The Higgs Sector of the NMSSM

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Abstract

I discuss the Higgs boson spectra of the Next-to-Minimal Supersymmetric Standard Model. The renormalisation group flow is used to motivate natural values of the parameters of the model. I present the qualitative features of the Higgs boson masses and demonstrate their dependence on how severely the Peccei–Quinn symmetry of the model is broken.

1 The Model

1.1 The NMSSM as a solution to the hierarchy problem

Supersymmetry is widely regarded as being one of the most likely candidates for physics beyond the Standard Model. It is naturally contained in many theories of physics at or beyond the GUT scale, and by making supersymmetry local, a theory of gravity naturally emerges. Furthermore, low energy supersymmetry solves the hierarchy problem, stabilising the Higgs boson mass at the electroweak scale as required for electroweak symmetry breaking. For these reasons the minimal supersymmetric standard model (MSSM) attracts much study. However, it is important that we consider models beyond the minimal version of supersymmetry. In particular, theories of physics at the Planck scale often involve higher gauge symmetry groups. These symmetries become broken at the TeV scale, often leaving behind extra residual symmetries and/or particles. It is important that the effects of such minor (and reasonable) additions to the low scale phenomenology be thoroughly investigated.

One such model is the next-to-minimal supersymmetric standard model (NMSSM) [1], which includes an extra Higgs singlet field in order to explain the μ problem of the MSSM. The MSSM Superpotential contains the term $-\mu H_1 \epsilon H_2$, where μ has dimensions of mass. One would naturally expect μ to be either zero or the Planck mass, but to fit phenomenological constraints it must be of the order of the electroweak scale. It is then natural to ask: why is μ so small but non-zero? The NMSSM postulates that μ originates from the vacuum expectation value of a new complex scalar Higgs field N. The NMSSM superpotential contains the term $-\lambda N H_1 \epsilon H_2$, and, by allowing the new singlet field to gain a non-zero expectation value comparable to that of the other Higgs bosons, the μ term naturally emerges at the required scale: $\mu = \lambda \langle N \rangle$. Of course, this still does not explain why the expectation values of the fields and consequently the electroweak scale itself are so much smaller than the Planck scale in the first place.

Notice that the NMSSM superpotential exhibits an extra $U(1)$ symmetry, known as the Peccei–Quinn symmetry [2], which is explicitly broken in the MSSM by the μ-term itself. This symmetry is spontaneously broken by the singlet Higgs field acquiring a vacuum expectation value and subsequently would lead to a massless $\mathcal{CP}$ odd Higgs field.

Although much of the allowed parameter space has been ruled out by searches for this "Peccei–Quinn axion", the parameter range $10^{-7} < \lambda < 10^{-10}$ is still allowed [3]. However, due to the very small value of λ, this would require a very large value of $\langle N \rangle$ and would therefore be a rather unsatisfactory solution of the μ problem.

Consequently one must explicitly break this symmetry, which is usually done by introducing a breaking term of the form $\frac{1}{3}\kappa N^3$ into the superpotential[1]. This extra term explicitly breaks the Peccei–Quinn symmetry, while violating no other wanted symmetry, and results in the NMSSM superpotential:

$$W = -\lambda N H_1 \epsilon H_2 + \frac{1}{3}\kappa N^3 + \dots \tag{1}$$

where the ellipsis denotes the usual MSSM superpotential minus the μ-term.

Of course, one should not ignore the domain wall problem which the NMSSM generates [4]. The superpotential described above still has a $\mathbb{Z}_3$ symmetry which would lead to domain walls in the early universe between regions which became causily disconnected during inflation. Attempts to break this $\mathbb{Z}_3$ symmetry by introducing explicit $\mathbb{Z}_3$ breaking operators were found to lead to quadratically divergent tadpoles and subsequently a very heavy singlet Higgs boson [5]. Recently, work has been done by a number of groups [6] to alleviate this problem by introducing new symmetries (e.g. $\mathbb{Z}_2$) which loop suppress the tadpole terms. These approaches lead to $\mathbb{Z}_3$ breaking (solving the domain wall problem) while preserving the mass hierarchy.

1.2 Constraining the parameters of the NMSSM Higgs sector

The Higgs potential of the NMSSM resulting from the above superpotential is [1],

$$V = V_F + V_D + V_{\rm soft} + \Delta V \tag{2}$$

where the F-terms, D-terms and soft supersymmetry breaking terms are given by

$$\begin{aligned}
V_D &= \frac{1}{8}g^2(H_1^\dagger \sigma H_1 + H_2^\dagger \sigma H_2)^2 + \frac{1}{8}g'(|H_1|^2 - |H_2|^2)^2, & (3)\\
V_F &= \lambda^2 |N|^2 (|H_1|^2 + |H_2|^2) + |-\lambda H_1 \epsilon H_2 + \kappa N^2|^2 \quad , & (4)\\
V_{\rm soft} &= m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_s^2 |N|^2 - [\lambda A_\lambda N H_1 \epsilon H_2 + \frac{1}{3}\kappa A_\kappa N^3 + \text{h.c.}], & (5)
\end{aligned}$$

respectively. ΔV represents higher order corrections from top and stop loops; its one-loop vacuum-expectation value is given by:

$$\langle \Delta V \rangle = \frac{3}{32\pi^2}\left[m_{\tilde{t}_1}^4 \left(\log\frac{m_{\tilde{t}_1}^2}{Q^2} - \frac{3}{2} \right) + m_{\tilde{t}_2}^4 \left(\log\frac{m_{\tilde{t}_2}^2}{Q^2} - \frac{3}{2} \right) - 2m_t^4 \left(\log\frac{m_t^2}{Q^2} - \frac{3}{2} \right) \right]. \tag{6}$$

The NMSSM Higgs sector is then defined by the parameters λ and κ, augmented by their associated soft supersymmetry breaking parameters A_λ and A_κ, and the soft

[1]One could in principle also include terms of the form $\kappa' N^2$ or $\kappa'' N$ to break the U(1) symmetry, but this would introduce further dimensionful couplings.

masses m_1, m_2 and m_s. By finding the minimum of the Higgs potential one obtains three equations relating these parameters to the vacuum-expectation-values of the fields. These equations can be used to eliminate the three soft masses in favour of the auxiliary parameters $\tan\beta$ (the ratio of the expectation values of the two Higgs doublets), μ (or equivalently $\langle N\rangle$) and the sum of the squares of the Higgs doublet vacuum expectation values $v^2 = v_1^2 + v_2^2$. Only six free parameters remain to be varied: λ, κ, $\tan\beta$, μ, A_λ and A_κ.

One can get some idea of the value of these parameters by investigating their renormalisation group running from higher scales [7]. This follows due to the presence of an infra-red quasi-fixed point in the model [8]; the renormalisation flow 'pulls' the parameters towards this quasi-fixed point as they run down from the GUT scale. While the presence of this fixed point may seem advantageous at this stage since it tells us what the likely values of the low scale parameters are, it will eventually prove to be awkward once the parameters are experimentally determined since it will prevent us from accurately extrapolating to their values at the high scale and thereby inhibit our investigation of the high scale theory [9].

This renormalisation group running is most usefully demonstrated by looking at two quantities: $\sqrt{\lambda^2+\kappa^2}$ and κ/λ. The dependence of $\sqrt{\lambda^2+\kappa^2}$ is shown in figure 1 (left), where one notes that even large values of this quantity tend to lead to small values at

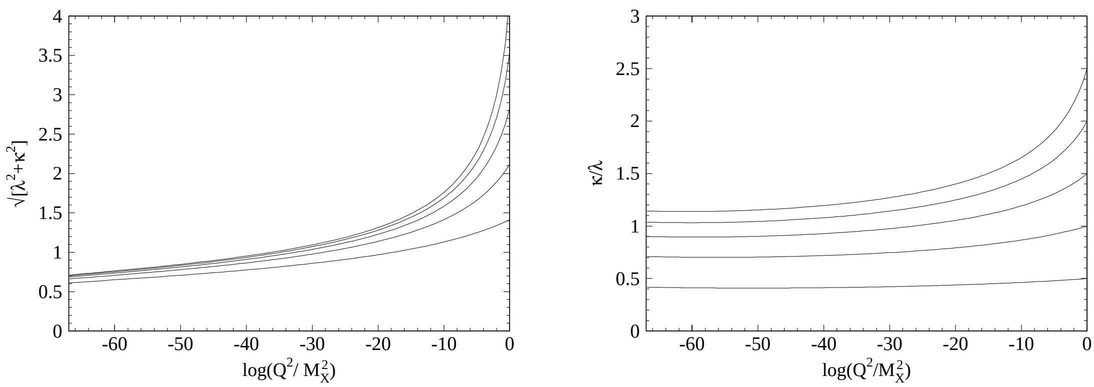

Figure 1: *The dependence of* $\sqrt{\lambda^2+\kappa^2}$ *(left) and* κ/λ *(right) on renormalisation scale. The different curves represent different values of* λ *and* κ *at the GUT scale* (M_X). *For the left-hand plot* $\lambda(M_X) = \kappa(M_X) = 1$, 1.5, 2, 2.5 *and* 3. *For the right hand plot* $\kappa(M_X)/\lambda(M_X) = 0.5$, 1, 1.5, 2 *and* 2.5, *with* $\lambda(M_X) = 1$.

lower scales. Indeed, the requirement of perturbativity up to the GUT scale leads to the restriction $\lambda^2 + \kappa^2 \lesssim 0.5$ at the electroweak scale. The running of the second quantity, seen in figure 1 (right), is not quite so pronounced, but again the quantity tends towards low values at the low scale. Additionally, the value of λ should not be too small if one wants to maintain a 'natural' theory. Since λ is related to the vacuum expectation value of the new singlet field and μ by $\lambda = \mu/\langle N\rangle$, a small value of λ implies a large value of $\langle N\rangle$, no longer comparable to the vacuum expectation values of the other Higgs fields.

Putting all this together demonstrates that the Peccei–Quinn U(1) symmetry is most likely to be only 'slightly' broken, with the value of κ/λ of order one or less. Of course, this is not to say that the theory must be of this form; only that the majority of plausible parameter choices at the high scale lead to low values of κ/λ at the low scale, making this the most likely scenario. I will therefore pay most attention to this case, but not ignore the other possibilities.

The renormalisation group running from high scales towards the infra-red fixed point also favours low values of $\tan\beta$. Although very low values of $\tan\beta$ (< 2.4) have already been ruled out in the MSSM by the LEP experiments [10], these experimental restrictions are not applicable to the NMSSM. Here I will adopt a value of $\tan\beta = 5$, suitable for both the NMSSM and the MSSM.

2 The Higgs Boson Spectra

When examining the Higgs boson mass spectra, it is useful to introduce a new parameter M_A to replace to soft supersymmetry breaking parameter A_λ. I define M_A to be the diagonal entry in the pseudoscalar mass matrix which does not vanish as the MSSM limit is approached (defined by letting κ and λ tend to zero while keeping κ/λ fixed). In this limit M_A becomes the mass of the pseudoscalar Higgs boson (hence my choice of name). At tree-level it is given by:

$$M_A^2 = \frac{2\mu}{\sin 2\beta}(A_\lambda + \mu\kappa/\lambda). \tag{7}$$

The extra complex scalar field results in two additional Higgs bosons, substantially complicating the Higgs boson mass matrices. The scalar mass matrix becomes a 3×3 matrix while the pseudoscalar becomes a 2×2 matrix. Consequently the analytic expressions for the physical masses become somewhat unwieldy and unenlightening. To aid the eye, it is useful to consider the approximate form of these equations under the assumption that M_A is large compared to the other scales, and $1/\tan\beta$ and κ/λ are $<< 1$. Making an expansion in the resulting small quantities leads to a rather crude but instructive approximation[2]. The tree-level $\mathcal{CP}$ even Higgs boson masses then take the approximate form:

$$M_{H_3}^2 \approx M_A^2 \left\{ 1 + \frac{1}{2}\lambda^2 v^2 \left(\frac{\sin 4\beta}{4\mu} \right)^2 \right\}, \tag{8}$$

$$M_{H_2}^2 \approx M_Z^2, \tag{9}$$

$$M_{H_1}^2 \approx \frac{\kappa\mu}{\lambda}\left(4\frac{\kappa\mu}{\lambda} - A_\kappa \right), \tag{10}$$

while the $\mathcal{CP}$ odd approximate tree-level masses are:

$$M_{A_2}^2 \approx M_A^2 \left\{ 1 + \frac{1}{2}\lambda^2 v^2 \left(\frac{\sin 2\beta}{2\mu} \right)^2 \right\}, \tag{11}$$

$$M_{A_1}^2 \approx 3\frac{\kappa\mu}{\lambda} \left\{ A_\kappa + \frac{3}{2}\lambda^2 v^2 \left(\frac{\sin 2\beta}{2\mu} \right) \right\}. \tag{12}$$

[2]It must be stressed that this approximation is used only for illustrative purposes and has not been used to calculate any of the Higgs spectra in this report.

No approximation is required for the tree-level charged Higgs boson mass, which is [1]:

$$M_{H^\pm}^2 = M_A^2 + M_W^2 - \frac{1}{2}\lambda^2 v^2. \tag{13}$$

Notice that the masses of the lightest Higgs bosons are governed by the value of κ/λ, i.e. how severely the Peccei–Quinn symmetry is broken. For 'slightly' broken Peccei–Quinn symmetry scenarios the lightest scalar and pseudoscalar will be rather light. The next heaviest scalar will be of a mass around the electroweak scale and the remaining Higgs states will lie close to the scale M_A.

2.1 The NMSSM with a Peccei–Quinn symmetry

It is instructive to first consider the case where $\kappa = 0$, leaving the Peccei–Quinn symmetry of the model unbroken in the superpotential. It is then only spontaneously broken by the vacuum, giving rise to a massless Goldstone boson, which is manifest as the extra pseudoscalar Higgs field. Since there is now no need for the soft supersymmetry-breaking term proportional to A_κ there are only 4 parameters in the model at tree-level: λ, μ, $\tan\beta$ and M_A.

The one-loop Higgs boson masses are plotted as a function of M_A with $\lambda = 0.5$ in figure 2. One immediately notices the restriction on M_A caused by the lightest scalar

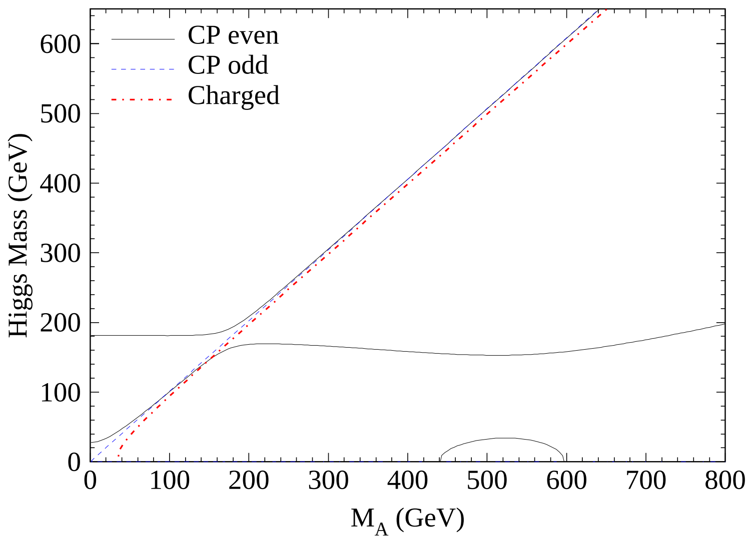

Figure 2: *The Higgs boson masses, plotted as a function of* M_A *for* $\lambda = 0.5$, $\kappa = 0$, $\mu = 100$ *GeV and* $\tan\beta = 5$.

Higgs boson. It is forced to be in the range of approximately $\mu\tan\beta$ (or more accurately $2\mu/\sin 2\beta = 520$ GeV in this case) to ensure the stability of the physical vacuum. Large values of μ and/or $\tan\beta$ would lead to a very large mass splitting between the heavy and light Higgs bosons. Even with quite low values of μ and $\tan\beta$ the restriction on M_A leads to a very distinct mass hierarchy. The charged Higgs boson, together with the heaviest

$\mathcal{CP}$ odd and $\mathcal{CP}$ even Higgs bosons remain at the scale of M_A. The lightest $\mathcal{CP}$ odd Higgs boson is massless (the Goldstone boson) and the lightest scalar is also very light. The remaining scalar has a mass of the order of the electroweak scale. This is exactly as seen in the crude approximation presented earlier. (The small mass of the lightest scalar is provided by terms which have been neglected in equation 10.)

The lightest states in this model are predominantly composed of the new degrees of freedom resulting from the new complex scalar field N. Since they have a small (or zero) mass and mix only very weakly with the other degrees of freedom, the intermediate and heavy Higgs bosons in this model have masses very reminiscent of the MSSM.

The presence of a massless pseudoscalar and light scalar immediately rules out this parameter choice. The direct production of the lightest scalar, H_1, together with a Z boson would have been possible at LEP for much of the allowed M_A range. However, the coupling g_{ZZH_1} passes through zero in this range, so such production alone does not rule out the model. The Z boson decay to the lightest scalar and pseudoscalar Higgs bosons, $Z \to H_1 A_1$, is more damaging, since the coupling is always large enough to allow detection which the choice of λ made here. This may only be remedied by forcing λ to become very small, so that the extra scalar and pseudoscalar fields decouple from the other Higgs bosons. However, this is somewhat academic since (as already discussed) cosmological and astrophysical observations have already ruled out all values of λ except for the window $10^{-7} < \lambda < 10^{-10}$ [3]. Although this window forces the required decoupling it is clearly unsuitable for explaining the μ-problem.

2.2 The NMSSM with a slightly broken Peccei–Quinn symmetry

Turning on a non-zero value of κ breaks the Peccei–Quinn symmetry and provides a mass for the lightest pseudoscalar Higgs boson. I consider the symmetry to be only slightly broken as long as $\kappa \lesssim \lambda$, as favoured by the renormalisation group flow. There are two extra parameters in the model as compared to the case with an unbroken Peccei–Quinn symmetry: κ and its associated soft supersymmetry-breaking parameter A_κ. For illustrative purposes I choose $A_\kappa = 100$ GeV, but the reader should be aware that one can easily change the masses of the two singlet dominated fields by altering this value, as indicted by the crude approximation of equations 10 and 12.

The one-loop masses of the Higgs bosons for $\lambda = 0.5$ and $\kappa = 0.35$ are shown as a function of M_A in figure 3. As before, the heavy Higgs bosons all have masses around the value of M_A and one of the scalar Higgs bosons remains at approximately the electroweak scale. The extra singlet dominated pseudoscalar is no longer massless. Its mass has been raised from zero by having the Peccei–Quinn symmetry explicitly broken by the term $\frac{1}{3}\kappa N^3$ in the superpotential. Once again, the value of M_A is bounded by the requirement that the physical vacuum be stable (i.e. $M_{H_1}^2 > 0$), although the restriction is now much looser.

Also shown are the restrictions on M_A imposed by the LEP experiments [10] for this parameter choice. These limits are dependent on the masses of the light Higgs bosons and their couplings to the Z. The light scalar escapes detection via Higgs-strahlung only when it has a sufficiently reduced coupling to the Z boson. Since this coupling passes through zero in the allowed range, one is unable to rule out all values of M_A. Furthermore,

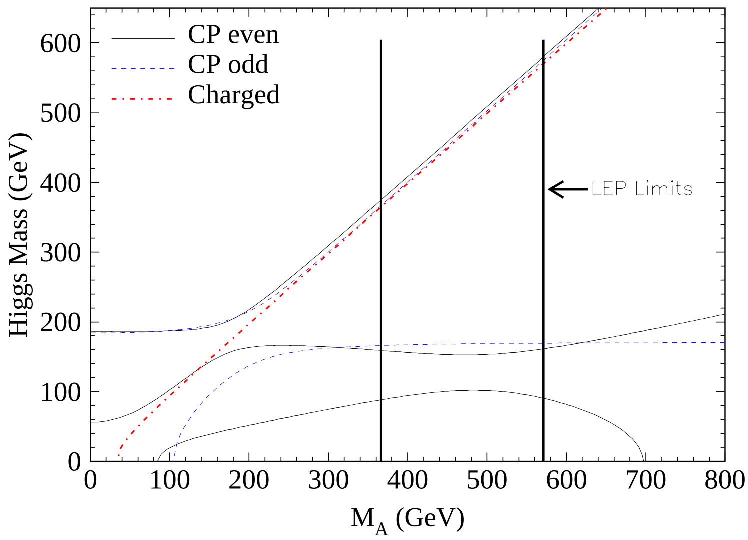

Figure 3: *The one-loop Higgs boson masses as a function of M_A for $\lambda = 0.5$, $\kappa = 0.35$, $\mu = 100$ GeV, $\tan\beta = 5$ and $A_\kappa = 100$ GeV.*

in contrast to the $\kappa = 0$ scenario, the lightest pseudoscalar is now rather heavy and the direct production of $e^+e^- \to H_1A_1$ would not have been accessible at LEP.

Varying the parameters μ, $\tan\beta$ and A_κ does little to change the qualitative picture presented here, although the values of the masses are changed. In particular, increasing μ or $\tan\beta$ causes the allowed region to move up to higher values of M_A, resulting in the increase of the heavy Higgs boson masses while leaving the lighter masses little affected. Changing the value of A_κ only varies the lightest pseudoscalar and scalar masses, leaving the rest of the spectrum unchanged.

2.3 The NMSSM with a severely broken Peccei–Quinn symmetry

When κ becomes much greater than λ, the Peccei–Quinn symmetry is severely broken. The extra pseudoscalar and scalar Higgs bosons gain large masses from the Peccei–Quinn breaking term $\frac{1}{3}\kappa N^3$ in the superpotential. This is so pronounced that the scalar Higgs boson which is comprised mainly of the new fields may no longer be the lightest scalar Higgs boson. That rôle is then taken over by a Higgs boson which is predominantly composed of the usual lightest Higgs boson of the MSSM. This switching of rôles leads to extra intermediate mass scalar and pseudoscalar states for all values of M_A. The one-loop masses of these Higgs bosons are shown as a function of M_A for the parameter choice $\lambda = 0.2$ and $\kappa = 0.5$ in figure 4. LEP limits for this scenario are not shown since they exclude only very low values of M_A, which are predominantly ruled out by vacuum stability (i.e. the requirement that $M^2_{A_1} > 0$).

As for the scenario with a slightly broken Peccei–Quinn symmetry, varying the parameters μ, $\tan\beta$ and A_κ does not drastically change the structure of the spectrum. Again A_κ has the most influence, allowing one to increase the mass of the singlet dominated

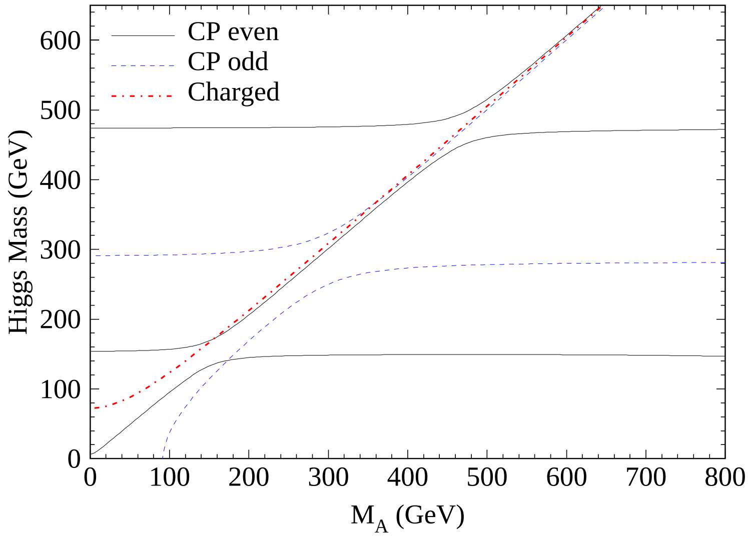

Figure 4: *The one-loop Higgs boson masses as as a function of M_A for the parameter choice $\lambda = 0.2$, $\kappa = 0.5$, $\mu = 100$ GeV, $\tan\beta = 5$ and $A_\kappa = 100$ GeV.*

scalar or pseudoscalar while decreasing the mass of the other.

This scenario is less attractive due to the renormalisation group flow, since large values of κ/λ at the GUT scale tend to be diluted when running down to the electroweak scale, as shown in figure 1 (right).

3 Summary and Conclusions

In this presentation I have investigated the Higgs sector of the Next-to-Minimal Supersymmetric Standard Model. This model attempts to explain the μ-problem of the MSSM by introducing a new singlet Higgs field, N, with a non-zero vacuum expectation value. The NMSSM also provides an interesting example of the effects of introducing new symmetries and particles into minimal supersymmetric models.

I have shown that the qualitative features of the Higgs boson masses are dependent on how severely the Peccei–Quinn symmetry of the model is broken. The renormalisation group running of the parameters from the high scale naturally leads to scenarios where the Peccei–Quinn symmetry is only slightly broken.

If the Peccei–Quinn symmetry is left explicitly unbroken in the Lagrangian (although broken by the structure of the physical vacuum), a massless Goldstone boson is present, which is the usual Peccei–Quinn axion. This axion rules out most of the allowed parameter space, only allowing scenarios which do not provide a valid solution of the μ-problem.

If the Peccei–Quinn symmetry is only slightly broken, the fields which are predominantly composed of the new singlet degrees of freedom remain light. The LEP limits on the lightest Higgs boson mass allow some of the parameter space to be ruled out. However, since the couplings to the Z can be very much reduced, much of the parameter space remains. It is therefore extremely important that future colliders search for light scalar and pseudoscalar Higgs bosons with reduced couplings.

In contrast, a severely broken Peccei–Quinn symmetry can lead to extra intermediate mass Higgs bosons, which are only weakly coupled to the Z.

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