Two–loop sbottom corrections to the Higgs boson masses in the MSSM

Pietro Slavich

Physikalisches Institut der Universität Bonn
Nussallee 12, D-53115 Bonn, Germany

We present in this talk the results of a recent computation of the two–loop sbottom corrections to the MSSM Higgs masses [1]. At the tree level, the masses of the neutral CP–even Higgs bosons of the MSSM can be computed in terms of three input parameters: the mass m_A of the neutral CP–odd particle, the mass m_Z of the weak neutral gauge boson, and the ratio of Higgs vacuum expectation values $\tan\beta \equiv v_2/v_1$. For $\tan\beta \ll m_t/m_b$, the dominant one–loop corrections are the $\mathcal{O}(\alpha_t)$ ones, where $\alpha_t \equiv h_t^2/(4\pi)$ and h_t is the superpotential top coupling. Such coupling controls both the top–Higgs Yukawa couplings and a number of cubic and quartic stop–Higgs scalar couplings, and leads to significant contributions from both top and stop loops [2]. The $\mathcal{O}(\alpha_b)$ one–loop corrections associated with the superpotential bottom coupling h_b, where $\alpha_b \equiv h_b^2/(4\pi)$, can be numerically non–negligible only for $\tan\beta \gg 1$ and sizeable values of the μ parameter. At the classical level $h_b/h_t = (m_b/m_t)\tan\beta$, thus we need $\tan\beta \gg 1$ to have $\alpha_b \sim \alpha_t$ in spite of $m_b \ll m_t$. Moreover, and in contrast with the top–stop case, numerically relevant contributions can only come from sbottom loops: those coming from bottom loops are always suppressed by the small value of the bottom mass. A sizeable value of μ is then required to have sizeable sbottom–Higgs scalar interactions in the large $\tan\beta$ limit.

We are now at the stage where the most important genuine two–loop corrections are being evaluated: general results have been obtained both for the $\mathcal{O}(\alpha_t\alpha_s)$ [3, 4, 5] and for the $\mathcal{O}(\alpha_t^2)$ [3, 6, 7] corrections. In Ref. [1] we moved one step further, computing the $\mathcal{O}(\alpha_b\alpha_s)$ corrections and discussing the $\mathcal{O}(\alpha_b^2)$ and $\mathcal{O}(\alpha_t\alpha_b)$ ones. For convenience, we evaluated two–loop effects directly in the physically relevant limit of large $\tan\beta$:

$$v_1 \to 0\,, \qquad v_2 \to v \equiv (\sqrt{2}\,G_\mu)^{-1/2}\,, \tag{1}$$

where G_μ is the Fermi constant. As a result, we obtained extremely compact analytical formulae. Keeping $v_1 \neq 0$ would only generate more complicated expressions, without adding any relevant information.

The momentum–independent part of the one–loop $\mathcal{O}(\alpha_b)$ and two–loop $\mathcal{O}(\alpha_b\alpha_s)$ corrections to the neutral CP–even Higgs boson mass matrix can be obtained by taking the second derivatives of the effective potential [1] at its minimum, or by performing appropriate substitutions and limits in the $\mathcal{O}(\alpha_t\alpha_s)$ results of [5]. In the limit of Eq. (1), we find:

$$\left(\Delta\mathcal{M}^2_S\right)^{\rm eff}_{11} = \frac{1}{2}\, h_b^2\, s^2_{2\theta_b}\left[A_b^2\left(F^{1\ell}+F^{2\ell}\right)+2\,A_b\,m_{\tilde g}\,G^{2\ell}\right], \tag{2}$$

$$\left(\Delta\mathcal{M}^2_S\right)^{\rm eff}_{12} = \frac{1}{2}\, h_b^2\, s^2_{2\theta_b}\left[\mu\,A_b\left(F^{1\ell}+F^{2\ell}\right)+\mu\,m_{\tilde g}\,G^{2\ell}\right], \tag{3}$$

$$\left(\Delta\mathcal{M}^2_S\right)^{\rm eff}_{22} = \frac{1}{2}\, h_b^2\, s^2_{2\theta_b}\,\mu^2\left(F^{1\ell}+F^{2\ell}\right). \tag{4}$$

Our conventions are such that, at the classical level, the top and bottom quark masses are given by $m_t = h_t v_2/\sqrt{2}$ and $m_b = h_b v_1/\sqrt{2}$, where the Yukawa couplings (h_t, h_b) and the VEVs (v_1, v_2) are all taken to be real and positive. In addition, we assume μ and A_b to be real, but we do not make any assumption on their sign, whereas we choose the gluino mass $m_{\tilde g}$ to be real and positive. At the classical level, the sbottom mixing angle $s_{2\theta_b} \equiv \sin 2\theta_{\tilde b}$ is given by

$$s_{2\theta_b} = \frac{\sqrt{2}\,h_b(A_b\,v_1+\mu\,v_2)}{m^2_{\tilde b_1}-m^2_{\tilde b_2}} \longrightarrow \frac{\sqrt{2}\,h_b\,\mu\,v}{m^2_{\tilde b_1}-m^2_{\tilde b_2}}, \tag{5}$$

where the arrow denotes the large $\tan\beta$ limit, and $m^2_{\tilde b_1} > m^2_{\tilde b_2}$ are the two eigenvalues of the sbottom mass matrix. Finally, the superscripts in the functions (F, G) indicate the order of the loop contribution. At one loop, and in the large $\tan\beta$ limit, the only relevant function is

$$F^{1\ell} = \frac{N_c}{16\,\pi^2}\left(2-\frac{m^2_{\tilde b_1}+m^2_{\tilde b_2}}{m^2_{\tilde b_1}-m^2_{\tilde b_2}}\,\ln\frac{m^2_{\tilde b_1}}{m^2_{\tilde b_2}}\right), \tag{6}$$

where $N_c = 3$ is a color factor. Notice that $F^{1\ell}$ is negative definite.

In Ref. [1], we started our discussion by presenting the results for $F^{2\ell}$ and $G^{2\ell}$ in the $\overline{\rm DR}$ scheme, i.e. assuming that the $\mathcal{O}(\alpha_b)$ one–loop contribution is written entirely in terms of $\overline{\rm DR}$ parameters (masses and couplings), evaluated at a certain renormalization scale Q. This way of presenting the results is convenient for analysing models that predict, via the MSSM renormalization group equations, the low–energy $\overline{\rm DR}$ values of the MSSM input parameters in terms of a more restricted set of parameters, assigned as boundary conditions at some scale much larger than the weak scale. General low–energy analyses of the MSSM parameter space, however, do not refer to boundary conditions at high scales. These analyses are usually performed in terms of parameters with a more direct physical interpretation, such as pole masses and appropriately defined mixing angles in the squark

[1]The effective potential for vanishing CP–odd fields was computed in [6]. To make contact with the physical m_A, the effective potential should be computed as a function of both CP–even and CP–odd fields, as in [5].

sector. Such an approach requires modifications of our two–loop $\overline{\rm DR}$ formulae, induced by the variation of the one–loop parameters when moving from the $\overline{\rm DR}$ scheme to a different scheme. We recall that, at the one–loop level, the two VEVs (v_1, v_2) and the mass parameter μ are not renormalized by the strong interactions. Therefore, the only parameters in the Higgs mass matrix that require a one–loop definition are $(h_b,\, A_b,\, s_{2\theta_b}, m_{\tilde{b}_1},\, m_{\tilde{b}_2})$, although only four of these are independent, because of the relation (5).

The sbottom masses $(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2})$ in Eq. (6) can be naturally identified with the pole masses. For the generic parameter x, we define the shift from the $\overline{\rm DR}$ value $\hat{x}$ as $\delta x \equiv \hat{x}-x$. According to this definition, we find $\delta m^2_{\tilde{b}_i} \equiv \Pi_{ii}(m^2_{\tilde{b}_i})$, where $\Pi_{ij}(p^2)$ denotes the real and finite part of the (ij) component of the sbottom self–energy $(i, j = 1, 2)$. Explicitly, in units of $g_s^2\, C_F/(16\pi^2)$:

$$\begin{aligned} \delta m^2_{\tilde{b}_1} \;\; = \;\; m^2_{\tilde{b}_1} \Bigg[& 3 \,\ln \frac{m^2_{\tilde{b}_1}}{Q^2} - 3 - c^2_{2\theta_b} \left(\ln \frac{m^2_{\tilde{b}_1}}{Q^2} - 1 \right) - s^2_{2\theta_b} \frac{m^2_{\tilde{b}_2}}{m^2_{\tilde{b}_1}} \left(\ln \frac{m^2_{\tilde{b}_2}}{Q^2} - 1 \right) \\ & -6 \frac{m^2_{\tilde{g}}}{m^2_{\tilde{b}_1}} - 2 \left(1 - 2 \frac{m^2_{\tilde{g}}}{m^2_{\tilde{b}_1}} \right) \ln \frac{m^2_{\tilde{g}}}{Q^2} - 2 \left(1 - \frac{m^2_{\tilde{g}}}{m^2_{\tilde{b}_1}} \right)^2 \ln \left| 1 - \frac{m^2_{\tilde{b}_1}}{m^2_{\tilde{g}}} \right| \Bigg] \,, \end{aligned} \qquad (7)$$

and $\delta m^2_{\tilde{b}_2}$ is obtained from Eq. (7) by the interchange $m^2_{\tilde{b}_1} \leftrightarrow m^2_{\tilde{b}_2}$.

The most convenient definition of $(h_b, A_b, s_{2\theta_b})$ is less easily singled out. To clarify this point, we recall the parallel case of the $\mathcal{O}(\alpha_t\alpha_s)$ corrections. In that case, besides the stop pole masses, the remaining independent parameters are chosen to be [4, 5] a conveniently defined stop mixing angle, $s_{2\theta_t}$, and the top Yukawa coupling h_t^{pole}, as defined by the top pole mass M_t via the relation $M_t \equiv h_t^{pole} v_2/\sqrt{2}$. Then, the stop counterpart of Eq. (5) is used to establish the one–loop definition of A_t in terms of the pole top and stop masses and of the stop mixing angle. In the case of the $\mathcal{O}(\alpha_b\alpha_s)$ corrections, a similar procedure is not appropriate since, as can be easily seen from Eq. (5), $s_{2\theta_b}$ is independent of A_b in the large $\tan\beta$ limit. A second complication arises from the large one–loop threshold corrections [8] proportional to v_2 that contribute to the pole bottom mass: for our calculation, the relevant ones are those $\mathcal{O}(\alpha_s)$, associated with one–loop SQCD diagrams with gluinos and sbottom quarks on the internal lines. As noticed in [9], a definition of A_b in terms of the pole bottom and sbottom masses through Eq. (5) would produce very large shifts in A_b with respect to its $\overline{\rm DR}$ value, $\delta A_b = \mathcal{O}(\alpha_s\, \mu^2\, \tan^2\beta/m_{\tilde{g}})$. A $\overline{\rm DR}$ definition for the parameters $(h_b, A_b, s_{2\theta_b})$ would avoid this problem, but would still suffer from the known fact that it does not make manifest the decoupling of heavy particles, for example a heavy gluino.

We then look for definitions of the relevant parameters that automatically include the decoupling of heavy gluinos, allow to disentangle the genuine two–loop effects from the large threshold corrections to the bottom mass, and provide a consistent prescription for A_b in the large $\tan\beta$ limit. A suitable definition of the mixing angle θ_b, with the virtue of being infrared (IR) finite and gauge–independent with respect to the strong interaction, is [10]:

$$\delta\theta_{\tilde{b}} = \frac{1}{2}\,\frac{\Pi_{12}(m^2_{\tilde{b}_1}) + \Pi_{12}(m^2_{\tilde{b}_2})}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}}\,, \tag{8}$$

where $\Pi_{12}(p^2)$ turns out to be independent of p^2 in the large $\tan\beta$ limit. Since v and μ are not renormalized by the strong interactions, Eqs. (5) and (7) can be used, in the large $\tan\beta$ limit, to translate the prescription for $\theta_{\tilde{b}}$ into a prescription for h_b. Explicitly, in units of $g_s^2\,C_F/(16\pi^2)$:

$$\frac{\delta h_b}{h_b} = -4 + 2\,\ln\frac{m^2_{\tilde{g}}}{Q^2} + \left[\frac{2\,m^2_{\tilde{b}_1}}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}}\left(2\,\ln\frac{m^2_{\tilde{b}_1}}{m^2_{\tilde{g}}} - \left(1 - \frac{m^2_{\tilde{g}}}{m^2_{\tilde{b}_1}}\right)^2 \ln\left|1 - \frac{m^2_{\tilde{b}_1}}{m^2_{\tilde{g}}}\right|\right) + (1 \leftrightarrow 2)\right]\,. \tag{9}$$

We stress that our renormalized h_b, as defined above, differs at the one–loop level both from the $\overline{\rm DR}$ quantity $\hat{h}_b$ and from the quantity h_b^{pole} that would be obtained by plugging the pole bottom mass, M_b, into the tree–level formula: $h_b \neq h_b^{pole} \equiv \sqrt{2}M_b/v_1$.

Concerning the definition of A_b, we observe that the Yukawa coupling h_b multiplying A_b can be absorbed in a redefinition of the trilinear soft–breaking term, $\widetilde{A}_b \equiv h_b A_b$. The shift in $\widetilde{A}_b$ could be defined via a physical process, e.g. one of the decays $\widetilde{b}_1 \to \widetilde{b}_2\,A$ or $A \to \widetilde{b}_1\,\widetilde{b}_2^*$, but such a definition would suffer from the problem of infrared (IR) singularities associated with gluon radiation. To overcome this problem, and given our ignorance of the MSSM spectrum, we find less restrictive to define $\delta\widetilde{A}_b$ in terms of the $(\widetilde{b}_1\widetilde{b}_2^*A)$ proper vertex, at appropriately chosen external momenta and including suitable wave function corrections, so that the resulting combination is IR finite and gauge–independent, and gives rise to an acceptable heavy gluino limit. Denoting the proper vertex $\widetilde{b}_1\widetilde{b}_2^*A$ with $i\Lambda_{12A}(p_1^2, p_2^2, p_A^2)$, we define [2]:

$$\begin{aligned}\delta\widetilde{A}_b \;&=\; -\frac{i}{\sqrt{2}}\left[\Lambda_{12A}(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_1}, 0) + \Lambda_{12A}(m^2_{\tilde{b}_2}, m^2_{\tilde{b}_2}, 0)\right] \\ &\quad + \frac{1}{2}\,\widetilde{A}_b\,\frac{\Pi_{11}(m^2_{\tilde{b}_1}) + \Pi_{22}(m^2_{\tilde{b}_1}) - \Pi_{11}(m^2_{\tilde{b}_2}) - \Pi_{22}(m^2_{\tilde{b}_2})}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}}\,. \end{aligned} \tag{10}$$

Writing $\delta\widetilde{A}_b = \delta h_b\,A_b + h_b\,\delta A_b$, we find, in units of $g_s^2\,C_F/(16\pi^2)$:

$$\delta A_b = 2\,m_{\tilde{g}}\left\{4 - 2\,\ln\frac{m^2_{\tilde{g}}}{Q^2} - \left[\left(1 - \frac{m^2_{\tilde{g}}}{m^2_{\tilde{b}_1}}\right)\ln\left|1 - \frac{m^2_{\tilde{b}_1}}{m^2_{\tilde{g}}}\right| + (1 \leftrightarrow 2)\right]\right\}\,. \tag{11}$$

With our one–loop specifications of h_b and A_b, Eqs. (9) and (11), the CP–even Higgs boson mass matrix takes again the form of Eqs. (2)–(4), but the one–loop part of the corrections must now be evaluated in our renormalization scheme, and the functions $F^{2\ell}$

[2]This definition is suitable at $\mathcal{O}(\alpha_s)$. It can be generalized to the case of Yukawa corrections by specifying a prescription for the A wave function.

and $G^{2\ell}$ read, in units of $g_s^2\, C_F\, N_c/(16\,\pi^2)^2$:

$$
\begin{aligned}
F^{2\ell} \;=& -(1+s^2_{2\theta_b})\left(2-\frac{m^2_{\tilde b_1}+m^2_{\tilde b_2}}{m^2_{\tilde b_1}-m^2_{\tilde b_2}}\ln\frac{m^2_{\tilde b_1}}{m^2_{\tilde b_2}}\right)^2-\left(\ln\frac{m^2_{\tilde b_1}}{m^2_{\tilde b_2}}\right)^2\\
&+4-2\left(\frac{m^2_{\tilde g}}{m^2_{\tilde b_1}}+\frac{m^2_{\tilde g}}{m^2_{\tilde b_2}}\right)+4\left(\ln\frac{m^2_{\tilde b_1}}{m^2_{\tilde g}}+\ln\frac{m^2_{\tilde b_2}}{m^2_{\tilde g}}\right)\\
&+4\frac{m^2_{\tilde b_1}+m^2_{\tilde b_2}-2\,m^2_{\tilde g}}{m^2_{\tilde b_1}-m^2_{\tilde b_2}}\left[\mathrm{Li}_2\left(1-\frac{m^2_{\tilde b_1}}{m^2_{\tilde g}}\right)-\mathrm{Li}_2\left(1-\frac{m^2_{\tilde b_2}}{m^2_{\tilde g}}\right)-\frac{1}{2}\ln\frac{m^2_{\tilde b_1}}{m^2_{\tilde b_2}}\right]\\
&-2\left[\left(1+\frac{2\,m^2_{\tilde b_1}}{m^2_{\tilde b_1}-m^2_{\tilde b_2}}-\frac{2\,m^4_{\tilde b_1}}{(m^2_{\tilde b_1}-m^2_{\tilde b_2})^2}\ln\frac{m^2_{\tilde b_1}}{m^2_{\tilde b_2}}\right)\left(1-\frac{m^2_{\tilde g}}{m^2_{\tilde b_1}}\right)^2\ln\left|1-\frac{m^2_{\tilde b_1}}{m^2_{\tilde g}}\right|+(1\leftrightarrow 2)\right],
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
G^{2\ell} \;=& \;4\,\ln\frac{m^2_{\tilde b_1}m^2_{\tilde b_2}}{m^4_{\tilde g}}+4\,\frac{m^2_{\tilde b_1}+m^2_{\tilde b_2}-2\,m^2_{\tilde g}}{m^2_{\tilde b_1}-m^2_{\tilde b_2}}\left[\mathrm{Li}_2\left(1-\frac{m^2_{\tilde b_1}}{m^2_{\tilde g}}\right)-\mathrm{Li}_2\left(1-\frac{m^2_{\tilde b_2}}{m^2_{\tilde g}}\right)\right]\\
&-2\left(2-\frac{m^2_{\tilde b_1}+m^2_{\tilde b_2}}{m^2_{\tilde b_1}-m^2_{\tilde b_2}}\ln\frac{m^2_{\tilde b_1}}{m^2_{\tilde b_2}}\right)\left[\left(1-\frac{m^2_{\tilde g}}{m^2_{\tilde b_1}}\right)\ln\left|1-\frac{m^2_{\tilde b_1}}{m^2_{\tilde g}}\right|+(1\leftrightarrow 2)\right].
\end{aligned}
\tag{13}
$$

Notice that, in this scheme, $F^{2\ell}$ and $G^{2\ell}$ do not depend explicitly on Q. We also stress that, in terms of our renormalized quantities $(m^2_{\tilde b_1}, m^2_{\tilde b_2}, h_b, A_b)$, the corrections have a smooth heavy gluino limit. In fact, in contrast with the case of the $\mathcal{O}(\alpha_t\alpha_s)$ corrections, the gluino decouples for $m_{\tilde g}\to\infty$, since $m_{\tilde g}\,G^{2\ell}\to 0$ and $F^{2\ell}$ reduces to the first line of Eq. (12).

Phenomenological analyses of the MSSM parameter space should exploit the experimental information on the bottom mass. Instead of expressing such information with the pole mass M_b, it is convenient to use directly the running mass, in the SM and in the $\overline{\rm DR}$ scheme, evaluated at the reference scale $Q_0 = 175$ GeV. Following a procedure outlined in [11], we take as input the SM bottom mass in the $\overline{\rm MS}$ scheme, $m_b(m_b)^{\overline{\rm MS}}_{\rm SM} = 4.23\pm0.08$ GeV, as determined from the Υ masses [12]; we evolve it up to the scale Q_0 by means of suitable renormalization group equations; finally, we convert it to the $\overline{\rm DR}$ scheme. The result, which accounts for the resummation of the universal large QCD logarithms, is $\overline{m}_b \equiv m_b(Q_0)^{\overline{\rm DR}}_{\rm SM} = 2.74\pm0.05$ GeV. The relation between $\hat h_b \equiv h_b(Q_0)^{\overline{\rm DR}}_{\rm MSSM}$ and $\overline{m}_b$ is given by:

$$
\hat h_b \equiv h_b(Q_0)^{\overline{\rm DR}}_{\rm MSSM} = \frac{\overline{m}_b\sqrt{2}}{v_1}\,\frac{1+\delta_b}{|1+\epsilon_b|}\,,
\tag{14}
$$

where

$$
\delta_b \;=\; \frac{\alpha_s}{3\pi}\left\{\frac{3}{2}-\ln\frac{m^2_{\tilde g}}{Q_0^2}\right.
$$

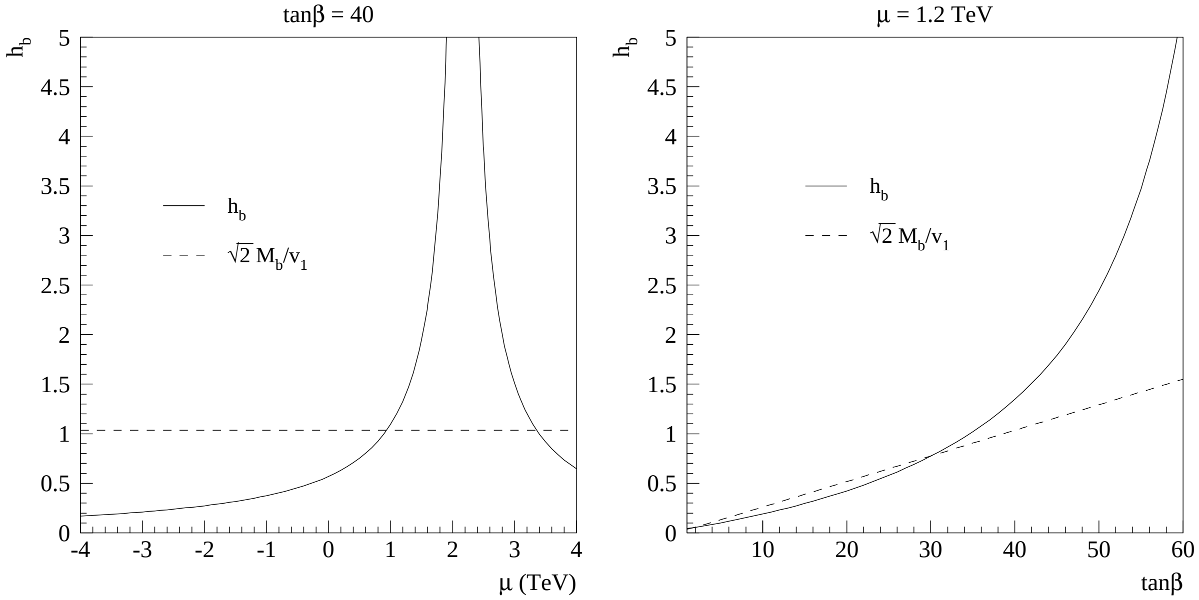

Figure 1: The Yukawa coupling h_b, as defined in Eq. (14): as a function of μ for $\tan\beta = 40$ (left panel); as a function of $\tan\beta$ for $\mu = 1.2$ TeV (right panel). The other parameters are $A_b = 2$ TeV, $m_Q = m_D = m_{\tilde{g}} = 1$ TeV. The quantity $h_b^{pole} \equiv \sqrt{2}\, M_b/v_1$ is also shown for comparison.

$$
+\frac{1}{2}\left[\frac{m^2_{\tilde{b}_1}}{m^2_{\tilde{g}} - m^2_{\tilde{b}_1}}\left(1 - \left(\frac{2\, m^2_{\tilde{g}} - m^2_{\tilde{b}_1}}{m^2_{\tilde{g}} - m^2_{\tilde{b}_1}} - \frac{4\, m_{\tilde{g}} A_b}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}}\right)\ln\frac{m^2_{\tilde{g}}}{m^2_{\tilde{b}_1}}\right) + (1 \leftrightarrow 2)\right]\Bigg\}\ , \qquad (15)
$$

and

$$
\epsilon_b \;=\; -\frac{2\,\alpha_s}{3\,\pi}\,\frac{m_{\tilde{g}}\,\mu\,\tan\beta}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}}\left[\frac{m^2_{\tilde{b}_1}}{m^2_{\tilde{b}_1} - m^2_{\tilde{g}}}\ln\frac{m^2_{\tilde{b}_1}}{m^2_{\tilde{g}}} - \frac{m^2_{\tilde{b}_2}}{m^2_{\tilde{b}_2} - m^2_{\tilde{g}}}\ln\frac{m^2_{\tilde{b}_2}}{m^2_{\tilde{g}}}\right]. \qquad (16)
$$

The running parameter $\hat{h}_b$ is the appropriate input quantity to be used with the $\overline{\rm DR}$ result, while the formulae obtained in our renormalization scheme should be used with $h_b = \hat{h}_b - \delta h_b$, evaluating Eq. (9) for $Q = Q_0$. Notice that in Eq. (14) the large $\mathcal{O}(\alpha_s)$ threshold corrections [8] parametrized by ϵ_b have been resummed to all orders as in [13].

We are now ready for some numerical examples. To prepare the ground, we study the variation of our renormalized h_b with respect to other parameters, keeping the reference bottom mass $\overline{m}_b$ fixed to 2.74 GeV. The left panel of Fig. 1 shows h_b as a function of μ (solid line), for $\tan\beta = 40$. The other relevant parameters are chosen as $A_b = 2$ TeV, $m_Q = m_D = m_{\tilde{g}} = 1$ TeV, where m_Q and m_D are soft supersymmetry–breaking masses. The quantity $h_b^{pole} = \sqrt{2}\, M_b/v_1$ is also shown as a dashed line. The curve corresponding to $\hat{h}_b$ would be very close to that of h_b, thus we do not display it. We see that having large values of $\tan\beta$ and μ is a necessary but not sufficient condition for having a sizeable h_b:

when the threshold contribution to the bottom mass dominates, $|\epsilon_b| \gg 1$, h_b must decrease for increasing values of $|\mu| \tan\beta$. We also see that, when there is an almost complete destructive interference between the two contributions to the bottom mass, $\epsilon_b \simeq -1$, the correct value of the bottom mass cannot be reproduced by the one–loop formula for h_b in the perturbative regime, and the corresponding set of MSSM parameters must be discarded. Finally, we can see that the renormalized h_b can be large only for positive [3] values of μ. We then focus our attention on the case in which μ is large and positive, so that h_b and the corresponding corrections to the Higgs masses can be sizeable.

For completeness, we should mention (for recent discussions and references, see e.g. [14]) that models with b–τ Yukawa coupling unification at the GUT scale favour, in our conventions, a positive sign of $\mu\, m_{\tilde{g}}$, which leads to a negative ϵ_b. For sufficiently small $|\mu|$, radiative B decays and the muon anomalous magnetic moment may favour a negative sign of $\mu\, M_2$, where M_2 is the $SU(2)$ gaugino mass, and a positive sign of $\mu\, A_t$. Similar but more model–dependent constraints can be extracted, with the help of additional assumptions on the soft supersymmetry–breaking terms, from the cosmological relic density.

The right panel of Fig. 1 shows h_b as a function of $\tan\beta$, for $\mu = 1.2$ TeV. Again, the curve for $\hat{h}_b$ would be practically indistinguishable and we do not show it. The other parameters are chosen as in the left panel, and the value of h_b^{pole} is also shown. We can see that, for this choice of parameters (to be taken in the following as a representative one), values of $\tan\beta$ much larger than 40–50 would imply a value of h_b beyond the perturbative regime. On the other hand, for low values of $\tan\beta$ the coupling h_b is even smaller than h_b^{pole}, and the corresponding corrections to the Higgs masses are expected to be negligible. For this reason, in the numerical examples of the $\mathcal{O}(\alpha_b\alpha_s)$ corrections we restrict ourselves to values of $\tan\beta$ between 25 and 45. Fig. 2 shows the light Higgs mass m_h as a function of $\tan\beta$ for $\mu = 1.2$ TeV. The left panel corresponds to $m_A = 120$ GeV and the right panel to $m_A = 1$ TeV . The other input parameters are chosen as $A_t = A_b = 2$ TeV, $m_Q = m_U = m_D = m_{\tilde{g}} = 1$ TeV. The curves in Fig. 2 correspond to the one–loop corrected m_h at $\mathcal{O}(\alpha_t)$ (long–dashed line) and at $\mathcal{O}(\alpha_t + \alpha_b)$ (dot–dashed line), and to the two–loop corrected m_h at $\mathcal{O}(\alpha_t\alpha_s)$ (short–dashed line) and at $\mathcal{O}(\alpha_t\alpha_s + \alpha_b\alpha_s)$ (solid line), respectively. We can see from Fig. 2 that, while the $\mathcal{O}(\alpha_t)$ prediction for m_h is practically independent of $\tan\beta$ for $\tan\beta > 25$, the $\mathcal{O}(\alpha_b)$ corrections lower m_h considerably when $\tan\beta$ increases. This effect is enhanced by the steep dependence of the renormalized coupling h_b on $\tan\beta$, depicted in Fig. 1. Comparing the solid and the short–dashed curves, we can see that the 'genuine' two–loop $\mathcal{O}(\alpha_b\alpha_s)$ corrections to the Higgs mass, given by Eqs. (2)–(4) and (12)–(13), are usually a small fraction of the $\mathcal{O}(\alpha_b)$ ones, but the former can still reach several GeV when the latter are very large. In particular, for small m_A the $\mathcal{O}(\alpha_b\alpha_s)$ corrections can be comparable in magnitude with the $\mathcal{O}(\alpha_t\alpha_s)$ ones. We stress that the absence of very large two–loop effects from the sbottom sector is a consequence of our renormalization prescription, which allows to set apart the $\tan\beta$–

[3] Our convention for the sign of μ is implicitly defined in Eq. (5).

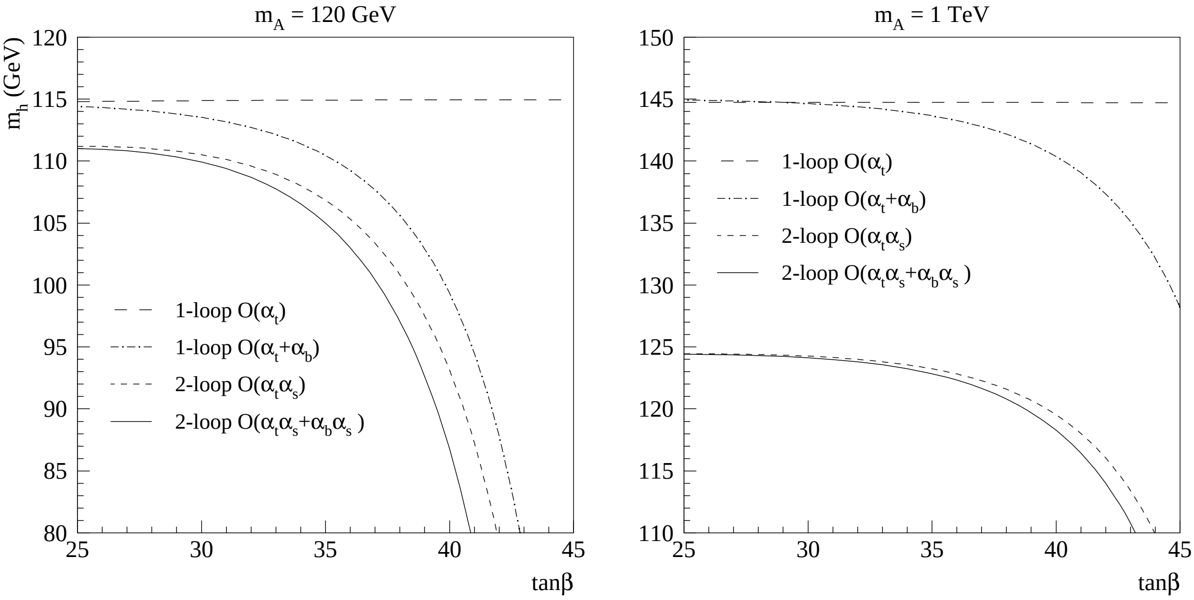

Figure 2: The mass m_h as a function of $\tan\beta$, for $m_A = 120$ GeV (left panel) or 1 TeV (right panel). Other parameters are $\mu = 1.2$ TeV, $A_t = A_b = 2$ TeV, $m_Q = m_U = m_D = m_{\tilde{g}} = 1$ TeV.

enhanced corrections, resummed to all orders in the renormalized coupling h_b. If we were to adopt for the sbottom sector the same renormalization scheme that we use for the stop sector, the dependence on $\tan\beta$ of the one–loop corrected m_h would be smoother, but very large corrections (growing as $\tan^2\beta$) would appear at two loops, questioning the validity of the perturbative expansion.

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