Quantum Effects to the Higgs boson self-couplings in the SM and in the MSSM.

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From works in collaboration with:

W. Hollik
Eur. Phys. J. C23, 163-172, 2002 , hep-ph/0108245

A. Dobado, M.J. Herrero and W. Hollik
In preparation , KA-TP-25-2001

SUSY02, DESY Hamburg
June 17-23, 2002

Plan of the talk

- Introduction
- Tree-level Higgs boson self-couplings
- One-loop contributions - Analytical studies
  - Higgs sector itself
  - Leading contributions from $t - \tilde{t}$ sector
- Numerical analysis for heavy top-squarks sector
- Conclusions

Higgs self-couplings

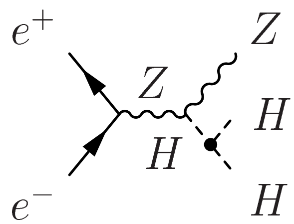
To establish the Higgs mechanism experimentally in an unambiguous way, the Higgs self-interaction potential must be reconstructed.

This task requires the measurement of the trilinear and quartic self-couplings, as predicted in the Standard Model or in supersymmetric theories.

TESLA physics programme at $\sqrt{s} = 500$ GeV

TESLA Technical Design Report, DESY 2001-011

Double Higgs-strahlung: $e^+e^- \to ZHH$

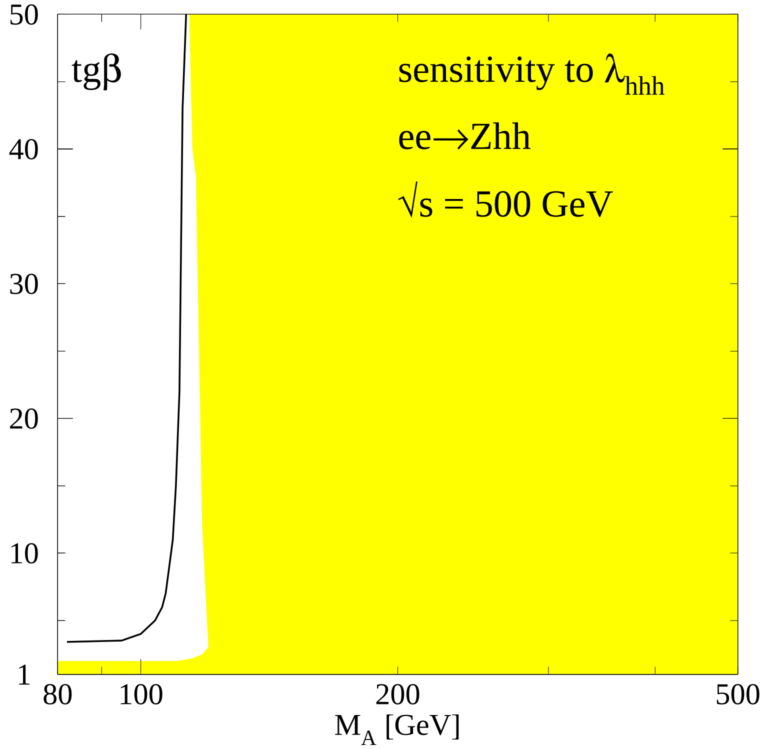


- For a SM-like Higgs boson with $m_h = 120$ GeV at $1000 fb^{-1}$, a precision of $\delta\lambda_{hhh}/\lambda_{hhh} = 23\%$ is possible.

  D.J.Miller *et al.*, hep-ph/0001194; C.Castanier *et al.*, hep-ex/0101028

- Regions of accessibility in MSSM parameters for MSSM h^0 Higgs self-couplings have been determined:

  R.Lafaye *et al.*, hep-ph/0002238; A.Djouadi, hep-ph/0001169



Radiative corrections to neutral Higgs self-couplings

- Use radiative corrections to obtain information for establishing the Higgs potential and thus the Higgs mechanism as the basic mechanism for generating the masses of the fundamental particles
- Three-point one-loop radiative corrections for the neutral Higgs system in the MSSM have been calculated within the effective potential approximation and in some limiting situations

  V. Barger et al., Phys.Rev. D45 (1992)
  P.Osland and P.N.Pandita, Phys.Rev. D59 (1999), hep-ph/9806351
- Phenomenological studies have addressed the issue of the measurements of some of the Higgs self-couplings in the MSSM

  M. Mühlleitner, hep-ph/0008127 and references therein

- Our intention is to investigate how far the MSSM Higgs potential reproduces the SM potential when the non-standard particles are heavy
- We want to explore decoupling behaviour, both numerically and analytically, of the radiative corrections to h^0 self-couplings at the one-loop level
- We have started by
  - Leading contributions from $t - \tilde{t}$ sector
    ($m_t > m_{h^0}$ in the MSSM)
  - Contributions from the Higgs sector itself.

Tree-level Higgs boson self-couplings

Trilinear and quartic SM and MSSM Higgs boson self-couplings

	ϕ	$\lambda_{\phi\phi\phi}$	$\lambda_{\phi\phi\phi\phi}$
SM	H	$\frac{3gM_H^2}{2\,M_W}$	$\frac{3g^2M_H^2}{4\,M_W^2}$
MSSM	h^o	$\frac{3gM_Z}{2c_W}\cos 2\alpha \sin(\beta+\alpha)$	$\frac{3g^2}{4c_W^2}\cos^2 2\alpha$

Decoupling limit in the Higgs sector,

Haber & Nill 1990

$M_A \gg M_Z$

$$M_{H^o} \simeq M_{H^\pm} \simeq M_A \gg M_Z$$
$$M_{h^o} \simeq M_Z |\cos 2\beta|$$

$\alpha \to \beta - \frac{\pi}{2} \quad \Rightarrow \quad \cos 2\alpha \to -\cos 2\beta \quad \sin(\beta+\alpha) \to -\cos 2\beta$

$$\Downarrow \quad \phi \equiv h^0$$

$$\lambda^0_{\phi\phi\phi} \simeq \frac{3g}{2\,M_W} M_{h^0}^{2\,\mathrm{tree}}, \quad \lambda^0_{\phi\phi\phi\phi} \simeq \frac{3g^2}{4\,M_W^2} M_{h^0}^{2\,\mathrm{tree}}$$

Tree-level couplings lead to equal results in the MSSM and in the SM in the *decoupling limit*

$\Rightarrow$ Decoupling at tree-level

One-loop contributions to the Higgs self-couplings

ANALYTICAL STUDIES

- Our intention is to investigate the differences between the corrections to the h^0 self-couplings with respect to the $H_{\rm SM}$ self-couplings
- We want to study the radiative corrections from heavy top-squarks and Higgs sector itself to the Higgs self-couplings at one-loop level
- Analytical results for the $n-$point renormalized vertex functions in the MSSM and in the SM
  Is there decoupling of heavy particles beyond tree-level?
- By using the standard on-shell renormalization procedure

A. Dabelstein, Z. Phys. **C67** (1995) 495; Nucl. Phys. **B456** (1995) 25;
M. Böhm, H. Spiesberger, W. Hollik, Fortsch. Phys. **34** (1986) 687;
W. Hollik, Fortsch. Phys. **38** (1990) 165.

- Generic R_ξ gauge
- We consider the decoupling limit

$$M_{H^0} \sim M_{H^\pm} \sim M_{A^0} \gg M_Z$$

while both the h^0 mass and the momenta of the external particles remain at the same low energy scale below M_{A^0}

and in $t - \tilde{t}$ sector:

$$m^2_{\tilde{t}_1}\,, m^2_{\tilde{t}_2} \gg M^2_Z\,, M^2_{h^0}$$

$$|m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}| \ll |m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}|$$

Higgs sector contributions

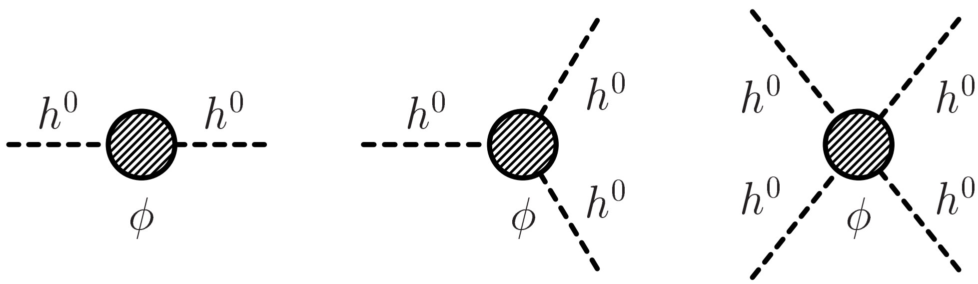
SIMPLIFICATIONS

1. We have checked that one-loop contributions from diagrams that have at least one gauge boson particles are the same in both models

   - pure gauge boson diagrams are exactly the same

   - $(\gamma, Z, W^\pm) + (H^0, H^\pm, A^0)$ one-loop diagrams are proportional to $\cos(\beta - \alpha) \to 0$ in the *decoupling limit*

2. We consider



$\phi \equiv H^0, H^\pm, A^0$ ⇔ Heavy Contributions

$\phi \equiv H^0, H^\pm, A^0$ and $G^0, G^\pm, h^0$ ⇔ Mixed Contributions

$\phi \equiv G^0, G^\pm, h^0$ (or $H^{\rm SM}$) ⇔ Light Contributions

(using path integral formulation, diagrammatic computation and *FeynArts*, *FormCalc* programs)

T.Hahn, hep-ph/0012260 , T.Hahn, M.Pérez-Victoria, Comput.Phys.Com.118 (1999) 153, hep-ph/9807565
http://www.feynarts.de

MSSM HIGGS SELF-INTERACTIONS IN THE DECOUPLING LIMIT

- One-loop contributions:

$$\Delta\Gamma^{(n)}_{h^0} = M_Z^2 \left[\mathcal{O}\left(\frac{1}{\epsilon}\right) + \mathcal{O}\left(\log\frac{M_{EW}^2}{\mu_0^2}\right) + \mathcal{O}\left(\log\frac{M_{A^0}^2}{\mu_0^2}\right) + \text{ finite terms} \right]$$
$$+ M_{A^0}^2 \left[\mathcal{O}\left(\frac{1}{\epsilon}\right) + \mathcal{O}\left(\log\frac{M_{A^0}^2}{\mu_0^2}\right) + \text{ finite terms} \right]$$

$M_{EW}^2 \equiv M_Z^2\,, M_W^2\,, M_{h^0}^2$

- All potential non-decoupling effects of heavy Higgs MSSM particles manifest as divergent contributions in $D = 4$ and some finite contributions, one of which is logarithmically dependent on M_{A^0} and the other one is quadratically dependent on M_{A^0}.

- Renormalized vertex:

$$\Delta\Gamma^{(2)}_{R\,h^0} = \Delta M_{h^0}^2\,,$$
$$\Delta\Gamma^{(3)}_{R\,h^0} = \frac{3g}{2M_Z c_W}\Delta M_{h^0}^2 + \frac{g^3}{64\pi^2 c_W^3} M_Z\,\Psi^{\rm rem}_{\rm MSSM}\,,$$
$$\Delta\Gamma^{(4)}_{R\,h^0} = \frac{3g^2}{4M_Z^2 c_W^2}\Delta M_{h^0}^2 + \frac{g^4}{64\pi^2 c_W^4}\Psi^{\rm rem}_{\rm MSSM}\,.$$

where:

$$\Delta M_{h^0}^2 = M_Z^2 \left[\mathcal{O}\left(\frac{1}{\epsilon}\right) + \mathcal{O}\left(\log\frac{M_{EW}^2}{\mu_0^2}\right) + \mathcal{O}\left(\log\frac{M_{A^0}^2}{\mu_0^2}\right) + \text{ finite terms} \right]$$

$$\Psi^{\rm rem}_{\rm MSSM} \sim \mathcal{O}\left(\log\frac{M_{EW}^2}{\mu_0^2}\right) + \text{ finite terms}$$

finite function dependent on β, and ξ-gauge dependent
come only from *light* contributions

- The quadratic heavy mass terms, $M_{A^0}^2$, disappear in the on-shell renormalization procedure

- The UV-divergence and the logarithmic dependence on $M_{A^0}^2$ can be absorbed in the redefinition of the Higgs boson mass $M_{h^0}^2$

- All potential non-decoupling effects of heavy Higgs bosons disappear, but $\Psi^{\rm rem}_{\rm MSSM}$ remains

SM HIGGS SELF-INTERACTIONS

- The renormalized Higgs propagator has a pole at $M^2_{H_{SM}}$

$$\Delta\Gamma^{(2)}_{R\,H_{SM}}(M^2_{H_{SM}}) = 0$$

- Renormalized $H_{\rm SM}$ self-couplings

$$\Delta\Gamma^{(3)}_{R\,H_{SM}} = \frac{g^3}{64\pi^2 c_W^3} M_Z\, \Psi^{\rm rem}_{SM}\,, \quad \Delta\Gamma^{(4)}_{R\,H_{SM}} = \frac{g^4}{64\pi^2 c_W^4} \Psi^{\rm rem}_{SM}\,.$$

where:

$\Psi^{\rm rem}_{SM} \;\; \longrightarrow$ finite function also ξ-gauge dependent

- Comments:

- All divergent terms dissappear in the on-shell renormalization procedure

- Some finite terms, included in $\Psi^{\rm rem}_{SM}$, remain
They are, in principle, different to $\Psi^{\rm rem}_{\rm MSSM}$

- HOWEVER, by identifying $M^2_{H_{SM}} \leftrightarrow M^{\rm tree^2}_{h^0} \simeq M_Z^2 C^2_{2\beta}$ in the *decoupling limit*, we have obtained that

$$\Psi^{\rm rem}_{SM} \longrightarrow \Psi^{\rm rem}_{\rm MSSM}$$

$\Rightarrow$ The EW-finite terms are common to both

h^0 and H_{SM}

(in the SM after renormalization of the trilinear and quartic couplings) if and only if $M_{A^0} \gg M_Z$.

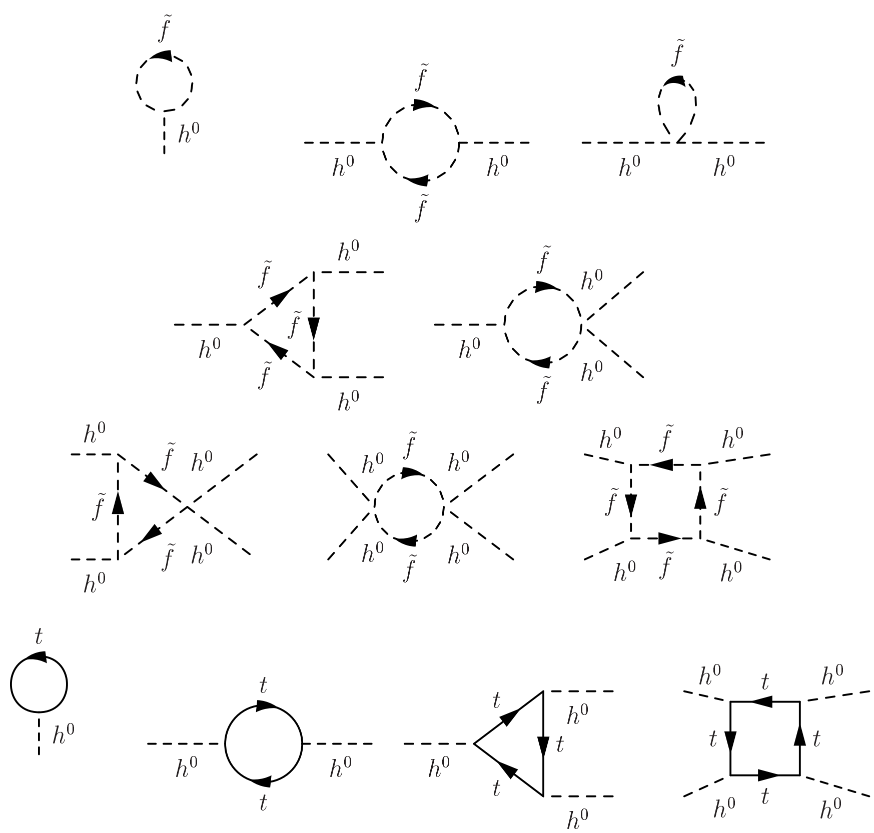
$\mathcal{O}(m_t^4)$ one-loop contributions

- We consider the large masses limit:

$$m^2_{\tilde{t}_1}\,, m^2_{\tilde{t}_2} \gg M^2_Z\,, M^2_{h^0}\,,$$

$$|m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}| \ll |m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}|\,.$$

- One-loop $t - \tilde{t}$ diagrams



Green Functions Counterterms: $\delta\Gamma^{(1)}_{h^0}\,, \delta\Gamma^{(2)}_{h^0}\,, \delta\Gamma^{(3)}_{h^0}\,, \delta\Gamma^{(4)}_{h^0}$

$$\Rightarrow\ \delta Z_{H_{1,2}}\,, \delta v\,, \delta g^2\,, \delta g'^2\,, \delta m_1^2\,, \delta m_2^2\,, \delta m_{12}^2$$

- MSSM Renormalized vertex functions

$$\Delta\hat{\Gamma}_{h^0}^{t,\tilde{t}\,(2)} = \Delta M_{h^0}^2\,,$$
$$\Delta\hat{\Gamma}_{h^0}^{t,\tilde{t}\,(3)} = \frac{3g}{2M_Z c_W}\,\Delta M_{h^0}^2 - \frac{3}{8\pi^2}\frac{g^3}{M_W^3}\,m_t^4\,,$$
$$\Delta\hat{\Gamma}_{h^0}^{t,\tilde{t}\,(4)} = \frac{3g^2}{4M_Z^2 c_W^2}\,\Delta M_{h^0}^2 - \frac{3}{4\pi^2}\frac{g^4}{M_W^4}\,m_t^4\,.$$

where: $\Delta M_{h^0}^2 = -\frac{3}{8\pi^2}\frac{g^2}{M_W^2}\,m_t^4\,\log\frac{m_t^2}{m_{\tilde{t}_1}m_{\tilde{t}_2}}$

- The UV-divergence cancel out in the renormalization procedure, such that the mass correction $\Delta M_{h^0}^2$ is finite

- The logarithmic terms in the heavy-squark masses disappear when the vertices are expressed in terms of the Higgs-boson mass $M_{h^0} \Rightarrow$ they decouple

- but lineal heavy mass terms $\mathcal{O}(m_t^4)$ remain

- Without the non-logarithmic top-mass term, the trilinear and quartic h^0 self couplings at the one-loop level have the same form as the tree level couplings, with the tree-level Higgs mass replaced by the corresponding one-loop mass $M_{h^0}^2 = M_{h^0}^{2\,\mathrm{tree}} + \Delta M_{h^0}^2$.

- SM Renormalized trilinear and quartic self-couplings

$$\Delta\hat{\Gamma}_H^{(3)} = -\frac{3g^3}{8\pi^2 M_W^3}\,m_t^4\,, \qquad \Delta\hat{\Gamma}_H^{(4)} = -\frac{3g^4}{4\pi^2 M_W^4}\,m_t^4\,.$$

The non-logarithmic top-mass terms are common to both h^0 and H_{SM} (in the SM after renormalization of the trilinear and quartic couplings).

SUMMARY

- Tree-level couplings are equal in the MSSM and in the SM in the *decoupling limit*
- Finite terms are the same in both models in the *decoupling limit* by identifying $M^2_{H_{SM}} \leftrightarrow M^{\mathrm{tree}^2}_{h^0} \simeq M_Z^2 C_{2\beta}^2$
- Difference between Renormalized MSSM and SM Higgs self-interactions

$$\Gamma^{(2)}_{R\ h^0}{}^{\mathrm{MSSM}} - \Gamma^{(2)}_{R\ H_{\mathrm{SM}}}{}^{\mathrm{SM}} = \Delta M^2_{h^0}\,,$$

$$\Gamma^{(3)}_{R\ h^0}{}^{\mathrm{MSSM}} - \Gamma^{(3)}_{R\ H_{\mathrm{SM}}}{}^{\mathrm{SM}} = \frac{3}{v}\Delta M^2_{h^0}\,,$$

$$\Gamma^{(4)}_{R\ h^0}{}^{\mathrm{MSSM}} - \Gamma^{(4)}_{R\ H_{\mathrm{SM}}}{}^{\mathrm{SM}} = \frac{3}{v^2}\Delta M^2_{h^0}\,.$$

- The one-loop MSSM contributions to the h^0 vertex functions in the asymptotic limit
  either represent a shift in the h^0 mass and in the h^0 triple and quartic self-couplings, which can be absorbed in M_{h^0},
  or reproduce the SM one-loop corrections.

$\Downarrow \quad \Downarrow \quad \Downarrow$

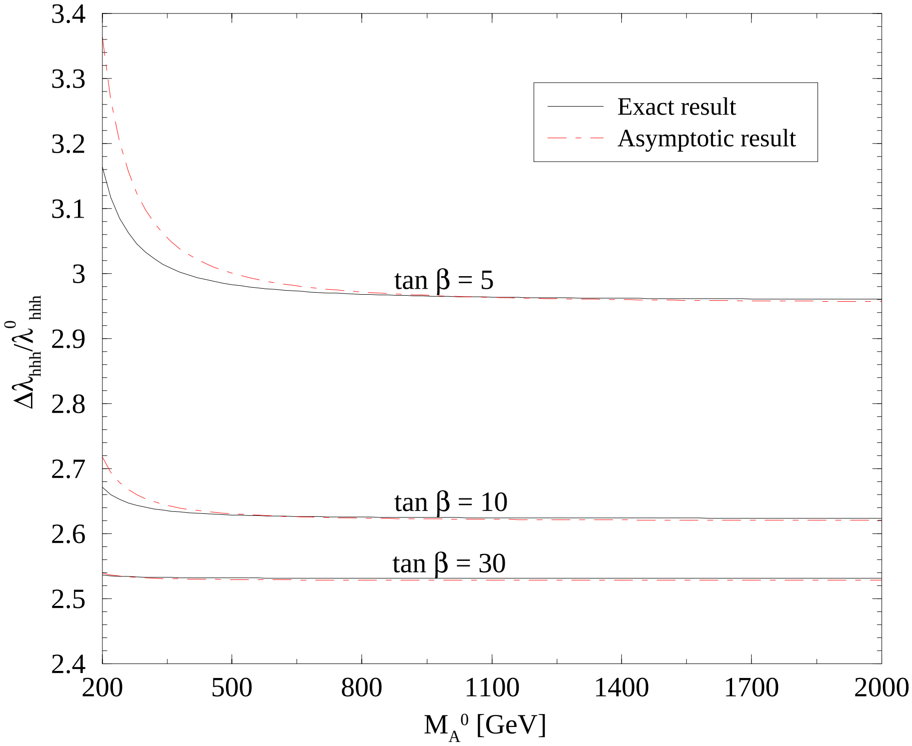
The triple and quartic h^0 couplings thereby acquire the structure of the SM Higgs-boson self-couplings.

- DECOUPLING IF AND ONLY IF $M_{A^0} \gg M_Z$

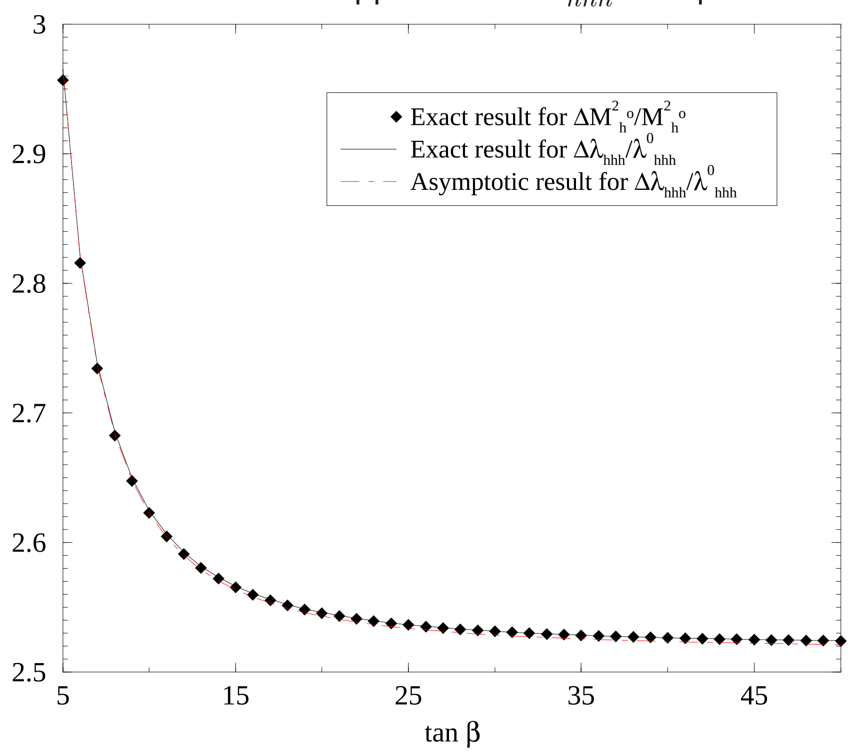
$\mathcal{O}(m_t^4)$ contributions to $\Delta\lambda_{hhh}/\lambda^0_{hhh}$

$M_{\tilde{Q}} \sim M_{\tilde{U}} \sim 15$ TeV, $\mu \sim |A_t| \sim 1.5$ TeV $\Rightarrow$ $|m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}| \ll |m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}|$

$\longrightarrow$ Large top-quark/squark corrections, even for large M_{A^0}



$\longrightarrow$ The radiative corrections dissappear when λ^0_{hhh} is expressed in terms of M_{h^0}



Trilinear h^0 self-couplings

- Exact analytical results for $t - \tilde{t}$ contributions to the trilinear h^0 self-couplings.

$$\Delta\lambda_{hhh} = \frac{3g^3}{32\pi^2}\frac{1}{M_W^3}\, m_t^4 \,\frac{\cos^3\alpha}{\sin^3\beta}\left\{3\log\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4} + \ldots\right\}$$

We agree with the results given in
V. Barger, M. S. Berger, A. L. Stange, R. J. Phillips, Phys. Rev. **D45** (1992) 4128;
P. Osland, P. N. Pandita, Phys. Rev. **D59** (1999) 055013; hep-ph/9911295; hep-ph/9902270

- Numerical analysis:

- The SUSY parameters have been taken to be

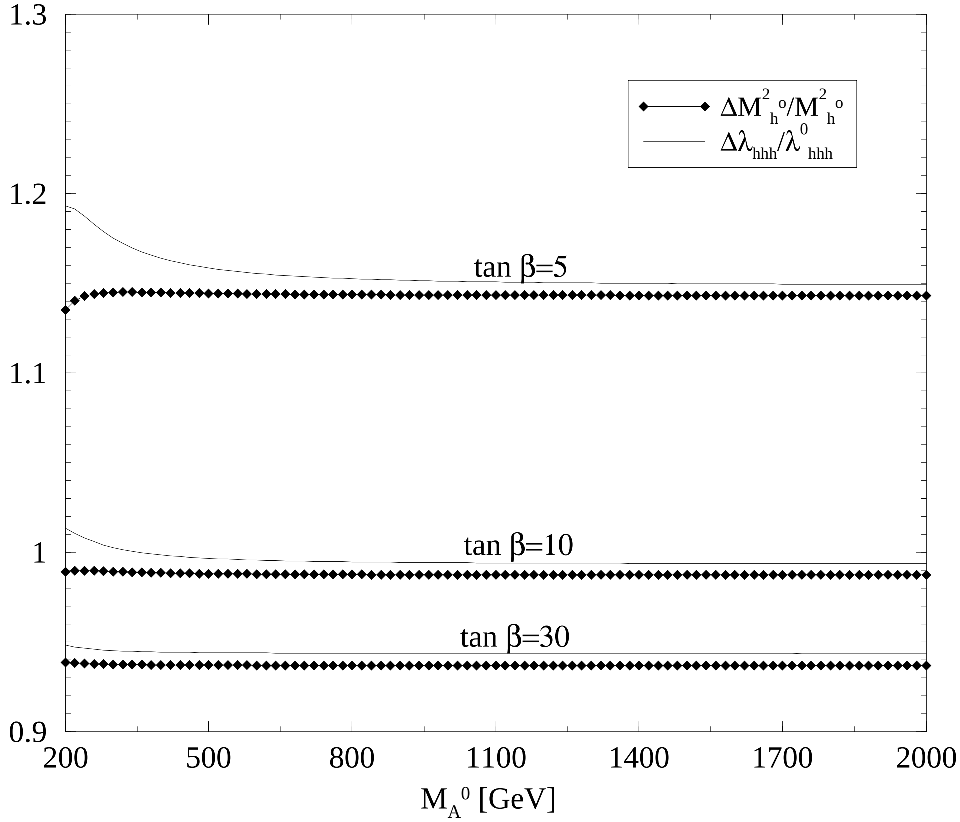
$$M_{\tilde{Q}} \sim 1 \text{ TeV}\,, \;\; M_{\tilde{U}} \sim \mu \sim |A_t| \sim 500 \text{ GeV}$$

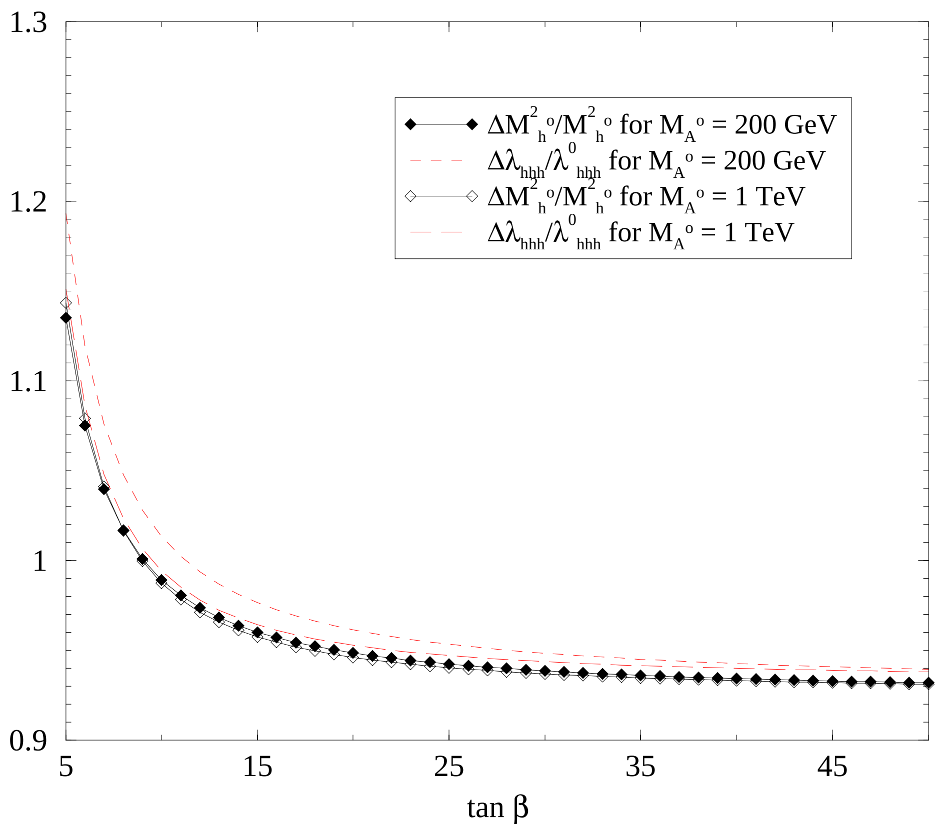
$$\Updownarrow$$

$m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$, are heavy as compared to the to the electroweak scale, but their difference is of $\mathcal{O}(M_{\tilde{U}})$

$$m_{\tilde{t}_1}^2\,, m_{\tilde{t}_2}^2 \gg M_Z^2\,, M_{h^0}^2\,,$$
$$|m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2| \simeq |m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2|\,.$$

- The radiative correction to the α angle is included.





- Large correction which decrease with $\tan\beta$
- The relation $\Delta\lambda_{hhh}/\lambda^0_{hhh} \approx \Delta M^2_{h^0}/M^{2\,\text{tree}}_{h^0}$ is fulfilled up to a small difference which remains also for large M_A

SUMMARY

- For heavy stop system with large mass splitting, $\mathcal{O}(M_{SUSY})$, the $\mathcal{O}(m_t^4)$ corrections to the trilinear h^0 self-couplings are large, but their main part can again be absorbed in the mass M_{h^0}.
- The genuine loop corrections to the triple couplings, after re-expressing them in terms of M_{h^0}, is of the order of a few per cent

  $\rightarrow$ They are largest for low $\tan\beta$ and M_{A^0}, typically 5%.

  $\rightarrow$ For large M_{A^0}, they decrease to the level of 1%.
- Not possible to measure at TESLA
- Similar results have been obtained for the quartic h^0 self-coupling.

CONCLUSIONS

- We showed analytically that Higgs sector and $\mathcal{O}(m_t^4)$ one-loop contributions to the h^0 self-couplings :

  - Decouple when the self-couplings are expressed in terms of the Higgs-boson mass, in the limit of large M_{A^0} and heavy top squarks, with masses close to each other.

  $\Rightarrow$ The triple and quartic h^0 couplings acquire the structure of the SM Higgs-boson self-couplings.

  Decoupling if and only if $M_{A^0} \gg M_Z$.

- For large mass splitting in the stop sector, the corrections to the triple couplings, after re-expressing them in terms of M_{h^0}, is of the order of a few per cent

  Examples: For low $\tan\beta$ and M_{A^0}, typically 5%.

  For large M_{A^0}, they decrease to the level of 1%.

  - Similar results have been obtained also for the quartic h^0 self-coupling.

  The h^0 self-interactions are very close to those of the SM Higgs boson for the heavy stop sector and would need high-precision experiments for their experimental verification. Not possible at TESLA

- IN PROGRESS:

  - Explore both numerically and analytically complete radiative corrections to h^0 self-couplings and Phenomenological implications