0$$

as one can expect from simple physical consideration (see below). This fundamental result is of great practical importance, because a crucial condition in HGHG FEL is that the output signal to noise ratio $(P_{\mathrm{s}}/P_{\mathrm{n}})_{\mathrm{out}}$ must be made much larger than unity, in order for the properties of the output radiation to be a map of the characteristics of the high-quality seed laser.

Simple physical consideration can lead directly to a crude approximation for the value of F. It should be noted that the method which can be applied to determine the output field perturbation is independent of a specific kind of HGHG technique. This calculation depends on the frequency multiplication factor N only. The field of amplified electromagnetic wave in the first FEL amplifier can be represented as

$$E_1 = E_{\mathrm{s}}\exp(\mathrm{i}\omega_0 t) + \sum_j u_j \exp(\mathrm{i}\omega_0 t + \Delta\omega_j t) + \mathrm{C.C.} \qquad (2)$$

where E_{s} is the amplitude of amplified seed signal. The quantities $u_j \exp(\mathrm{i}\omega_0 t + \mathrm{i}\Delta\omega_j t)$ represent the (small) signal changes due to the shot noise. Starting with a field at fundamental harmonic in the first stage (2) and omitting an inessential common factor, we find for the field in the main undulator at frequency $N\omega_0$:

$$E_{\rm N} = E_{\rm s}^N {\rm e}^{{\rm i}N\omega_0 t} + NE_{\rm s}^{N-1}{\rm e}^{{\rm i}(N-1)\omega_0 t} \times \sum_j u_j {\rm e}^{{\rm i}\omega_0 t}{\rm e}^{\Delta\omega_j t} + {\rm C.C.} \quad (3)$$

where we have assumed that at the end of each stage the amplitude of a higher harmonic density modulation is small. It is obvious, this is the situation that is encountered in any HGHG scheme. As we have seen in Section 2, the amplitude of the nth harmonic of the beam density modulation, a_n, is proportional to the nth power of the field amplitude at the fundamental frequency, i.e. $a_n \propto (E_1)^n$ at $a_n \ll 1$. Therefore, the field amplitude $E_{\rm N}$ in the main undulator is proportional to $E_{\rm N} \propto a_N \propto (E_1)^N$. When derived (3), we also required the output signal to noise ratio to be much larger than unity. These two assumptions are quite general and do not reduce significantly the practical applicability of the result obtained. However, for simplicity presented derivation is limited to the case where only first FEL amplifier have significant contribution to the noise output.

In the frame of approximations discussed above the output signal to noise ratio can be represented as

$$\frac{\langle (E_{\rm s}^N {\rm e}^{{\rm i}N\omega_0 t} + {\rm C.C.})^2 \rangle}{\langle (NE_{\rm s}^{N-1}{\rm e}^{{\rm i}(N-1)\omega_0 t} \sum_j u_j {\rm e}^{{\rm i}\omega_0 t}{\rm e}^{\Delta\omega_j t} + {\rm C.C.})^2 \rangle} = \frac{1}{N^2}\left(\frac{P_{\rm s}}{P_{\rm n}}\right)_{\rm in}$$

where symbol $\langle \cdots \rangle$ means the ensemble average over bunches. Thus, frequency multiplication by N degrades the signal/noise ratio by N^2. Here we illustrated the essential shot noise effects on the basis of a simple model. Indeed, the present derivation assumes shot noise influence in the second, third and other stages to be negligible compared to the shot noise influence in the 1st stage. Nevertheless, the problem of calculation of HGHG FEL noise factor is more complicated and there are situations where, for instance, shot noise in the second stage can provide comparable or even larger output signal perturbation than shot noise in the first stage. As a result, we can conclude only that in practical situations noise factor satisfies inequality $F > N^2$.

4. Numerical study of the harmonic generation process in a multistage HGHG FEL

In this section we illustrate operation of the HGHG FEL scheme described in the previous sections. To be specific, we consider numerical example for its possible realization at the TESLA Test Facility at DESY [11]. However, this specific example highlights all general properties of the HGHG FEL schemes.

Numerical results presented in this paper are obtained with a version of code FAST [12] upgraded for simulation of higher harmonics. This code allows one to perform three-dimensional simulations of the FEL process taking into account diffraction, space-charge, energy spread and slippage effects, and shot noise in the electron beam. In the present simulations we assumed both transverse and longitudinal profile of the electron beam to be gaussian. The simulation procedure of full HGHG scheme was described in Ref. [13].

Parameters of the electron beam are: energy 900 MeV, bunch charge 1 nC, rms bunch length 50 μm, rms energy spread 1 MeV, and rms normalized emittance 2π mm-mrad. The HGHG FEL is seeded by the laser with the wavelength of 260 nm, pulse duration of 5 ps, 50 μJ energy per pulse. Numerical results are illustrated for two cases: with and without taking into account shot noise in the electron beam. When we trace the behaviour of an idealized case (without taking into account shot noise), we obtain perfect quality radiation pulse (both in temporal and spectral domain) at the exit of HGHG FEL. However, taking into account noise effect results in a degradation of the quality of the radiation pulse from stage to stage. As we mentioned in the previous section, the HGHG FEL scheme, as well as any frequency multiplication scheme possesses significant intrinsic disadvantage. Namely, contribution of the shot noise power to the signal grows quadratically with the number of harmonic to be generated. The results of numerical simulations are in good agreement with simple physical

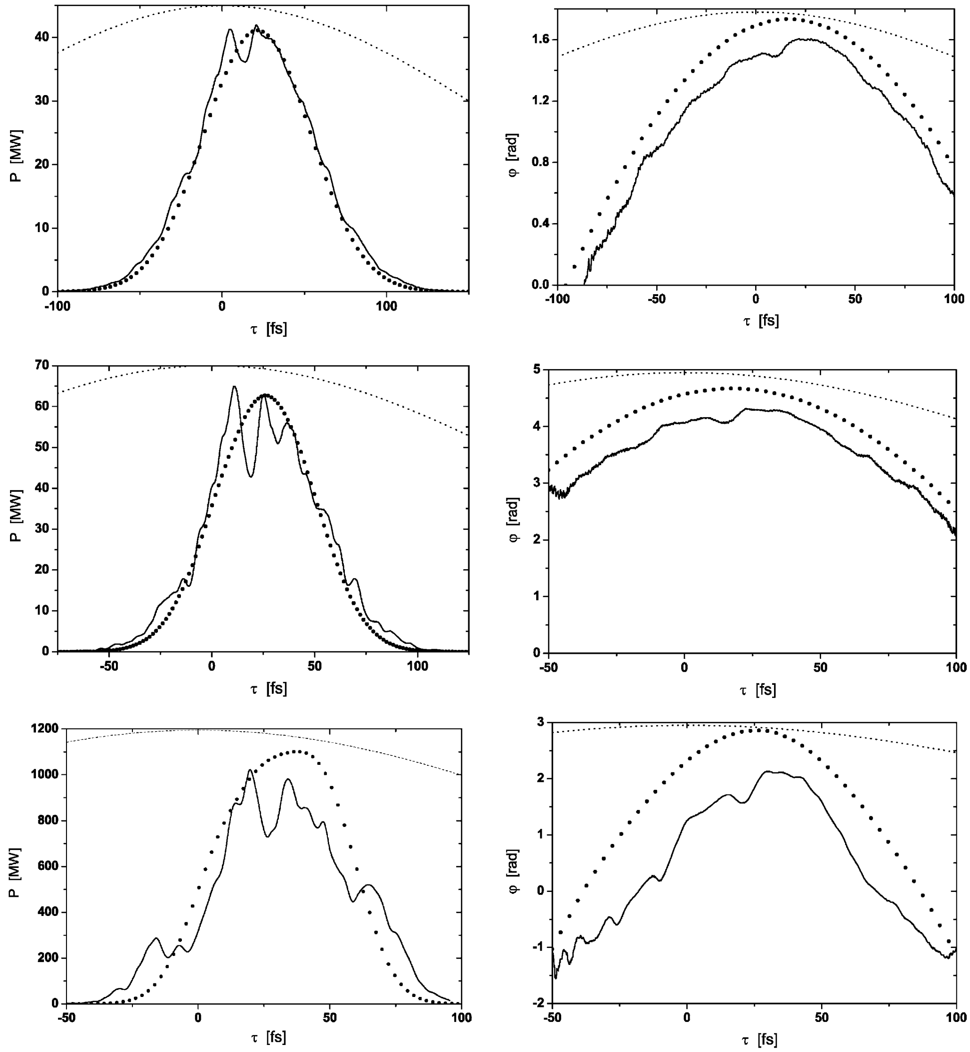

Fig. 3. Time structure of radiation power (left column) and phase of output radiation (right column) at the exit of different stages of HGHG scheme (upper, middle, and lower plots correspond to the fourth and fifth stages, and main undulator, respectively). Solid curves are calculated taking into account shot noise in the electron beam, and the circles present the results without noise effect. Dashed curves denote longitudinal profile of the electron bunch. The first stage is seeded by a long laser pulse.

estimations presented in the previous section. The growth of the noise contribution is clearly illustrated with the plots for the phase of the radiation field (see Fig. 3). Analysis of the present numerical example shows that the shot noise in the first stage gives relatively small contribution to the final value of the noise to signal ratio at the exit of the HGHG scheme. This is achieved by means of increase of the power of seed radiation. However, HGHG scheme does not allow to do this for the next stages when a small beam density modulation serves as input signal. In our case the main contribution of noise degradation comes from the second stage. The third stage also gives visible contribution to the noise.

The most critical issue of the single-bunch HGHG scheme is growth of uncorrelated energy spread. However, parameters of the stages can be optimized in such a way that this effect almost does not lead to degradation of output radiation. Calculations show that the increase of uncorrelated energy spread is 20% only (see Ref. [13] for more details).

The results presented above allows us to make conclusion that shot noise degradation does not allow to reach very short wavelengths with the HGHG FEL scheme. A limit imposed by this fundamental effect seems to be around 8 nm when HGHG procedure starts from UV seeding laser (multiplication factor of about 30). Practical limit is reached for even longer wavelength. The reason for this is that HGHG FEL scheme is extremely sensitive to fluctuations of the beam and seeding radiation parameters.

5. Discussion

In conclusion we would like to discuss some general aspects of noise influence on the HGHG FEL operation. It should be emphasized that despite the theory of HGHG FEL was developed over a decade, there are no papers devoted to the analysis of the noise properties of these sources. Here, it is relevant to remember that the analysis of the frequency multiplier chains and their effects on source noise in radar and similar systems has always been an important problem. The majority of communication and radar engineers are familiar with the fact that inserting the amplifier prior to frequency multiplication has the disadvantage that the phase noise contribution of the amplifier is multiplied by n^2, where n is a frequency multiplication factor (see, for example, Ref. [14]). In the case of HGHG FEL this means that the effect of frequency multiplication by a factor of N multiplies the first FEL amplifier noise power to carrier ratio by N^2. This prevents operation of HGHG FEL at very short wavelength range.

Let us give a more detailed discussion of problems relating to the X-ray HGHG FEL noise output. Frequency multiplication process produces a noise degradation proportional to at least the square of the multiplication ratio. As a result, HGHG FEL starting from optical wavelength range ($\lambda_{\mathrm{in}} > 2000$ Å) cannot produce coherent radiation spanning to Angstrom wavelength range. The main problem is that the contribution of the noise to the output power increases drastically when approaching the X-ray band. Consider firstly an idealized case where only the first FEL amplifier has significant contribution to the noise output. Then the signal to noise ratio after an ideal frequency multiplier chain is smaller than the signal to noise ratio at the fundamental frequency by a factor of $F \simeq N^2 \simeq 10^7$ at $\lambda_{\mathrm{out}} \simeq 1$ Å. If we want to make the ratio $(P_{\mathrm{s}}/P_{\mathrm{n}})_{\mathrm{out}}$ much larger than unity, $(P_{\mathrm{s}}/P_{\mathrm{n}})_{\mathrm{in}}$ must reach values of about 10^9. To estimate the required value of the peak power of the seed laser pulse it is convenient to introduce the notion of an effective power of shot noise P_{sh}, which is usually used for numerical simulation of the SASE FEL with steady-state codes (see for example Ref. [15]). For the visible range of the spectrum the effective shot noise power for usual SASE FEL parameters is about $P_{\mathrm{sh}} \simeq 10^2$ W. This means that successful operation of the X-ray HGHG FEL requires a seed power of about 100 GW, but this value is beyond the output power of the FEL amplifier at saturation ($P_{\mathrm{sat}} \simeq 10$ GW). In principle, in this situation a laser pulse of 100 GW power level and a first undulator with a few periods only could be used. However, this does not solve the problem of the noise degradation, because the present analysis is based on assuming an ideal frequency multiplier chain. Considering the other contributions to the noise output, it is obvious that the 100 GW level power is not attainable in the radiator undulator operating at the second (or higher) harmonic. This means that the condition $(P_{\mathrm{s}}/P_{\mathrm{n}})_{\mathrm{out}} \gg 1$ will be violated when taking into account of the second HGHG stage contribution to the noise output.

Recently, various HGHG schemes have been proposed to improve the performance of X-ray FEL. The basic theory of these schemes does not take into account the shot noise effect, meanwhile it leads to a dramatic degradation of the quality of the output radiation when applying frequency multiplication schemes. The arguments discussed above, based on our results of numerical simulations, seem to be strong enough to suggest that the HGHG FEL schemes for reaching hard X-rays proposed in the literature so far will not work.

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