[Hollik, Kraus, Roth, Rupp, Sibold, DS, 2002]
[A. Freitas, DS, 2002]

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$\tan\beta$

- $\tan\beta$: central input parameter
- must be renormalized

Aim: Find a good renormalization scheme for $\tan\beta$

Important: gauge dependence of $\tan\beta$

Outline:

- gauge dependence of well-known schemes
- gauge-indep. schemes $\longrightarrow$ drawbacks
- "No-Go"-Theorem, no ideal scheme
- Conclusions

$\tan\beta$

Lowest order: $\tan\beta = \frac{v_2}{v_1}$

Higher orders: renormalization scheme determines

- relation to observables
- numerical value $(\tan\beta)_{\text{exp}}$
- gauge dependence, ...

Three desirable properties of ren. schemes:

- Gauge independence
- Numerical stability
- Process independence

Process dependent: e.g.

$$\tan^2\beta \stackrel{!}{=} \text{const} \times \Gamma(H^+ \to \tau^+ \nu_\tau)$$

Gauge dependence

Known schemes:

$\overline{DR}$:	$\delta \tan\beta^{\text{fin}} \stackrel{!}{=} 0$
Dabelstein, Chankowski et al	$\hat{\Sigma}_{A^0 Z}(M_A^2) \stackrel{!}{=} 0$

Good: $\tan\beta$ defined in the Higgs sector, technically easy

Bad: no direct relation to observables,
$\longrightarrow$ gauge dependent?

Gauge dependence

Determination of ξ-dependence:

extended STI $\tilde{S}(\Gamma) = S(\Gamma) + \chi\partial_\xi\Gamma \stackrel{!}{=} 0$

$\Rightarrow$ $(\tan\beta)_{\text{exp}}$ ξ-indep.

ξ-dependence of the $\overline{DR}$-scheme

$$\begin{array}{llll} \overline{DR}: & & (\forall\xi) & \delta\tan\beta^{\text{fin}} \stackrel{!}{=} 0 \\ \text{but} & \tilde{S}(\Gamma) = 0 \Rightarrow & & \partial_\xi\delta\tan\beta \neq 0 \end{array}$$

$\Rightarrow$ Contradiction!

Dabelstein, . . . : similarly

Good luck:

no contradiction in $\overline{DR}$, R_ξ-gauge, 1-Loop

Result:

$\overline{DR}$, Dabel,. . .-schemes are ξ-dep.!

(for 2-Loop: *[Yamada01]*)

ξ-independent schemes

Alternative (?): Devise ξ-indep. schemes:

(1) Tadpole scheme:

$$\delta t_\beta^{\text{fin}} \stackrel{!}{=} \text{const.} \left(\frac{\delta t_1}{v_1} + \frac{\delta t_2}{v_2} \right)$$

(2) m_3-scheme:

$$\delta m_3^{\text{fin}} \stackrel{!}{=} 0$$

(3) HiggsMass-scheme:

$$\cos^2(2\beta) \stackrel{!}{=} \frac{M_h^2 M_H^2}{M_A^2(M_h^2 + M_H^2 - M_A^2)}$$

$\Rightarrow \tilde{S}(\Gamma) = 0$, $(\tan\beta)_{\text{exp}}$ ξ-indep.

Result:
Process-indep. and gauge-indep. schemes exist

Numerical properties

Drawback of ξ-indep. schemes

- e.g. $\bar{\mu}$-dependence

$\frac{d}{d \log \bar{\mu}} \tan\beta$: $(m_h^{\text{max}} -$ Scenario$)$

	$\overline{DR}$	(1)	(2)
$\tan\beta = 3$	-0.1	4.5	0.8
$\tan\beta = 50$	-0.2	370.7	285.3

Result:
Numerically unstable, useless in practice !

- M_h (1-Loop): $(m_h^{\text{max}}, \tan\beta = 3)$

$\overline{DR}$	Dabel,...	(2)	(3)
134.63	134.44	173.45	119.58

“No-Go”-Theorem

Generalization:

If $\tan\beta$ is defined in the Higgs sector,

$$\delta\tan\beta^{\text{fin}} = \text{linear combination of}$$
$$\Big(\Sigma_{A^0A^0}(M_A^2), \Sigma_{A^0G^0}(M_A^2), \Sigma_{A^0Z}(M_A^2),$$
$$\Sigma_{HH}(M_H^2), \Sigma_{hh}(M_h^2), \Sigma_{Hh}(M_{H,h}^2),$$
$$\Sigma'_{A^0A^0}(M_A^2), \Sigma'_{HH}(M_H^2), \Sigma'_{hh}(M_h^2),$$
$$\delta t_1, \delta t_2\Big)^{\text{fin}}$$

and gauge-independent, then
always a numerical instability is introduced

[Freitas, DS, 02]

Alternative

Three desirable properties of ren. schemes:

(G) Gauge independence
(N) Numerical stability
(P) Process independence

Negative results:

($\overline{DR}$, Dabel,...): not (G)

(G), (P) schemes: exist

(G), (P) schemes: numerically unstable

Alternative (?): process-dep. schemes: (G),(N)

e.g. $\tan^2\beta \stackrel{!}{=} \text{const.} \times \Gamma(A^0 \to \tau\tau)$

or $\tan^2\beta \stackrel{!}{=} \text{const.} \times \Gamma(H^+ \to \tau^+\nu)$

Drawbacks: technically complicated (IR-Div. QED-corrections in 2nd process), flavour dependent, not unique

Conclusions

Gauge-indep. schemes in the Higgs sector useless

$\overline{DR}$, Dabel,... useful but gauge dependent

NB: $\overline{DR}$, R_ξ-gauge: gauge-indep. at 1-Loop

[Yamada, 01]

$\overline{DR}$: numerically most well-behaved

[Frank, Heinemeyer, Hollik, Weiglein, 02]

$(A^0 \to \tau\tau)$-scheme possible but has drawbacks

best compromise: $\overline{DR}$-scheme

- numerically most well-behaved,
- gauge indep. (1-Loop, R_ξ-gauge)
- technically easiest