

# OVERVIEW OF $\tan\beta$ DETERMINATION AT A LINEAR $e^+e^-$ COLLIDER

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## Abstract

The ratio of the vacuum expectation value of the two Higgs doublets,  $\tan\beta$ , is an important parameter of the general 2-Higgs-Doublet Model (2HDM) and the Minimal Supersymmetric extension of the Standard Model (MSSM). The expected uncertainty on the determination of  $\tan\beta$  at a Linear Collider (LC) of at least 500 GeV center-of-mass energy and high luminosity is reviewed based on studies of neutral and charged Higgs boson production.

## Introduction

Various methods to determine  $\tan\beta$  at a LC exist and they have in common that a physical observable depends on  $\tan\beta$ :

- The pseudoscalar Higgs boson, A, could be produced via radiation off a pair of b-quarks:  $e^+e^- \rightarrow b\bar{b} \rightarrow b\bar{b}A \rightarrow b\bar{b}b\bar{b}$ . The  $b\bar{b}A$  coupling is proportional to  $\tan\beta$  and thus the expected production rate is proportional to  $\tan^2\beta$ .
- The  $b\bar{b}b\bar{b}$  rate from the pair-production of the heavier scalar, H, in association with the pseudoscalar Higgs boson  $e^+e^- \rightarrow HA \rightarrow b\bar{b}b\bar{b}$  can be exploited. While the HA production rate is almost independent of  $\tan\beta$  the sensitivity occurs via the variation of the decay branching ratios with  $\tan\beta$ .
- The value of  $\tan\beta$  can also be determined from the H and A decay widths, which can be obtained from the previously described reaction.
- The  $t\bar{t}t\bar{t}$  rate from charged Higgs boson production can contribute to the determination of  $\tan\beta$  from the reaction  $e^+e^- \rightarrow H^+H^- \rightarrow t\bar{t}t\bar{t}$  because of the charged Higgs boson branching ratio variation with  $\tan\beta$ .
- In addition, the charged Higgs boson total decay width depends on  $\tan\beta$ .

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## The $b\bar{b}A \rightarrow b\bar{b}b\bar{b}$ bremsstrahlung process

The experimental challenge of this study is the low expected production rate and the large irreducible background for a four-jet final state, as discussed in a previous simulation [1]. The expected background rate for a given  $b\bar{b}A \rightarrow b\bar{b}b\bar{b}$  signal efficiency is shown in Fig. 1. Taking a working point of 10% efficiency, we estimate the statistical error in determining  $\tan\beta$  by  $\Delta \tan^2\beta / \tan^2\beta = \Delta S/S = \sqrt{S+B}/S = \sqrt{200}/100 \approx 0.14$ , resulting in an error on  $\tan\beta = 50$  of 7%. In the MSSM, the  $b\bar{b}h$  signal would essentially double the number of signal events and have exactly the same  $\tan\beta$  dependence, yielding  $\Delta \tan^2\beta / \tan^2\beta \approx \sqrt{300}/200 \approx 0.085$  for  $\tan\beta = 50$  and the  $\tan\beta$  error would be about 4%. Systematic errors arising from interference with the  $hA \rightarrow b\bar{b}b\bar{b}$  reaction can be controlled [2].

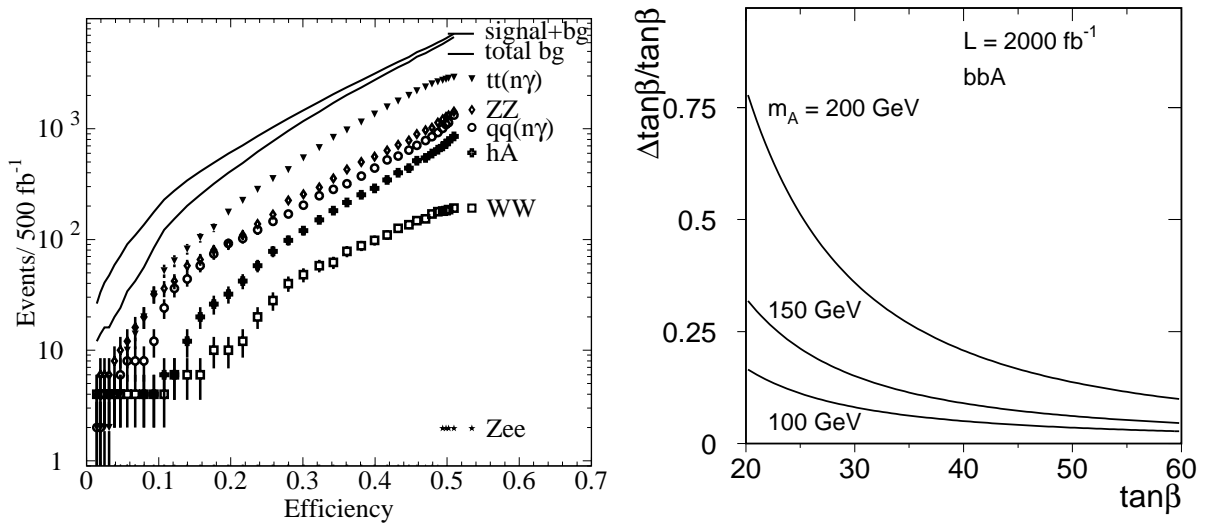


Figure 1: Left: Final background rate versus  $b\bar{b}A$  signal efficiency for  $m_A = 100$  GeV,  $\sqrt{s} = 500$  GeV and  $\mathcal{L} = 500$  fb $^{-1}$ . Right: Corresponding  $\tan\beta$  statistical error for  $\mathcal{L} = 2000$  fb $^{-1}$  and  $m_A = 100, 150, 200$  GeV. For both plots, we take a fixed value of  $m_b = 4.62$  GeV.

## HA production: branching ratios and decay widths

The branching ratios for H, A decay to various allowed modes vary rapidly with  $\tan\beta$  in the MSSM when  $\tan\beta$  is below 20. Consequently, if these branching ratios can be measured accurately,  $\tan\beta$  can be determined with good precision in this range. As the H and A decay rates depend on the MSSM parameters, two cases are considered. In scenario (I), SUSY decays of the H and A are kinematically forbidden. Scenario (II) is taken from [3] in which SUSY decays (mainly to  $\tilde{\chi}_1^0\tilde{\chi}_1^0$ ) are allowed. We assume event selection criteria with an event selection efficiency of 10% and negligible background, based on the expected b-tagging performance and kinematic event selection. The expected  $HA \rightarrow b\bar{b}b\bar{b}$  event rates and  $1\sigma$  statistical bounds are shown in Fig. 2 as a function of  $\tan\beta$  for  $\sqrt{s} = 500$  GeV and  $\mathcal{L} = 2000$  fb $^{-1}$ . The resulting bounds for  $\tan\beta$  are plotted in Fig. 3 (right) for MSSM scenarios (I) and (II).

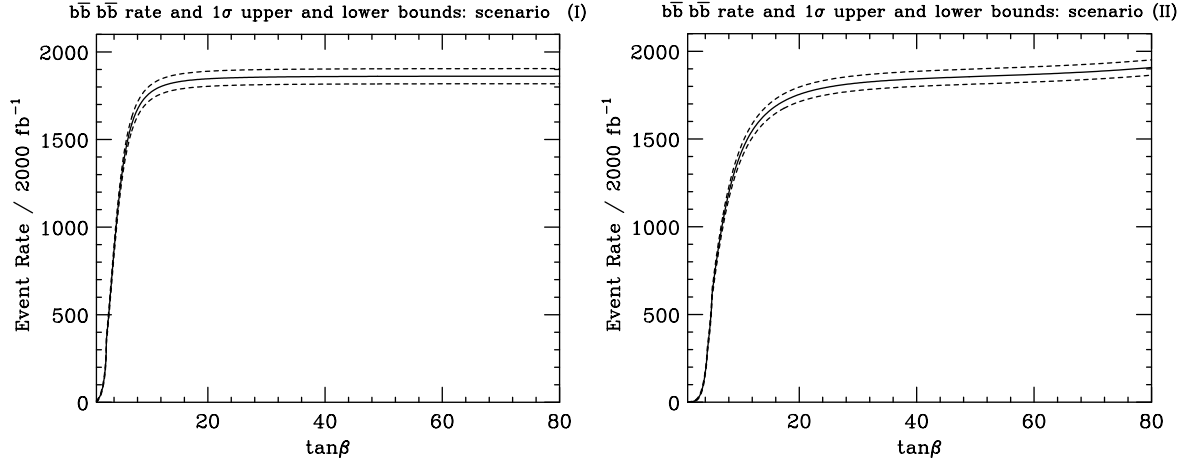


Figure 2: Expected  $e^+e^- \rightarrow HA \rightarrow b\bar{b}b\bar{b}$  event rates for 10% efficiency and  $\pm 1\sigma$  bounds in scenarios (I) and (II) in the MSSM for  $m_A = 200$  GeV,  $\sqrt{s} = 500$  GeV and  $\mathcal{L} = 2000 \text{ fb}^{-1}$ .

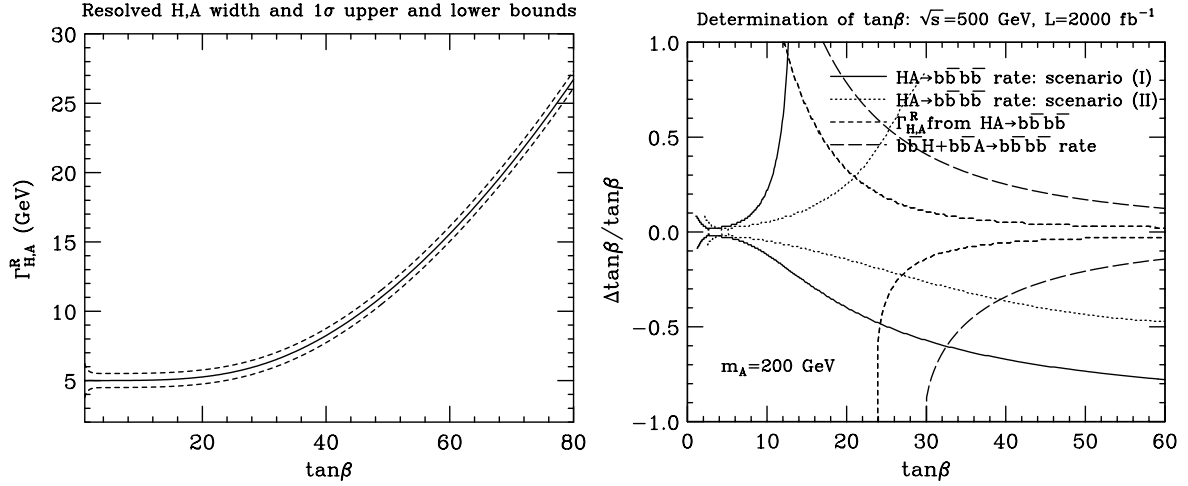


Figure 3: Left: Expected resolved width  $\Gamma_{H,A}^R$ , Eq. (1), for scenario (I) and  $1\sigma$  upper and lower bounds with 10% selection efficiency. The statistical bounds include an additional efficiency factor of 0.75 for keeping only events in the central mass peak and assume a detector resolution of  $\Gamma_{\text{res}} = 5$  GeV with a 10% uncertainty. Right: Expected precision on  $\tan\beta$  ( $1\sigma$  bounds) for  $m_A = 200$  GeV,  $\sqrt{s} = 500$  GeV and  $\mathcal{L} = 2000 \text{ fb}^{-1}$  based on the  $e^+e^- \rightarrow b\bar{b}A + b\bar{b}H \rightarrow b\bar{b}b\bar{b}$  rate, the  $e^+e^- \rightarrow HA \rightarrow b\bar{b}b\bar{b}$  rate and  $\Gamma_{H,A}^R$ .

A previous HA simulation [5] indicates that about 25% of the time wrong jet-pairings are made, which are attributed to the wings of the mass distribution. The  $m_{b\bar{b}}$  values from H and A decays are binned in a single distribution, since the H and A mass splitting is typically substantially smaller than the detector resolution of  $\Gamma_{\text{res}} = 5$  GeV for the large  $\tan\beta$  values considered. Our effective observable is the resolved average width defined by

$$\Gamma_{H,A}^R = \frac{1}{2} \left[ \sqrt{[\Gamma_{\text{tot}}^H]^2 + [\Gamma_{\text{res}}]^2} + \sqrt{[\Gamma_{\text{tot}}^A]^2 + [\Gamma_{\text{res}}]^2} \right]. \quad (1)$$

Its dependence on  $\tan\beta$  is shown in Fig. 3 for  $m_H \approx m_A = 200$  GeV in MSSM scenario (I) and it is very similar for scenario (II).

In order to extract the implied  $\tan\beta$  bounds, we must account for the fact that the detector resolution will not be precisely determined. There will be a systematic uncertainty which we have estimated at 10% of  $\Gamma_{\text{res}}$ , i.e. 0.5 GeV. This systematic uncertainty considerably weakens our ability to determine  $\tan\beta$  at the lower values of  $\tan\beta$  for which  $\Gamma_{\text{tot}}^{\text{H}}$  and  $\Gamma_{\text{tot}}^{\text{A}}$  are smaller than  $\Gamma_{\text{res}}$ . This systematic uncertainty should be carefully studied as part of future experimental analyses. Figure 3 shows also the expected  $\pm 1\sigma$  experimental errors based on the measurement of  $\Gamma_{\text{H,A}}^{\text{R}}$ . An excellent determination of  $\tan\beta$  will be possible at high  $\tan\beta$ . The  $\text{b}\bar{\text{b}}\text{H/A}$  and  $\text{H/A}$  width methods are nicely complementary in their  $\tan\beta$  coverage to the  $\tan\beta$  determination based on the  $\text{HA} \rightarrow \text{b}\bar{\text{b}}\text{b}\bar{\text{b}}$  rate method at lower  $\tan\beta$ .

## $\text{H}^+\text{H}^-$ production: branching ratios and decay widths

The reaction  $\text{e}^+\text{e}^- \rightarrow \text{H}^+\text{H}^- \rightarrow \text{t}\bar{\text{b}}\text{t}\bar{\text{b}}$  can be observed at a LC [6] and recent high-luminosity simulations [7] show that precision measurements can be performed. As soon as the charged Higgs boson decay into  $\text{tb}$  is allowed this decay mode is dominant. Nonetheless,  $\text{BR}(\text{H}^\pm \rightarrow \text{tb})$  varies significantly with  $\tan\beta$ , especially for small values of  $\tan\beta$  where the  $\text{tb}$  mode competes with the  $\tau\nu$  mode. The  $\text{H}^+ \rightarrow \text{t}\bar{\text{b}}$  branching ratio and width are sensitive to  $\tan\beta$  in the form  $\Gamma(\text{H}^\pm \rightarrow \text{tb}) \propto m_{\text{t}}^2 \cot^2\beta + m_{\text{b}}^2 \tan^2\beta$ . As in the previous section, we use HDECAY [4] (which incorporates the running of the b-quark mass) to evaluate the charged Higgs boson branching ratios and decay widths. The  $\text{tb}$  partial width and the corresponding branching ratio have a minimum in the vicinity of  $\tan\beta \approx 6 - 8$ . In contrast to the variation of the branching ratio, the cross section for  $\text{e}^+\text{e}^- \rightarrow \text{H}^+\text{H}^-$  production is largely independent of  $\tan\beta$ .

Our procedures for estimating errors for the  $\text{t}\bar{\text{b}}\text{t}\bar{\text{b}}$  rate and for the total width are similar to those given earlier for  $\text{HA}$  production rate and width in the  $\text{b}\bar{\text{b}}\text{b}\bar{\text{b}}$  channel. For  $m_{\text{H}^\pm} = 300$  GeV at  $\sqrt{s} = 800$  GeV, a  $\text{H}^+\text{H}^-$  study [7] finds that the  $\text{t}\bar{\text{b}}\text{t}\bar{\text{b}}$  final state can be isolated with an efficiency of 2.2%. For  $m_{\text{H}^\pm} = 200$  GeV and  $\sqrt{s} = 500$  GeV, we have adopted the same 2.2% efficiency and negligible background. Figure 4 shows the resulting  $\text{t}\bar{\text{b}}\text{t}\bar{\text{b}}$  rates and  $1\sigma$  bounds for MSSM scenarios (I) and (II). The corresponding bounds on  $\tan\beta$  are shown in Fig. 5 (right).

For the total width determination, we assume that we keep only 75% of the events after cuts (i.e. a fraction  $0.75 \times 0.022$  of the raw event number), corresponding to throwing away wings of the mass peaks, and each  $\text{t}\bar{\text{b}}\text{t}\bar{\text{b}}$  event is counted twice since we can look at both the  $\text{H}^+$  and the  $\text{H}^-$  decay. We define a resolved width which incorporates the detector resolution  $\Gamma_{\text{res}} = 5$  GeV:

$$\Gamma_{\text{H}^\pm}^{\text{R}} = \sqrt{[\Gamma_{\text{tot}}^{\text{H}^\pm}]^2 + [\Gamma_{\text{res}}]^2}. \quad (2)$$

Estimated errors are based on the width measurement for 10% systematic error in  $\Gamma_{\text{res}} = 0.5$  GeV. The resolved width  $\Gamma_{\text{H}^\pm}^{\text{R}}$  for scenario (I) is given in Fig. 5 and it is very similar for scenario (II). It also shows resulting  $\tan\beta$  bounds. In comparison to the neutral Higgs boson methods (Fig. 3), we observe that for MSSM scenario (I) the  $\text{t}\bar{\text{b}}\text{t}\bar{\text{b}}$  rate measurement gives a  $\tan\beta$  determination that is quite competitive with that from  $\text{HA}$  production in

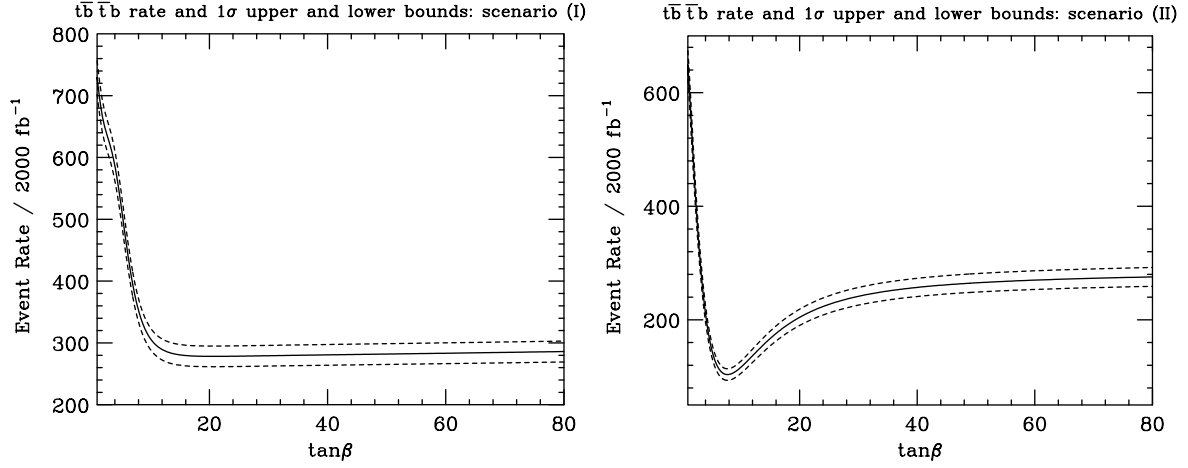


Figure 4: Expected  $e^+e^- \rightarrow H^+H^- \rightarrow t\bar{t}t\bar{t}$  event rates for 2.2% efficiency and  $\pm 1\sigma$  bounds in scenarios (I) and (II) in the MSSM for  $m_A = 200$  GeV,  $\sqrt{s} = 500$  GeV and  $\mathcal{L} = 2000 \text{ fb}^{-1}$ .

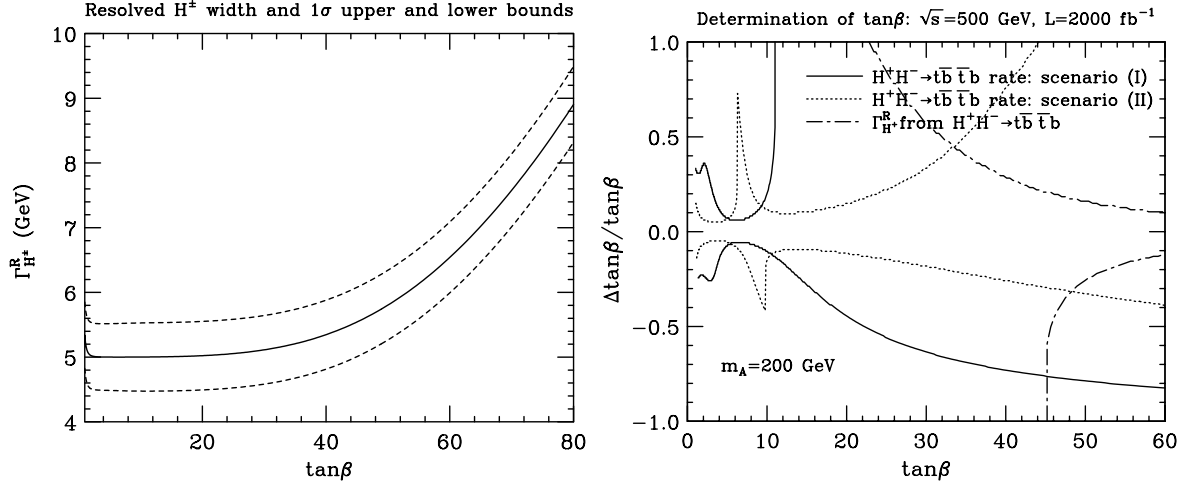


Figure 5: Left: Expected resolved width  $\Gamma_{H^\pm}^R$ , Eq. (2), for scenario (I) and  $1\sigma$  upper and lower bounds with 2.2% selection efficiency. The statistical bounds include an additional efficiency factor of 0.75 for keeping only events in the central mass peak and assume  $\Gamma_{\text{res}} = 5$  GeV with a 10% uncertainty. Right: Expected precision on  $\tan\beta$  ( $1\sigma$  bounds) for  $m_{H^\pm} \approx m_A = 200$  GeV,  $\sqrt{s} = 500$  GeV and  $\mathcal{L} = 2000 \text{ fb}^{-1}$  based on the  $e^+e^- \rightarrow H^+H^- \rightarrow t\bar{t}t\bar{t}$  rate and  $\Gamma_{H^\pm}^R$ .

the  $b\bar{b}b\bar{b}$  final state. For MSSM scenario (II), the  $t\bar{t}t\bar{t}$  rate gives an even better  $\tan\beta$  determination than does the  $b\bar{b}b\bar{b}$  rate. On the other hand, the width measurement from the  $t\bar{t}t\bar{t}$  final state of  $H^+H^-$  production is much poorer than that from the  $b\bar{b}b\bar{b}$  final state of  $HA$  production.

By combining the  $\tan\beta$  errors from all processes in quadrature we obtain the expected net errors on  $\tan\beta$  shown in Fig. 6 for MSSM scenarios (I) and (II). The Higgs sector will provide an excellent determination of  $\tan\beta$  at small and large  $\tan\beta$  values. However, larger bounds are expected for moderate  $\tan\beta$  in scenario (II) where SUSY decays of

the  $A, H, H^\pm$  are not significant. Further information on  $\tan\beta$  could be obtained from the reaction  $e^+e^- \rightarrow t\bar{t} \rightarrow tbH^\pm \rightarrow tb\tau\nu$ , further Higgs decay branching ratios (e.g.  $H \rightarrow WW, ZZ, hh$ ;  $A \rightarrow Zh$ ;  $H, A \rightarrow \text{SUSY particles}$ ), the  $H/A$  decay width from  $b\bar{b}H/A$  production, and the polarization of scalar taus.

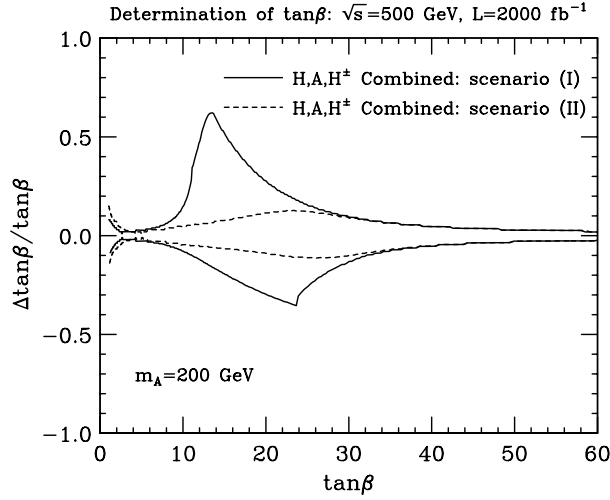


Figure 6: Expected combined precision on  $\tan\beta$  ( $1\sigma$  bounds) for  $m_{H^\pm} \approx m_A = 200$  GeV,  $\sqrt{s} = 500$  GeV and  $\mathcal{L} = 2000 \text{ fb}^{-1}$  based on combining (in quadrature) the results shown in Figs. 3 and 5 for the  $b\bar{b}H/A$  rate, the  $HA \rightarrow b\bar{b}b\bar{b}$  rate,  $\Gamma_{H,A}^R$ , the  $t\bar{t}t\bar{t}$  rate and  $\Gamma_{H^\pm}^R$ . Higher order calculations in the MSSM could influence the combination of the different  $\tan\beta$  methods.

## Conclusions

A high-luminosity linear  $e^+e^-$  collider will provide a precise measurement of the value of  $\tan\beta$  throughout most of the large possible  $\tan\beta$  range  $1 < \tan\beta < 60$ . In particular, we have demonstrated the complementarity of employing: a) the  $b\bar{b}A + b\bar{b}H \rightarrow b\bar{b}b\bar{b}$  rate; b) the  $HA \rightarrow b\bar{b}b\bar{b}$  rate; c) the average  $H, A$  total width from  $HA$  production; d) the  $H^+H^- \rightarrow t\bar{t}t\bar{t}$  rate; and e) the  $H^\pm$  total width from  $H^+H^- \rightarrow t\bar{t}t\bar{t}$  production. Experimental challenges will be the required high total luminosity, an excellent b-tagging performance, and precision detector resolution and selection efficiency determinations.

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