Consistency in Impact Parameter Descriptions of Multiple Hard Partonic Collisions

T.C. Rogers[1]†, *A.M. Staśto*[2], *M.I. Strikman*[2]
[1]Department of Physics and Astronomy,
Vrije Universiteit Amsterdam,
NL-1081 HV Amsterdam, The Netherlands
[2]Department of Physics, Pennsylvania State University,
University Park, PA 16802

Abstract
We discuss the role of the perturbative QCD inclusive dijet cross section in describing multiple partonic collisions in high energy pp scattering. Assuming uncorrelated partons, we check for consistency between an impact parameter description of multiple hard collisions and extrapolations of the total inelastic profile function. We emphasize the availability of parameterizations to experimental data for the impact parameter dependence of hard collisions.

1 Introduction

A satisfactory description of the complex hadronic final states expected at the LHC must certainly incorporate a description of multiple partonic collisions. However, models of multiple collisions necessarily use techniques that mix perturbative and nonperturbative processes. It is therefore important to incorporate as much experimentally available input about the structure of the proton as possible. Information about the impact parameter dependence of hard collisions can be obtained from parameterizations of generalized parton distribution functions (GPDs). The gluon GPD can be measured experimentally in electroproduction of light vector mesons at small-x or in photoproduction of heavy vector mesons. Because it is a universal objects, the gluon GPD can then be used in the impact parameter description of multiple hard collisions in pp scattering. Furthermore, it is possible to make direct use of the relationship between inclusive and total cross sections to obtain consistency constraints. In this contribution, we give a summary of the steps presented in [1] for comparing a description of multiple hard scattering that utilizes GPDs with extrapolations of the total inelastic cross section. This allows us to obtain constraints on the minimum value of the lower transverse momentum cutoff in the perturbative QCD (pQCD) formula for inclusive dijet production.

2 Total Inelastic Cross Section in Impact Parameter Space

The standard way of describing the total pp cross section in impact parameter space is to use the profile function, defined in terms of the elastic amplitude $A(s,t)$ as

$$\Gamma(s,b) = \frac{1}{2is(2\pi)^2} \int d^2\mathbf{q}\, e^{i\mathbf{q}\cdot\mathbf{b}} A(s,t). \qquad (1)$$

† speaker

The optical theorem then allows the total, elastic, and inelastic cross sections to be expressed in terms of the profile function:

$$\sigma_{\rm tot}(s) = 2\int d^2\mathbf{b}\, \mathrm{Re}\,\Gamma(s,b), \tag{2}$$

$$\sigma_{\rm el}(s) = \int d^2\mathbf{b}\; |\Gamma(s,b)|^2\,, \tag{3}$$

$$\sigma_{\rm inel}(s) = \int d^2\mathbf{b}\left(2\,\mathrm{Re}\,\Gamma(s,b) - |\Gamma(s,b)|^2\right) \tag{4}$$

$$= \int d^2\mathbf{b}\,\Gamma^{\rm inel}(s,b), \tag{5}$$

The last line defines the inelastic profile function, $\Gamma^{\rm inel}(s,b)$. If the amplitude is dominantly imaginary, then unitarity requires $\Gamma, \Gamma^{\rm inel} \leq 1$.

Experimental measurements at currently accessible energies find a slow growth for the total cross section and a slow broadening of the profile function with increasing energy (see e.g. [2] and references therein). In a standard fit to the profile function of the form $\sim e^{-b^2/2B(s)}$ with $B(s) = B_0 + \alpha' \ln s$, comparisons with data then yields $\alpha' \approx 0.25$ GeV^{-2}, and a slope at LHC energies (14 TeV) of about $B \approx 21.8$ GeV^{-2}. As illustrated in [3], there are only small variations between different model extrapolation.

In the next few sections, we will address the issue of consistency between such extrapolations and descriptions of multiple hard collisions that utilize GPDs. For the purpose of illustration we will work with the model for the profile function obtained in [4].

3 Inclusive Hard Collisions in Impact Parameter Space

In most perturbative or semiperturbative treatments of multiple collisions, the basic input is the lowest order inclusive perturbative QCD (pQCD) expression for the dijet production:

$$\sigma_{\rm 2jet}^{\rm inc}(s;p_t^c) = \int_{p_t^{c\,2}}^{\infty} d\,p_t^2 \frac{d\hat{\sigma}}{dp_t^2}\, f_{i/p_1}(x_1;p_t) \otimes f_{j/p_2}(x_2;p_t). \tag{6}$$

Implicit but not shown are a sum over parton types, a K factor, and any necessary symmetry factors. The hard partonic differential cross section is for $2 \to 2$ partonic scattering between partons of type i and j. The symbol $\otimes$ represents convolutions in momentum fraction. The parton distribution functions (PDFs) are evaluated at a hard scale which for dijet production should be approximately equal to the relative transverse momentum p_t of the produced dijet pair. For pQCD to be valid, the p_t integral in Eq. (6) must be cut off from below by some scale p_t^c. Because Eq. 6) diverges at low p_t, The value of $\sigma_{\rm 2jet}^{\rm incl}(s;p_t^c)$ is quite sensitive to the precise value of this cutoff. It should be chosen large enough for perturbation theory to be safe, but small enough to incorporate the maximum possible range of kinematics.

A description of where hard collisions take place in impact parameter space can be extracted directly from experimental measurements of the gluon GPD. The GPD describes non-diagonal transitions in the target arising from the exchange of two t-channel gluons, as illustrated

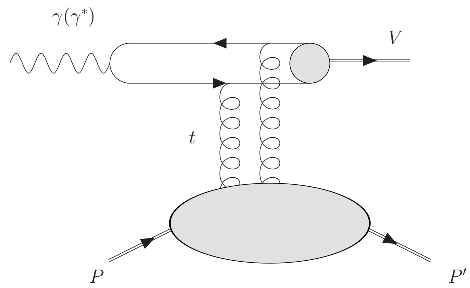

Fig. 1: The basic graphical structure in heavy vector meson photoproduction (or light vector meson small-x electroproduction) with two gluons exchanged in the t-channel. The lower bubble represents the GPD with P and P' labeling the different states that appear in the non-diagonal correlator.

in Fig. 1. It is related to the standard gluon PDF via the relation

$$xf_g(x,t;\mu) = xf_g(x;\mu)F_g(x,t;\mu) \tag{7}$$

where $F_g(x,t;\mu)$ parameterizes the t-dependence and is referred to as the *two-gluon form factor*. The GPD is evaluated at a hard scale μ, and it reduces to the standard gluon PDF at $t=0$. Fourier transforming Eq. (7) into transverse coordinate space gives the impact parameter dependent GPD,

$$\mathcal{F}_g(x,\rho;\mu) = \int d^2\mathbf{\Delta}\, F_g(x,t;\mu)\, e^{-i\Delta\cdot\rho}, \qquad t \equiv -\Delta^2. \tag{8}$$

Because the GPD in Eq. (7) is a universal object [5], it can be combined directly with Eq. (6) to yield a description of the impact parameter dependent inclusive dijet cross section in pp scattering. If we define the overlap function,

$$P_2(b,x_1,x_2;\mu) = \int d^2\rho_{\mathbf{1}}\, \mathcal{F}_g(x,|\rho_1|;\mu)\mathcal{F}_g(x,|\mathbf{b}-\rho_1|;\mu), \tag{9}$$

then the probability for a single hard collision with $\mu \approx p_t$ at impact parameter $\mathbf{b}$ is

$$\mathcal{N}_2(s,b;p_t^c) = \sigma_{\rm 2jets}^{inc}(s;p_t^c)P_2(s,b;p_t^c). \tag{10}$$

The subscript 2 refers to the production of a dijet pair. Using a dipole form to fit the two-gluon form factor, one obtains an analytic expression for the overlap function,

$$P_2(s,b;p_t^c) = \frac{m_g^2(\bar{x};p_t^c)}{12\pi}\left(\frac{m_g(\bar{x};p_t^c)b}{2}\right)^3 K_3(m_g(\bar{x};p_t^c)b). \tag{11}$$

(See [1] and [6] for more details on the above steps.) Here $x_1 \approx x_2 \approx \bar{x} = 2p_t^c/\sqrt{s}$. The parameter $m_g(\bar{x};p_t^c)$ is a mass that determines the radius of $P_2(s,b;p_t^c)$ and may depend on both the energy and on the hard scale. For $m_g(\bar{x};p_t^c)$ we will use the parameterization obtained in [6].

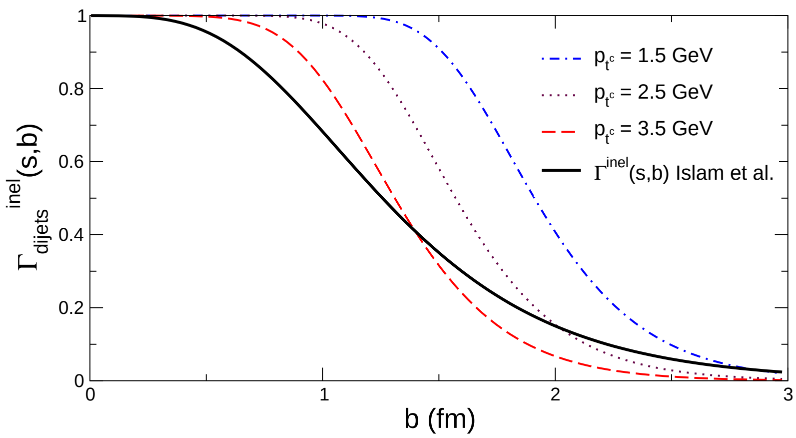

Fig. 2: The solid line shows the model extrapolation of the total inelastic profile function. The other three curves are the contributions from dijets to the total inelastic profile function obtained using Eq. (14) with the generalized parton distribution and three different values for the lower cutoff on transverse momentum.

4 Multiple Hard Collisions

For the case of uncorrelated partons, one can determine the dijet contribution to the total inelastic profile function (the non-diffractive contribution) from Eq. (10) by simply using the definition of the total inclusive inelastic cross section [7]. To see very generally how this works, we start with the exact formula obtained in [1] for the total inelastic profile function, written as a series of contributions from higher numbers of collisions:

$$\Gamma^{\rm inel}_{\rm jets}(s,b;p_t^c) = \sum_{n=1}^{\infty}(-1)^{n-1}\mathcal{N}_{2n}(s,b;p_t^c)\ . \tag{12}$$

For $n > 1$, $\mathcal{N}_{2n}(s,b;p_t^c)$ is the probability function analogous to Eq. (10) but for an n parton collision. For collisions involving identical uncorrelated partons

$$\mathcal{N}_{2n}(s,b;p_t^c) = \frac{1}{n!}\mathcal{N}_2(s,b;p_t^c)^2. \tag{13}$$

With this conjecture, Eq. (12) is a geometric series that becomes simply,

$$\Gamma^{\rm inel}_{\rm jets}(s,b;p_t^c) = 1 - \exp\left[-\mathcal{N}_2(s,b;p_t^c)\right]\ . \tag{14}$$

Hence, the assumption of uncorrelated partons results in what is typically referred to as the eikonal model. In a complete model of multiple partonic collisions, the effect of soft interactions is usually incorporated by including extra soft eikonal factors in the exponential of Eq. (14).

Consistency between extrapolations of the total inelastic profile function in Eq. (5) and Eq. (12) requires,

$$\Gamma^{\rm inel}_{\rm jets}(s,b;p_t^c) < \Gamma^{\rm inel}(s,b). \tag{15}$$

Now we can check directly whether Eq. (15) is satisfied for a particular extrapolation of the total profile function. As an example, we show in Fig. 2 the model of [4] at $\sqrt{s} = 14$ TeV. We compare this with Eq. (14) calculated using the parameterization for the two-gluon form factor taken from [6] for the b-dependence of the hard collisions. The total inclusive cross section is calculated directly from Eq. (6) using the CTEQ6M parameterizations [8] for the parton distribution functions. The calculation is shown for three sample values of p_t^c.

For very small b it is not that surprising that Eq. (15) is violated since this is the region where at very high energies the gluon density becomes large and nonlinear gluon recombination effects are expected to lead to taming of the gluon distribution. However, the plot in Fig. 2 shows that for $p_t^c \lesssim 3.5$ GeV, there is even a problem with Eq. (15) at rather large $b \sim 1.5$ fm where the uncorrelated assumption would naively be expected to be a good approximation. This implies that a rather large choice for p_t^c is needed to maintain consistency between a description of multiple hard collisions in terms of the gluon GPD and the total inelastic profile function. We note that a value of p_t^c between 3 GeV and 4 GeV is consistent with the parameter constraints reported by the Herwig++ group [9].

We note that it is certainly possible that the actual high energy total inelastic profile function is much different from current extrapolations. Whether this is true will be answered as higher energy data become available. However, as mentioned in Sect. 2 there is little variation between different extrapolations, and there would have to be a rather large deviation from general theoretical expectations in order to bring the total inelastic profile function into agreement with Eq. (15) with a small value for p_t^c. Regardless of what the true form of the high energy extrapolation profile function is, the consistency requirement of Eq. (15) should somehow be enforced.

Assuming for now that we have a roughly correct description of the total inelastic profile function for pp scattering, a violation of Eq. (15) for a given p_t^c implies a breakdown of one of the basic assumptions. Either the uncorrelated assumption of Eq. (13) is badly violated, or Eq. (10) is not an accurate description of the basic hard scattering. Hence, an improved description of the low-p_t region at large b likely requires some modeling of correlations. A general procedure for including transverse correlations has recently been proposed in [10]. An approach that goes beyond the standard pQCD description of the hard part by resumming soft gluons is suggested in [11]. A characteristic of the second method is that the width of the hard scattering overlap function becomes much narrower than what is expected from the 2-gluon form factor at high energies.

Using a narrower radius for the hard profile function ultimately allows total and inelastic cross sections to be fitted with smaller values for p_t^c (see, for example, [12]). We remark, however, that a narrower width for the hard part implies that $\mathcal{N}_2(s, b; p_t^c)$ grows large with energy very quickly at small-b. In deep inelastic scattering this would correspond to a very rapid approach to the unitarity limit. Thus, if the width of the hard part is too narrow, there is a danger that it will violate constraints from HERA data on the approach to the saturation limit. Furthermore, an extremely narrow b-distribution in the hard overlap function would correspond to a t-dependence for the 2-gluon form factor that is too weak. As an alternative approach, we suggest directly modifying the uncorrelated assumption in Eq. (13).

5 Conclusion

We have illustrated that, by describing the hard profile function in multiple collisions using parameterizations of the GPD and requiring consistency with model extrapolations of the total inelastic profile functions, we may obtain constraints on the allowed minimum transverse momentum cutoff p_t^c in the inclusive hard scattering cross section. For the case of uncorrelated hard collisions, we find that a rather large value for p_t^c is needed.

Acknowledgments

We thank R. Godbole and G. Pancheri for useful discussions. This work was partly supported by the U.S. D.O.E. under grant number DE-FGO2-93ER-40771. T. Rogers was partly supported by the research program of the "Stichting vorr Fundamenteel Onderzoek der Materie (FOM)", which is financially supported by the "Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO)". Fig. 1 was made using Jaxdraw [13].

References

[1] T. C. Rogers, A. M. Stasto, and M. I. Strikman, Phys. Rev. **D77**, 114009 (2008).

[2] M. M. Block, F. Halzen, and B. Margolis, Phys. Rev. **D45**, 839 (1992);
M. M. Block, F. Halzen, and T. Stanev, Phys. Rev. **D62**, 077501 (2000);
V. A. Khoze, A. D. Martin, and M. G. Ryskin, Eur. Phys. J. **C18**, 167 (2000).

[3] L. Frankfurt, C. E. Hyde, M. Strikman, and C. Weiss, Phys. Rev. **D75**, 054009 (2007).

[4] M. M. Islam, R. J. Luddy, and A. V. Prokudin, Mod. Phys. Lett. **A18**, 743 (2003).

[5] J. C. Collins, L. Frankfurt, and M. Strikman, Phys. Rev. **D56**, 2982 (1997).

[6] L. Frankfurt, M. Strikman, and C. Weiss, Phys. Rev. **D69**, 114010 (2004).

[7] L. Ametller and D. Treleani, Int. J. Mod. Phys. **A3**, 521 (1988).

[8] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. M. Nadolsky, and W. K. Tung, JHEP **0207**, 012 (2002).

[9] M. Bahr, S. Gieseke, and M. H. Seymour, arXiv:0809.2669 [hep-ph].

[10] G. Calucci and D. Treleani, arXiv:0809.4217 [hep-ph].

[11] A. Grau, G. Pancheri, and Y. Srivastava, Phys. Rev. **D60**, 114020 (1999);
R. M. Godbole, A. Grau, G. Pancheri, and Y. Srivastava, Phys. Rev. **D72**, 076001 (2005);
A. Achilli, R. Hedge, R. M. Godbole, A. Grau, G. Pancheri, and Y. Srivastava, Phys. Lett. **B659**, 137 (2008).

[12] R. Engel, T. K. Gaisser, and T. Stanev, *Extrapolation of hadron production models to ultra-high energy*, in *Procedings of the 27th International Cosmic Ray Conference (ICRC 2001), Hamburg, Germany, 7-15 Aug 2001*.

[13] D. Binosi and L. Theussl, Comput. Phys. Commun. **161**, 76 (2004);
C. K. D. Binosi, J. Collins and L. Theussl, arXiv:0811.4113 [hep-ph].