Let's begin by recalling how the Standard Model looked before Grand Unification was proposed.

Especially, what were the fermions?
THE LEFT-HANDED FERMIONS OF ONE GENERATION ARE

\[
\begin{pmatrix}
    \nu \\
    e^-
\end{pmatrix}_{-1}, \quad
\begin{pmatrix}
    u \\
    d
\end{pmatrix}_{1/3}, \quad
\bar{u}_{-4/3}, \quad \bar{d}_{2/3}, \quad e^+_{1/2}
\]

MANY BITS AND PIECES

- STRANGE FRACTIONS
SO IT WAS AN ILLUMINATION IN 1973 WHEN GEORGEL-GLAAS UNVEILED THE UNIFIED SU(5) MODEL, REDUCING A STANDARD MODEL GENERATION TO TWO PIECES

\[
\begin{pmatrix}
\bar{d} \\
\bar{d} \\
\bar{d} \\
\bar{d} \\
\bar{d}
\end{pmatrix}
\oplus
\begin{pmatrix}
\bar{u} \\
\bar{u} \\
\bar{u} \\
\bar{u} \\
\bar{u}
\end{pmatrix}
\]

AND EXPLAINING THE FRACTIONS.
To achieve this, it was necessary to put quarks and leptons (and their antiparticles) in the same multiplet ... and (as had been anticipated by Pati and Salam) this led to a prediction of proton decay.
In fact the $SU(5)$ gauge bosons make a matrix

$$
\begin{pmatrix}
\text{QCD} & x, y \\
\text{Gluons} & X, Y \\
X, Y & W, Z
\end{pmatrix}
$$

And the $x, y$ bosons mediate processes such as

$$p \rightarrow e^+ \pi^0$$
THERE ARE A COUPLE OF IMMEDIATE PROBLEMS

* IF THE STRONG INTERACTIONS ARE UNIFIED WITH THE WEAK AND ELECTROMAGNETIC INTERACTIONS, WHY ARE THEY SO MUCH STRONGER?

** WILL THE PROTON BE SUFFICIENTLY LONG-LIVED?
These issues were addressed in 1974 by Georgi, Quinn, and Weinberg, who calculated the "renormalization group running of the strong, weak, and electromagnetic couplings".
They found that unification was possible, provided the weak mixing angle had the right value $\sin^2 \theta_W \sim .2$ and the unification scale was $\sim 10^{15}$ GeV.

These were very nice results, since the value of the weak angle was about right, and the value of the unification scale was fortuitous.
In fact, $10^{15}\text{ GeV}$ was close enough to the Planck scale as to suggest a unification with gravity... and big enough to make the proton long-lived... a proton lifetime of about $10^{30}$ years was predicted.

The model would have failed if the computed unification scale were $< 10^{15}\text{ GeV}$ and would have been strange if it were $> 10^{19}\text{ GeV}$.
IT WAS NICE IN SU(5) TO REDUCE THE MESS OF A STANDARD MODEL GENERATION TO JUST TWO PIECES ... BUT CAN ONE DO BETTER? IT WAS SOON SEEN (GEORG1) THAT BY GOING TO THE LARGER GROUP SO(10), ALL QUARKS AND LEPTONS OF ONE GENERATION FIT NEATLY IN A SINGLE IRREDUCIBLE REPRESENTATION
There is a price, though—
you have to add a righthanded neutrino. Since it is a standard model singlet, it is natural for it to get a GUT scale mass, and this led to the idea of the "seesaw" mechanism for neutrinos (G.R.S., Yanagida).
The simplest models of this type give a mass matrix

\[
\begin{pmatrix}
\nu_L \\
\bar{\nu}_R
\end{pmatrix}
\begin{pmatrix}
0 & m \\
M & M
\end{pmatrix}
\]

where \( m \) comes from the electroweak Higgs effect, and \( M \) can be of order \( M_{\text{GUT}} \). If we take \( m \sim m_W, m_Z \), we get a light neutrino mass of order
\[ m_\nu \sim \frac{(100 \text{GeV})^2}{10^{15} \text{GeV}} \]

\[ \sim .01 \text{ eV} \]

This estimate, which was made in the late '70s, gave a big impetus to the search for neutrino masses and oscillations.
HAVING COME THIS FAR,
CAN WE GO FARTHER AND

* UNIFY THREE GENERATIONS
OF QUARKS AND LEPTONS
IN ONE IRREDUCIBLE REPRESENTATION
OF A LARGER GAUGE GROUP?

* UNIFY HIGGS BOSONS WITH
GAUGE FIELDS, OR WITH
QUARKS AND LEPTONS?
THE ANSWER TO THESE QUESTIONS WAS "NO" IN THE FRAMEWORK OF FOUR-DIMENSIONAL GRAND UNIFICATION.

FOR EXAMPLE, THERE IS NO CANDIDATE GUT GROUP THAT PUTS SEVERAL CHIRAL FERMION FAMILIES IN ONE IRREDUCIBLE REPRESENTATION - WITHOUT "ANTIFAMILIES" OF OPPOSITE CHIRALITY.
Likewise, in four-dimensional GUTs, one can't really unify Higgs particles with quarks and leptons or gauge bosons (or for that matter, unify quarks and leptons with gauge bosons).

The closest try uses supersymmetry plus the E6 model I'll get to shortly.
The other recognized problem of GUT's in this period was the "Gauge Hierarchy Problem".

Why is

\[ \frac{M_W}{M_X} \sim \frac{10^2 \text{GeV}}{10^{14} \text{GeV}} \sim 10^{-12} \]

so small?
I've stressed the SU(5) and SO(10) models which made sense of the fermion quantum numbers and led to predictions of

* Proton decay
* Neutrino masses

Are there any bigger groups that teach us more?
THERE ISN'T ANY FOUR-DIMENSIONAL MODEL THAT DOES BETTER - BUT THERE IS ONE MORE MODEL WORTHY OF NOTE — THE E6 MODEL (GURSEY, RAMOND, ETC.)

WHAT IS E6?

FOR GAUGE THEORY, WE NEED TO PICK A GAUGE GROUP.

THERE ARE INFINITE FAMILIES

\[ \{ \text{SO}(N) \}, \text{SU}(N), \text{Sp}(N) \} \]

OF POSSIBLE GROUPS, TWO OF WHICH — SU(5) AND SO(10) — ARE USED IN MODELS I MENTIONED SO FAR
AND THERE ARE FIVE EXCEPTIONS

\[ G_2, F_4, E_6, E_7, E_8 \]

NATURE IS EXCEPTIONAL, SO WHY NOT DESCRIBE IT USING AN EXCEPTIONAL LIE GROUP?

THERE IS ONE THAT BEAUTIFULLY WORKS — THE \( E_6 \) MODEL

IT CAPTURES THE SUCCESSES OF THE SO(10) MODEL "EXCEPTIONALLY"
But why $E_6$?
Why would nature go halfway down the chain of exceptional groups?

$G_2 < F_4 < E_6 < E_7 < E_8$

If nature likes exceptional groups, why stop halfway?

But $E_6$ is the only exceptional group that works for four-dimensional GUTs....
THE FOUR-DIMENSIONAL GUT
GROUPS THAT WORK FIT IN A CHAIN

\[ SU(5) \subset SO(10) \subset E_6 \subset E_7 \subset E_8 \]

AND, IN FOUR-DIMENSIONAL GUTS, WE CAN ONLY GO HALFWAY DOWN THIS CHAIN BECAUSE OF THE V-A STRUCTURE OF WEAK INTERACTIONS
The next developments I'll mention were experimental.

* More accurate measurements showed that $\sin^2 \theta_W$ is close to the GUT value, but not close enough.

* The proton lifetime turned out to be longer than the GUT value.
Both of these problems were neatly addressed by including 

**Supersymmetry**

Supersymmetry raises the GUT prediction for $\sin^2 \theta_W$ - which becomes very close to the modern measurement.
It also raises the GUT scale, making the proton lifetime long enough, and lowering the gap between $M_{\text{GUT}}$ and $M_{\text{Planck}}$.

Finally - SUSY stabilizes the hierarchy $\frac{M_{W}}{M_{X}} < 10^{-12}$, canceling large radiative corrections to the Higgs boson mass.
So, since the early 1980's, susy-gut's have been the attractive form of gut's
Going back to theory, it was soon realized (Ev, 1983) that although one cannot unify three generations in four dimensions, one can readily do so if one starts above four dimensions.

For example:

- An SO(12) model in six dimensions
- Or an SO(16) model in ten dimensions
A bigger change came in 1984 with Green-Schwarz anomaly cancellation and construction of the (Cross, Harvey, Martinec, Rob)

Heterotic String;

it became feasible to combine GUT's with string theory and thus unify all the forces, including gravity.
In this framework, one has to unify all the forces plus three generations of quarks and leptons plus Higgs bosons in one SUSY-Multiplet. Because that is all there is.

One also has to start in ten dimensions with $E_8 \times E_8$ (or $SO(32)$) because those are the only allowed ten-dimensional gauge group.
So one is forced to continue the GUT chain to the end:

\[ SU(5) \rightarrow SO(10) \rightarrow E_{6} \rightarrow E_{7} \rightarrow E_{8} \]

And to unify the three fermion generations...

Two things that didn't work in four-dimensional GUTs.
THE MODEL IS CONSTRUCTED
BY STARTING WITH

\[ \mathbb{R}^4 \times K \]

MINKOWSKI SPACE

COMPACT SIX-MANIFOLD CHosen TO PRESERVE FOUR-DIMENSIONAL SUSY

AND THEN TO OBEY THE EQUATIONS OF MOTION, ONE IS FORCED TO INTRODUCE VEV'S FOR GAUGE FIELDS ON K, BREAKING E_8 TO A SUBGROUP
The simplest choices for this break $E_8$ to one of the usual GUT groups:

$E_8 \xrightarrow{SU(5)} SO(10) \xrightarrow{} E_6$

- We don't know which choice to make! - and the light fermions occur in just the right representation with

Number of generations $=$ Topological invariant of
These models are not really four-dimensional GUTs, since unification really only occurs in ten dimensions.

They are similar to four-dimensional GUTs, with some differences that I won't stress today.

I'll just draw attention to one difference — the usual quantization of electric charge is generally not obeyed — there are superheavy unconfined particles with fractional electric charge....
DARK MATTER CANDIDATE?

But the predictions for $\sin^2 \theta W$, the proton lifetime, and the quantum numbers of the light fermions are similar to those of four-dimensional SUSY-GUTs.
Another thing that happened is that the gap between $M_{GUT}$ and $M_{Planck}$ dropped again, in the original Georgi-Quinn-Weinberg calculation the gap was a factor of $10^4$–$10^5$.

But it has since been reduced twice:
THE PERTURBATIVE HETEROTIC STRING, GIVEN MEASURED VALUES OF $G_{\text{newton}}$ AND $\alpha = \frac{e^2}{\kappa c}$, PREDICT A SUSY-GUT SCALE THAT DIFFERS BY A FACTOR OF 20 FROM THE VALUE INFERRED FROM LOW ENERGY MEASUREMENTS OF $\alpha_5$ AND $\sin^2 \theta_W$. 
THE DISCREPANCY IS REAL, BUT STILL A CLOSE TRY. IT WAS ALREADY A MAJOR ADVANCE WHEN GEORGI, QUINN, WEINBERG IN 1974 CAME WITHIN A FACTOR OF $10^4$!

THE TWO IMPROVEMENTS WERE BY PRODUCTS OF OTHER ADVANCES.

LATER IT WAS FOUND THAT GOING TO THE STRONGLY COUPLED HETERO TIC STRING (HORAVA AND EW) IS ONE WAY ONE MIGHT REDUCE OR ELIMINATE THE DISCREPANCY
WHAT HAS HAPPENED SINCE?

ONE IMPORTANT DEVELOPMENT IS CERTAINLY THAT OSCILLATIONS IN SOLAR AND ATMOSPHERIC NEUTRINOS HAVE POINTED TO NEUTRINO MASSES (OR AT LEAST MASS DIFFERENCES) IN ROUGHLY THE RANGE THAT HAD BEEN PREDICTED FROM GUT'S 20 YEARS EARLIER.
Astronomy has given other clues -

The acceleration of cosmic expansion should eventually be an important clue about SUSY-GUT's ...

And the observations of small anisotropies in the cosmic microwave radiation (COBE satellite plus balloons)
HAVE FOR THEIR SIMPLEST INTERPRETATION AN EARLY INFLATIONARY PERIOD OF THE UNIVERSE AT A SCALE NEAR THE SUSY-GUT SCALE.....

SO THIS PLUS NEUTRINOS MAY BE TWO OBSERVATIONS OF THE GUT-SCALE
ANOTHER DEVELOPMENT IS THAT THE TOP QUARK MASS TURNED OUT TO BE LARGE - AS ASSUMED IN SUPERSYMMETRIC MODELS OF ELECTROWEAK SYMMETRY BREAKING THAT WERE FORMULATED IN THE EARLY 1980's
AND OF COURSE, INCREASINGLY PRECISE TESTS OF THE STANDARD MODEL HAVE BEEN QUITE CONSISTENT WITH THE SUSY-BASED APPROACH TO THE HIERARCHY PROBLEM - WHILE ADDING TO THE CHALLENGES FACED BY OTHER APPROACHES TO THAT PROBLEM.
SO IN SHORT THE GUT-BASED APPROACH TO PHYSICS HAS BEEN ATTRACTIVE SINCE THE 1973 PAPER OF GEORGII AND GHASHOW; IT HAS BEEN ENRICHED BY NEW IDEAS, NOTABLY SUSY AND STRINGS AND THERE ARE REAL HINTS THAT IT IS ON THE RIGHT TRACK, NOTABLY FROM $\sin^2 \theta W$ AND NEUTRINO MASSES.
IF THIS APPROACH IS RIGHT, WHAT MAY WE FIND AT ACCELERATORS?

THE HIGGS BOSON REALLY SHOULD BE IN REACH—PERHAPS AT 115 GeV AS HINTED AT LEP—SINCE GUT-THEORIES DON'T WORK WITHOUT IT.

IT SHOULD APPEAR AT FERMILAB IF THE LUMINOSITY IS REACHED...
Supersymmetric particles should be in reach of the LHC — and maybe of Fermilab — since the supersymmetric approach to the hierarchy problem doesn't make sense if they are too heavy.
But would the superworld really look like? I don't think we have a convincing picture of this, partly because of problems in nonconservation of flavor, baryon number, CP due to SUSY particles.
WE HAVE A HUNCH THAT
SUPERSYMMETRY WILL BE FOUND
BUT IF IT IS FOUND, THE
DETAILS WILL BE SURPRISING,
at least to me, since no
MODEL OF THE TeV SCALE
SUPERWORLD IS REALLY
CONVINCING ... THAT IS
PART OF WHAT MAKES THE
SEARCH SO EXCITING
Moreover, exploration of the superworld will be a long project, because of the numerous new particles and new interactions.
It will require high precision (electron colliders, such as TESLA) as well as high energy (proton colliders like Fermilab and the LHC)