# Single and pair production of MSSM Higgs bosons as a probe of scalar-pseudoscalar mixing at $e^+e^-$ colliders

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### Abstract

We study the associated production of the $A^0$ neutral CP–odd Higgs boson with a neutral gauge boson $Z$ as well as single production of $A^0$ via $e^+e^- \to \nu_e \bar\nu_e A^0$ at the one loop level in the Minimal Supersymmetric Standard Model (MSSM). We show that the MSSM cross–section may be enhanced by light SUSY particles. Then we study the production processes $e^+e^- \to H_i^0 Z$, $H_i^0 \nu_e \overline{\nu}_e$ and $H_i^0 H_j^0$ in the context of the MSSM with scalar-pseudoscalar mixing. In a given channel we show that the cross–section for all $i$ $(= 1, 2, 3)$ can be above 0.1 fb provided $M_{H_{2,3}} \lesssim 300$ GeV. This should be detectable at a Next Linear Collider and would provide evidence for scalar–pseudoscalar mixing.

**1.** Recently, the phenomenology of the MSSM with complex SUSY parameters has received growing attention [1]. Such phases give new sources of CP violation which may provide: electroweak baryogenesis scenarios and CP violating phenomena in K and B decays [2]. It has been shown that by assuming universality of the gaugino masses at a high energy scale, the effects of complex soft SUSY parameters in the MSSM can be parametrized by two independent CP phases: the phase of the Higgsino mass term $\mu$ $(\mathrm{Arg}(\mu))$ and the phase of the trilinear scalar coupling parameters $A = A_f$ $(\mathrm{Arg}(A_f))$ of the sfermions $\widetilde{f}$. The presence of large SUSY phases can give contributions to the electric dipole moments of the electron and neutron (EDM) which exceed the experimental upper bounds. In a variety of SUSY models such phases turn out to be severely suppressed by such constraints i.e. $\mathrm{Arg}(\mu) < (10^{-2})$ for a SUSY mass scale of the order of few hundred GeV [3].

However, the possibility of having large CP violating phases can still be consistent with experimental data in any of the following three scenarios: i) Effective SUSY models [3], ii) Cancellation mechanism [4] and iii) Non-universality of trilinear couplings $A_f$ [5]. It is well known that the presence of SUSY CP violating phases induces mixing between the CP–even and CP–odd scalars, resulting in the 3 mass eigenstates $H_1^0$, $H_2^0$ and $H_3^0$ which do not have a definite CP parity. This mixing affects their phenomenology at present and future colliders, both in production mechanisms and decay partial widths [6, 7, 8].

In this study we will consider various production mechanisms for neutral Higgs bosons of the MSSM (with and without scalar-pseudoscalar mixing) in the context of a NLC with $\sqrt{s} = 500$ GeV and 800 GeV.

**2.** The study of the various production mechanisms of the CP–odd $A^0$ Higgs boson is well motivated since the discovery of such a particle would signify that the electroweak symmetry breaking is introduced by more that one Higgs doublet. The CP-odd $A^0$ possesses no tree–level coupling $A^0ZZ$ and $A^0WW$, and so it cannot be produced at tree level neither via the Higgsstrahlung process nor via W-W fusion. Both of those processes can be generated at one-loop order [9, 10, 11]. The one-loop diagrams for $e^+e^- \to ZA^0$ and $e^+e^- \to \nu_e\bar{\nu}_e A^0$ can be found respectively in [9, 11]. For both processes, the calculation was performed within the dimensional regularisation scheme with the help of FeynArts and FormCalc [12].

For our numerical evaluation, we will take into account the following considerations: The CP conserving MSSM Higgs sector is parametrized by the CP-odd mass $M_A$ and $\tan\beta$, taking into account radiative corrections to the lightest Higgs boson. We limit ourselves to the case where $\tan\beta \geq 2.5$. The chargino/neutralino sector can be parametrized by the usual $M_1$, $M_2$ and $\mu$. We assume that $M_1 \approx M_2/2$ and $\mu > 0$. In our analysis we will take into account the following constraints when the SUSY parameters are varied: i) the extra contribution $\delta\rho$ to the $\rho$ parameter [13] should not exceed the current limits from experimental measurements $\delta\rho \lesssim 10^{-3}$, ii) $m_{\tilde{t}_1,\tilde{b}_1} > 100$ GeV, $m_{\chi_1^\pm} > 103$ GeV, $m_{\chi_1^0} > 50$ GeV and $m_h > 110$ GeV.

Following the approach of [14] we assume a detection threshold of 0.1 fb for $e^+e^- \to A^0Z$. This would give 50 events before experimental cuts for the expected luminosities of 500 fb$^{-1}$. In the THDM this criterion would require $\tan\beta \leq 0.3$, even for a light pseudoscalar [9].

We start by recalling that the THDM contribution to $e^+e^- \to A^0Z$ [9] in the small $\tan\beta$ regime is enhanced by the top quark contribution, leading to cross–sections of order 0.04 fb. In the large $\tan\beta$ regime the cross–section is suppressed and does not attain observable rates. In the MSSM we limit ourself to the case where $\tan\beta \geq 2.5$, and consequently the THDM contribution is suppressed to the order of $\approx 0.002$ fb at $\sqrt{s} = 500$ GeV. The light SUSY particles (charginos) can slightly enhance the cross section, see Fig. 1 (left pannel), but due to a cancellation between the vertex and box diagrams the light SUSY enhancement turns out to be not very promising. The maximum cross-section for light $M_A$ is about 0.005 fb for $\tan\beta = 2.5$. As can be seen in Fig. 1, the cross-section can also be enhanced by polarizing the electron and positron beams.

Let us now discuss the single CP-odd Higgs boson production $e^+e^- \to \nu_e\bar{\nu}_e A^0$. This one-loop process has contributions from: i) one-loop W-W fusion to $A^0$, ii) the one loop process $e^+e^- \to A^0Z$ followed by Z decay to $\nu_e\bar{\nu}_e$, and iii) box diagrams [11]. In our study we have omitted the five point-functions, since these five point-functions do not have any enhancement factor, like a top quark or a scalar top quark inside the loop. Hence their contribution is expected to be smaller. Ref. [10] evaluated the top-bottom contribution to $e^+e^- \to \nu_e\bar{\nu}_e A^0$ coming from the one-loop W-W fusion to $A^0$.

In Fig. 1 (right panel), we plot the cross-section of $e^+e^- \to \nu_e\bar{\nu}_e A^0$ in the 2HDM for 800 GeV center of mass energy. For the Higgs couplings and masses we use the MSSM values with radiative corrections to the lightest Higgs boson mass. We found cross-sections of order 0.01 fb (resp 0.0006 fb) for small $\tan\beta = 0.5$ (resp $\tan\beta = 2.5$) and light $M_A < 400$ GeV.

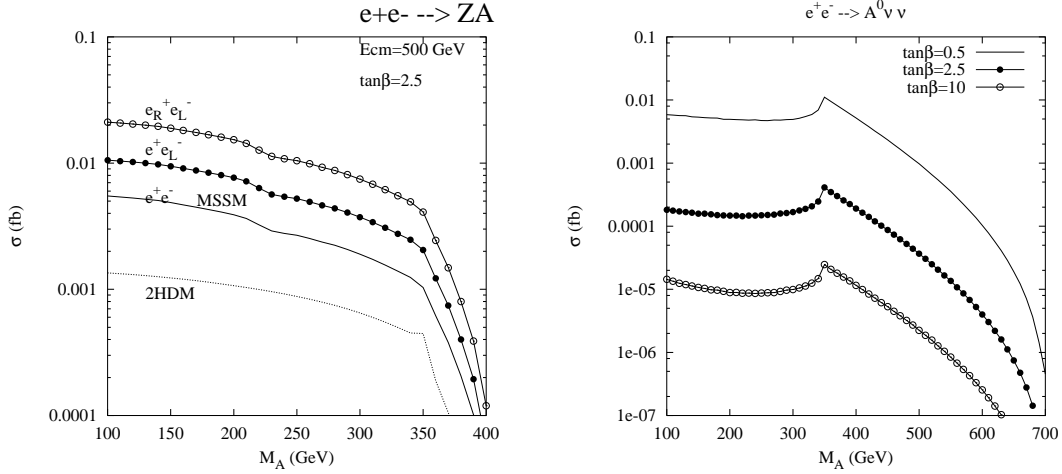Numerically, in the MSSM with real parameters, we found that even in the optimistic

Figure 1: Total cross-section for $e^+e^- \to A^0Z$ as function of $M_A$ at $\sqrt{s} = 500$ GeV (left panel) and for $e^+e^- \to \nu_e\overline{\nu}_e A^0$ (right panel) at $\sqrt{s} = 800$ GeV

scenario where all SUSY particles are light (of order 200 GeV), $\tan\beta = 2.5$, large $A_{t,b}$ and large $\tan\beta$, the cross section does not receive a substantial enhancement. One concludes that the cross sections for $e^+e^- \to ZA^0$ and $e^+e^- \to \nu_e\bar{\nu}_e A^0$ are well below 0.1 fb.

**3.** We now consider the effect of SUSY CP violating phases on the Higgs bosons production mechanisms. We will study the processes $e^+e^- \to Z^* \to H_i^0 Z$ and $e^+e^- \to H_i^0\nu_e\overline{\nu}_e$ in the context of a NLC. Both mechanisms are mediated by the tree–level effective couplings $H_i^0VV$ ($\equiv C_i$), but their cross–sections have different phase space and $\sqrt{s}$ dependence. In the CP conserving case one of these couplings would be zero, corresponding to the absence of the coupling $A^0VV$. We will also study the production of neutral Higgs pairs, $e^+e^- \to H_i^0 H_j^0$ ($i \neq j$). In the CP conserving MSSM, only the vertices $Zh^0A^0$ and $ZH^0A^0$ exist at tree–level while in the CP violating scenario all three couplings $ZH_1^0H_2^0$, $ZH_1^0H_3^0$ and $ZH_2^0H_3^0$ are generated at tree–level. Therefore an observable signal for all $i(= 1, 2, 3)$ in a given mechanism ($e^+e^- \to ZH_i^0$, $e^+e^- \to H_i^0 H_j^0$ or $e^+e^- \to H_i^0\nu_e\overline{\nu}_e$), would be a way of probing CP violation in the Higgs sector. We will calculate the tree–level rates of the above mechanisms in the context of the MSSM with scalar-pseudoscalar mixing, showing that in the most favourable scenarios this way of probing scalar–pseudoscalar mixing can be effective if $M_{H_{2,3}} \lesssim 300$ GeV.

It has been shown [1] that sizeable scalar–pseudoscalar mixing is possible for large $|\mu|, |A_t| > M_{SUSY}$. In [1] the mass matrix $\mathcal{M}_{ij}^2$ is evaluated to one–loop order using effective potential techniques and includes large two–loop non–logarithmic corrections induced by one-loop threshold effects on the top and bottom quark Yukawa couplings. The public code which we will employ in our numerical analysis can be found in [15].

In the CP conserving MSSM, $C_1 = \sin(\beta - \alpha)$ while one of $C_2, C_3$ is identified as $\cos(\beta - \alpha)$, and the other is identically zero. $C_i \equiv 0$ corresponds to the couplings $A^0ZZ$ and $A^0WW$, which will take on a non–zero value at 1-loop. Hence to lowest order in the MSSM only $H_1^0$ and *one* of $H_2^0, H_3^0$ can be produced in the Higgsstrahlung and $WW$ fusion mechanisms, $e^+e^- \to Z^* \to H_i^0 Z$ and $e^+e^- \to H_i^0\nu_e\overline{\nu}_e$. In the presence of SUSY phases all $H_i^0$ may be produced at tree–level via $e^+e^- \to Z^* \to H_i^0 Z$, and an observable signal

for all three $H_i^0$ would be evidence for CP violation in the Higgs sector. We stress here that the smallest of $\sigma(e^+e^- \to Z^* \to H_i^0 Z)$ (resp $\sigma(e^+e^- \to W^*W^* \to \nu_e\bar{\nu}_e H_i^0)$) should exceed the maximum rate for $e^+e^- \to A^0 Z$ (resp $\sigma(e^+e^- \to \nu_e\bar{\nu}_e A^0)$) in the context of the CP conserving MSSM, since the latter would constitute a "background" to any interpretation as scalar–pseudoscalar mixing. The previous section showed that these cross-sections in the CP conserving MSSM are expected to be comfortably below 0.1 fb. Note that the process $e^+e^- \to H_i^0 \nu_e \overline{\nu}_e$ proceeds via the same couplings $C_i$, but possesses a different phase space and $\sqrt{s}$ dependence. This mechanism is competitive with the Higgsstrahlung process, and becomes the dominant one as $\sqrt{s}$ increases. We will also consider the mechanism $e^+e^- \to H_j^0 H_k^0$, which proceeds via the coupling $ZH_j^0 H_k^0$, where $ZZH_i^0 = ZH_j^0 H_k^0$ for $i \neq j \neq k$.

In the MSSM (with or without SUSY phases), the properties of the lightest eigenstate $H_1^0$ become very similar to that of the SM Higgs boson in the decoupling region of $M_{H^\pm} \geq$ 250 GeV. In this region, $C_1$ is very close to 1, and so the sum $C_2^2 + C_3^2$ is constrained to be small (a consequence of the sum-rules). Therefore we expect that $M_{H^\pm} \leq 250$ GeV will allow larger values for the sum $C_2^2 + C_3^2$, and thus observable rates for both $H_2^0$ and $H_3^0$ in the above mechanisms. Distinct signals for all three $H_i^0$ in a given channel would be evidence for scalar–pseudoscalar mixing [†]. We now present our numerical results which we will generate with the fortran program cph.f [15]. We note that this program does not include $\chi^+$–$W$–$H^\pm$ contributions to the 1–loop neutral Higgs mass matrix, which have been shown to be sizeable in some regions of parameter space [16].

We are concerned with a NLC collider which has the ability to probe $\sigma(e^+e^- \to H_i^0 Z, H_i^0 H_j^0, H_i^0 \nu_e \overline{\nu}) \geq 0.1$ fb, and so we are also interested in smaller values for $C_2^2, C_3^2$ and larger $M_{H_i}$. We shall be presenting results for the production cross–sections of all the above mechanisms. Note that the cross–sections for the processes $e^+e^- \to Zh^0/H^0$, $e^+e^- \to Ah^0/H^0$, $e^+e^- \to h^0 H^0$, $e^+e^- \to ZA^0$ and $e^+e^- \to \nu_e\overline{\nu}_e h^0/H^0$ in the CP conserving MSSM are accurately known [9, 17, 18]. Deviations from these rates would be evidence for scalar–pseudoscalar mixing.

In our numerical analysis we will choose the CP violating benchmark scenario (CPX) which maximizes the CP violating effects: $\widetilde{M}_Q = \widetilde{M}_t = \widetilde{M}_b = M_{SUSY} = 1$ TeV, $\mu = 4$ TeV, $|A_t| = |A_b| = 2$, $|m_{\widetilde{B}}| = |m_{\widetilde{W}}| = 0.3$ TeV. Note that $\mu$ will be taken real while we allow a CP phase in the soft tri-linear parameters $A_t$ and $A_b$. The CP phases of $A_t$ and $A_b$ are chosen to be equal. In addition, we choose the charged Higgs mass and $\tan\beta$ as free parameters.

Our strategy to probe the scalar–pseudoscalar mixing requires the identification of the Higgs signals as distinct resonances. The inclusion of the phases in $A_t$ and $A_b$ breaks the near degeneracy among $M_{H_2}$ and $M_{H_3}$ [1], and gives sufficient splittings to allow identification of separate resonances for $H_2^0$ and $H_3^0$. These splittings may be $> 10$ GeV, which is sufficiently large for a NLC [19] to resolve the separate peaks. This will lead to three different peaks in the Higgsstrahlung and $WW$ fusion processes and motivates us to present the individual cross–sections for $e^+e^- \to ZH_i^0$ and $e^+e^- \to \nu\overline{\nu}_e H_i^0$ for $i = 1, 2, 3$. It has been shown in [1] that the inclusion of SUSY phases may drastically change the size of the couplings $ZZH_1^0$ and $ZH_1^0 H_2^0$ for low and intermediate $\tan\beta$. In such cases

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[†]Of course, other models with more than 2 Higgs doublets and/or additional singlets could provide multiple signals in these channels, e.g. the CP conserving NMSSM.
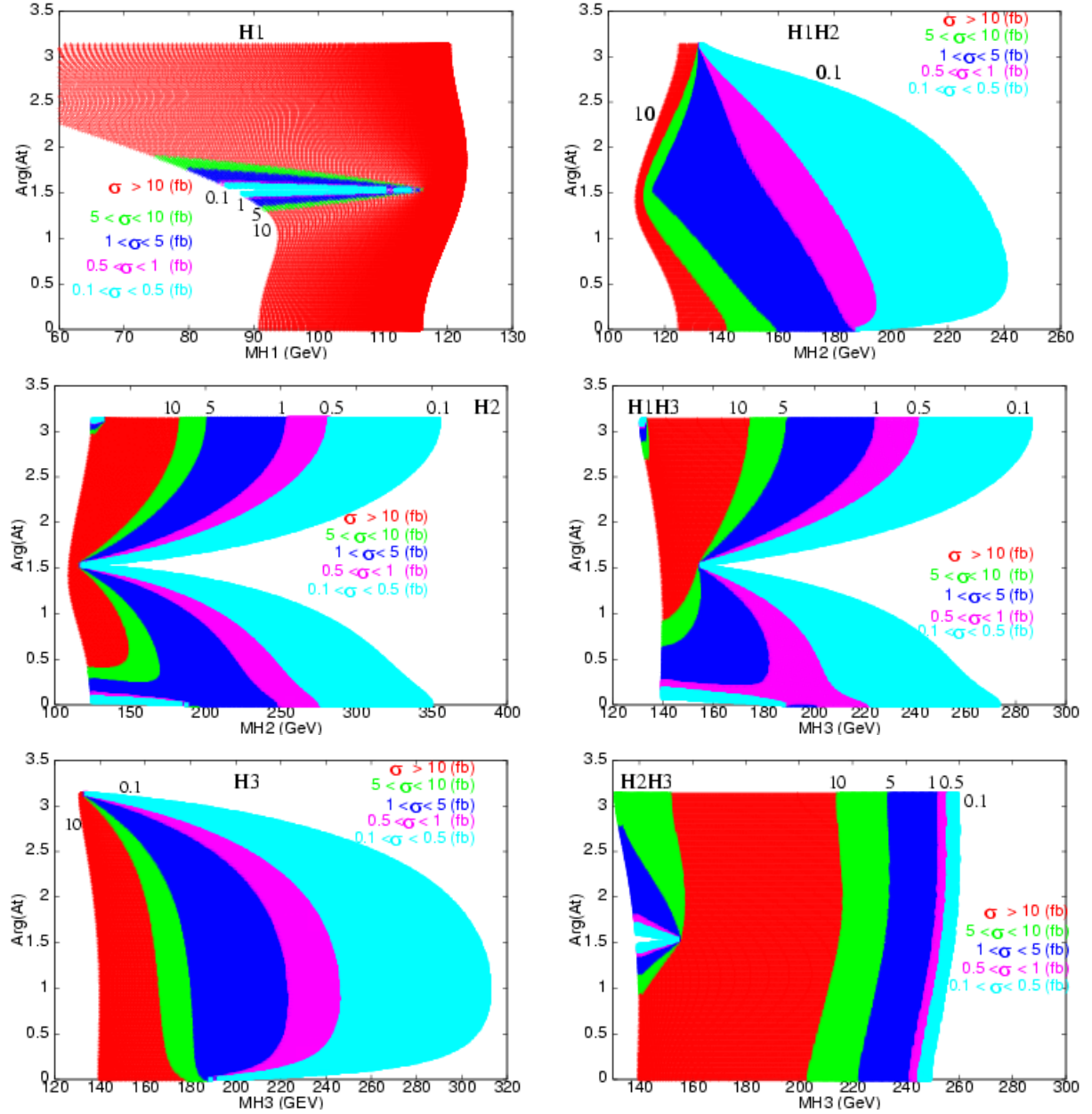
Figure 2: (left) $\sigma(e^+e^- \rightarrow H_i^0 \nu\overline{\nu})$ at $\sqrt{s} = 800$ GeV in $(M_{H_i}, \text{Arg}(A_t))$ plane, (right) $\sigma(e^+e^- \rightarrow H_i^0 H_j^0)$ at $\sqrt{s} = 500$ GeV in $(M_{H_i}, \text{Arg}(A_t))$ plane

the bound on the light Higgs boson obtained at LEPII may be weakened to $\lesssim 60$ GeV for large CP violation in the MSSM Higgs sector. We study the potential of a NLC to discover such a weakly coupled Higgs.

In Fig. 2 the left plots depict regions of $\sigma(e^+e^- \rightarrow \nu_e\bar{\nu}_e H_i^0)$ in the plane $(M_{H_i}, \text{arg}(A_t))$ for $\sqrt{s} = 800$ GeV, $\tan\beta = 6$, and $M_{SUSY} = 1$ TeV. In all plots the charged Higgs mass has been varied in increments from $140 \rightarrow 400$ GeV, which determines the values of $M_{H_i}$. As it can be seen in the plot, $H_1^0$ discovery is possible over most of the $(M_{H_1}, \text{arg}(A_t))$

plane, with small unobservable regions where $\sigma(e^+e^- \to ZH_1^0) < 0.1$ fb which occur for $\arg(A_t) \approx 1.5$ and $M_{H_1} \lesssim 105$ GeV.

$ZZH_2^0$ is maximized for $M_{H_2} \lesssim 150$ GeV and is minimized for $\arg(A_t) \approx 1.5$ and $M_{H_2} \gtrsim 150$ GeV. For $H_2^0$ and $H_3^0$, both $\sigma(e^+e^- \to \nu_e \bar{\nu}_e H_{2,3}^0)$ can be observable over a wide region of the plane, even up to relatively large mass values e.g. for $\arg(A_t) = 1$, $\sigma(e^+e^- \to \nu_e \bar{\nu}_e H_{2,3}^0) \geq 0.1$ fb for $M_{H_2} \leq 275$ and $M_{H_3} \leq 310$ GeV. Note that the scalar–pseudoscalar composition of $H_2^0$ and $H_3^0$ can change with increasing $M_{H_i}$ e.g. for low $\arg(A_t)$, one can see that $H_3^0$ is dominantly CP-even scalar for low masses, and has a much larger cross–section than for that for $H_2^0$. As $M_{H_{2,3}}$ increases, $H_2^0$ has the larger CP-even scalar component and may be produced with an observable rate for $M_{H_2} \leq 350$ GeV. The cross section for $e^+e^- \to ZH_i$ has a similar shape as the one for $e^+e^- \to \nu_e \bar{\nu}_e H_i$, but with this process one can probe scalar-pseudoscalar mixing only up to $M_{H_2} \leq 250$ GeV and $M_{H_3} \leq 270$ GeV both at $\sqrt{s} = 500$ and $800$ GeV.

In Fig. 2 (right panels) we show $\sigma(e^+e^- \to H_i^0 H_j^0)$ in the plane $(M_{H_i}, \arg(A_t))$ for $\tan\beta = 6$, and $M_{SUSY} = 1000$ GeV. Due to its phase space suppression, this mechanism offers smaller cross–sections than the Higgsstrahlung and $WW$ fusion processes, and consequently it is not as effective at probing the scalar–pseudoscalar mixing. Using the fact that $C_i^2 = C_{jk}^2$ for $i \neq j \neq k$, the behaviour of pair production $e^+e^- \to H_i^0 H_j^0$ can be roughly understood from the rate of the WW fusion process $e^+e^- \to \nu \bar{\nu} H_k^0$. As can be seen from the plots, there are some similarities between $e^+e^- \to H_i^0 H_j^0$ and $e^+e^- \to \nu \bar{\nu} H_k^0$, for $i \neq j \neq k$. However, $\sigma(e^+e^- \to H_1^0 H_{2,3}^0)$ has an observable rate in the region where the Higgsstrahlung and $WW$ fusion processes have very suppressed rates.

**4.** To conclude, we have studied various production mechanisms for neutral Higgs bosons in the MSSM in the context of a high–energy $e^+e^-$ collider. We computed the cross–section for the production mechanisms $e^+e^- \to A^0 Z$ and $e^+e^- \to \nu_e \bar{\nu}_e A^0$ in the framework of the MSSM. Such processes proceed via higher order diagrams and are strongly model dependent. In the MSSM light SUSY particles may give an enhancement to the cross–sections but such not sufficient to be observable at NLC.

We then studied the production processes $e^+e^- \to H_i^0 Z$, $H_i^0 H_j^0$ and $H_i^0 \nu_e \bar{\nu}_e$ in the context of the MSSM with SUSY CP violating phases. We showed that in a given channel the cross–section for all $H_i^0$ ($i = 1, 2, 3$) can be observable at a Next Linear Collider and would provide evidence for scalar–pseudoscalar mixing. At $\sqrt{s} = 500$ GeV the coverage of $e^+e^- \to H_i^0 Z$ and $H_i^0 \nu_e \bar{\nu}_e$ are comparable, with observable cross–sections for $M_{H_2} \leq 250$ GeV and $M_{H_3} \leq 270$ GeV for the most favourable choice of $\arg(A_t)$. At $\sqrt{s} = 800$ GeV, the process $e^+e^- \to H_i^0 \nu_e \bar{\nu}_e$ offers superior coverage, with a reach up to $M_{H_{2,3}} \leq 300$ GeV in the most favourable cases. The scalar–pseudoscalar mixing causes a mass splitting between $H_2^0$ and $H_3^0$ which should be sufficient for separate peaks to be resolved at a NLC. The mechanism $e^+e^- \to H_1^0 H_{2,3}^0$, while less effective for probing scalar–pseudoscalar mixing, can comfortably detect $H_1^0$ in the region of suppressed coupling $VVH_1^0$.

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