Higgs boson mass limits in perturbative unification theories *

Kazuhiro Tobe

Michigan Center for Theoretical Physics, Department of Physics,
University of Michigan, Ann Arbor, MI 48109, USA

Motivated in part by recent demonstrations that electroweak gauge coupling unification may occur at a low scale, we analyze the requirements on the Higgs mass if the gauge coupling unification is to be perturbative. We consider an infinite set of possible unification scenarios parametrized by the value of $\sin^2\theta_W$ at the unification scale Λ, and show the Higgs boson mass limits based on the minimal standard model, the standard model with the fourth generation, the minimal SUSY standard model, and the next-to-minimal SUSY standard model with an additional singlet field. One of our interesting results is that the Higgs boson can be as heavy as 500 GeV in the non-SUSY and non-minimal SUSY models if the unification scale is low ($\Lambda \sim 1-10$ TeV). Future Higgs discovery will tell us which unification scenarios are consistent with the perturbativity of the theory.

I. INTRODUCTION

The standard model (SM) has been tested by various experiments up to energy scale of order of a few hundred GeV, and it has been remarkably successful. However, the Higgs boson has not been discovered yet. Precision measurements have constrained the Higgs boson mass to be [2, 3]:

$$\log_{10}(m_h/\mathrm{GeV}) = 1.93^{+0.21}_{-0.23}, \quad \text{or} \quad m_h < 196 \text{ GeV} \ \ (95\% \text{ C.L.}). \tag{1}$$

Direct searches at LEP experiments have constrained the Higgs boson to have mass above 114.1 GeV at the 95 % C.L. [2, 4]. The remaining window of possible Higgs boson mass is relatively narrow. Nevertheless, a light Higgs boson below 196 GeV is not guaranteed. Precision electroweak data is sensitive mostly to the logarithm of the Higgs boson. From Eq. (1) we can deduce that 3σ (4σ) upper bound on the Higgs boson mass is about 363 GeV (589 GeV). Therefore, a heavier Higgs is not totally excluded. Furthermore, it is possible that a much heavier Higgs boson can conspire with new states to be compatible with the precision electroweak data [5]. For example, in the existence of the fourth generation fermions, a heavier Higgs ($m_h \sim 500$ GeV) is compatible with the precision electroweak data [6, 7]. Therefore, at this stage, it will be important to discuss how heavy (or light) Higgs is required from some theoretical considerations.

---

* This talk is based on work in collaboration with James D. Wells [1].

Unfortunately, the Higgs boson mass cannot be predicted in the SM. Since the Higgs boson mass is expressed by a certain coupling constant in general, we can obtain a Higgs mass limit if we can constrain the Higgs coupling constant.

In our analysis, in order to constrain the Higgs coupling, we consider the "perturbativity" of the theory. Within the framework of the gauge coupling unification theories, the perturbativity of the theory is important in order to successfully predict low energy gauge coupling constants from the relation among the gauge couplings at unification scale. There are several interesting gauge coupling unification models. The most well-known unification scenario is the grand unified theory (GUT) in which all gauge couplings are unified at very high scale and $\sin^2\theta_W = 3/8$ ($g_2 = \sqrt{5/3}g'$) at the unification scale. Another interesting scenario is $SU(3)$ electroweak unification [8–11] (for some early attempt, see [12–14]) in which the hypercharge and $SU(2)_L$ couplings are unified at a few TeV, that is, $\sin^2\theta_W = 1/4$ ($g_2 = \sqrt{3}g'$) at the unification scale. Here we consider an infinite set of possible unification scenarios parametrized by the value of $\sin^2\theta_W$ at the unification scale Λ. Most of the previous discussions on Higgs boson mass limits have relied strongly on high-energy unification ($\Lambda \sim 10^{16}$ GeV). In our analysis, the unification scale Λ ranges from ~ 1 TeV to $\sim 10^{18}$ GeV, and we show the Higgs boson mass limits requiring that the theory should be perturbative up to the unification scale Λ.

II. NON-SUPERSYMMETRIC UNIFICATION

A. Minimal standard model

In the SM, the Higgs mass squared is proportional to the Higgs self-coupling λ:

$$m_h^2 = \lambda v^2 \tag{2}$$

where v is a vacuum expectation value of Higgs field. Requiring the theory remain perturbative up to some scale Λ implies that the Higgs self-coupling $\lambda < \lambda_0$, where λ_0 is a non-perturbative value for the coupling constant. This perturbativity provides the Higgs boson mass upper limit.

To define the perturbativity of the Higgs self-coupling, we consider its β function:

$$\mu\frac{d\lambda}{d\mu} = \sum_i \beta_\lambda^{(i\,\mathrm{loop})} = \frac{L_1}{16\pi^2}\lambda^2 + \frac{L_2}{(16\pi^2)^2}\lambda^3 + \frac{L_3}{(16\pi^2)^3}\lambda^4 + \cdots, \tag{3}$$

where in the $\overline{\mathrm{MS}}$ scheme, $L_{1,2,3} = 12$, -78, and $897 + 504\zeta(3) \simeq 1503$, respectively [15, 16]. We then identify the onset of non-perturbativity as when any higher loop order contribution to the β function exceeds the value of any lower loop order contribution. That is,

$$\left|\beta_\lambda^{(j>i)}\right| < \left|\beta_\lambda^{(i)}\right| \quad \text{(perturbativity condition)} \tag{4}$$

implies perturbative coupling, and violation of the condition implies non-perturbative coupling. Using up to three loop results, we obtain $\lambda_0 = 8.2$.

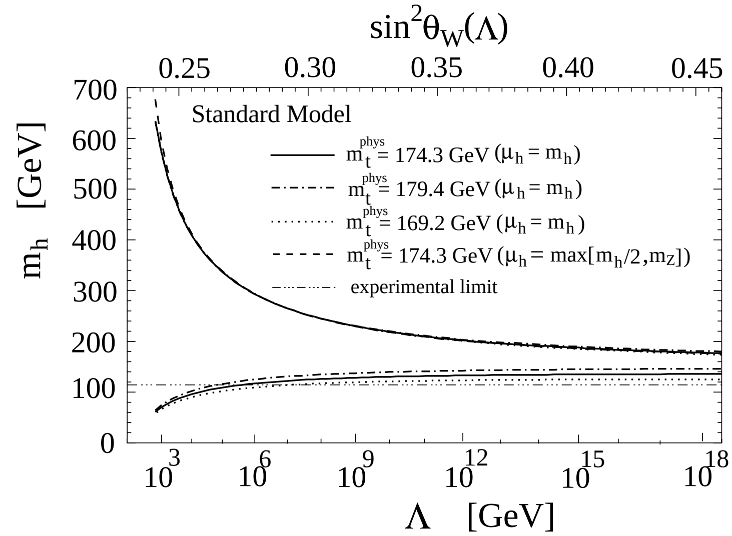


FIG. 1: The upper curve is the limit on the Higgs boson mass within the Standard Model such that the Higgs self-coupling remains perturbative, i.e., $\lambda < \lambda_0 = 8.2$, up the scale Λ. The lower curve is the Higgs limit such that $\lambda > 0$ for all scales below Λ. The x-axis can be equivalently expressed as Λ or the directly correlated value of $\sin^2\theta_W(\Lambda)$ which is labeled above. For detail, see Ref. [1].

In Fig. 1, we show the upper limit on the Higgs boson mass that can be expected in a theory that remain perturbative (i.e., $\lambda < \lambda_0 = 8.2$) up to the unification scale Λ. Similar SM analyses can be found in [17], where the perturbativity condition is different from ours. The correspondence between scale Λ and the value of $\sin^2\theta_W(\Lambda) = g'^2(\Lambda)/(g'^2(\Lambda)+g_2^2(\Lambda))$ at that scale is plotted on this same graph. There are three cases of particular interest in this graph. These are the perturbative upper limit for m_h when $\Lambda = M_{\rm Pl} = 2.4\times10^{18}$ GeV; the perturbative range for m_h at the scale where $\sin^2\theta_W = 3/8$, which should be close to the unification scale of simple $SU(5)$ or $SO(10)$ GUTs [27]; and, the perturbative upper limit for m_h at the scale where $\sin^2\theta_W = 1/4$, which is relevant for $SU(3)$ electroweak unification. We summarize the results of these three possibilities,

$$\sin^2\theta_W = 1/4 \;\Rightarrow\; \Lambda = 3.8\,\mathrm{TeV}, m_h < 460\,\mathrm{GeV}$$
$$\sin^2\theta_W = 3/8 \;\Rightarrow\; \Lambda \simeq 10^{13}\,\mathrm{GeV}, m_h < 200\,\mathrm{GeV}$$
$$\Lambda = M_{Pl} \;\Rightarrow\; m_h < 180\,\mathrm{GeV}.$$

B. Standard model with fourth generation

It has been pointed out that in the existence of the fourth generation of lepton (N, E) and quark (U, D), a Higgs boson as heavy as 500 GeV is allowed by the precision electroweak data [6, 7]. Here our question is: "What kind of the perturbative unification scenario can accommodate such a heavy Higgs boson?" Applying the same perturbativity condition as in Eq. (4), we obtain the Higgs boson mass limit in Fig. 2. As suggested in Ref. [7], the electroweak fit is compatible with $m_h = 500$ GeV, $m_N = 500$ GeV, $m_E = 100$ GeV, $m_U - m_D = 85$ GeV, and $m_U + m_D = 500$ GeV. Thus in Fig. 2, we take $m_N = 500$ GeV, $m_E = 100$ GeV, $m_U - m_D = 85$ GeV, and $m_U + m_D = 500$ or 550 GeV for the fourth generation fermion masses.

As can be seen from Fig. 2, we cannot have perturbative description of the theory above 10^6 GeV because of the existence of large Yukawa couplings in this model. An interesting point is that in order to have a heavier Higgs, low energy unification is needed. For example, in order to accommodate a 500 GeV Higgs, the unification scale should be as low as a few TeV.

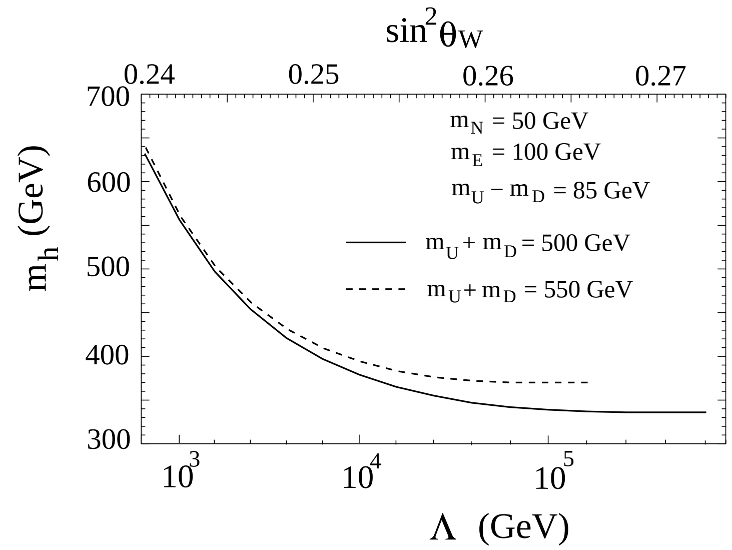


FIG. 2: Higgs boson mass limit as a function of the unification scale (Λ) and $\sin^2\theta_W(\Lambda)$ labeled above in the SM with the fourth generation fermions. Here we take the fourth generation fermion masses to be $m_N = 500$ GeV, $m_E = 100$ GeV, $m_U - m_D = 85$ GeV, $m_U + m_D = 500$ GeV for solid line and $m_U + m_D = 550$ GeV for dashed line.

III. SUPERSYMMETRIC UNIFICATION

A. Minimal supersymmetric standard model

In the minimal supersymmetric standard model (MSSM), the Higgs coupling is expressed by known gauge couplings due to the supersymmetry, and hence the Higgs mass is well constrained [18]. Perturbativity of the theory only constrains the range of $\tan\beta$. The lightest Higgs mass can be written in the following equation:

$$m_h^2 = m_Z^2 \cos^2 2\beta + \eta \frac{3G_F m_t^4}{\sqrt{2}\pi^2} \ln \frac{\Delta_S^2}{m_t^2}. \tag{5}$$

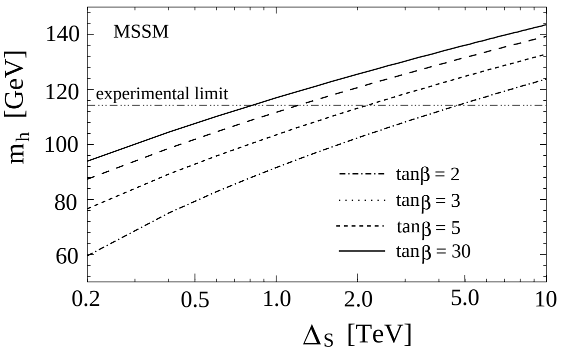
Since it has been known that $O(\alpha\alpha_s)$ two-loop contributions to the MSSM lightest Higgs mass reduce the one-loop upper limit on m_h [19], we introduce a suppression factor η in Eq. (5). To fix η, we match our expression Eq. (5) with the one in Ref. [19] at $\Delta_S^2 = m_{\tilde{t}}^2 \equiv (m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)/2 = (1\,\mathrm{TeV})^2$ assuming no stop mixing, and then we get $\eta = 1 - \frac{2\alpha_s}{\pi}\left(\ln\frac{m_{\tilde{t}}^2}{m_t^2} - \frac{2}{3}\right) = 0.78$. The numerical value of Δ_S is therefore a good indicator of the scale of superpartner masses.

In Fig. 3 we plot the lightest Higgs mass in the MSSM as a function of the supersymmetry mass scale Δ_S. We also show the relationship between Δ_S and $m_{\tilde{t}} \equiv \sqrt{(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)/2}$. Although presented in a slightly different way here, the results of this plot are well known. Low $\tan\beta$ requires large supersymmetry breaking mass in order to evade the current experimental limits on the lightest Higgs boson mass. Large $\tan\beta$ enables the MSSM to be comfortably within all experimental constraints for moderately small supersymmetry breaking mass. As can be seen from Fig. 3, the lightest Higgs boson mass is smaller than about 140 GeV for $\Delta_S < 10$ TeV. Therefore the MSSM predict a relatively light Higgs boson [18].

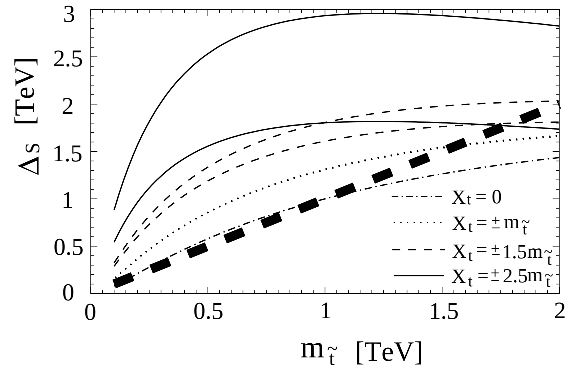
B. The next-to-minimal supersymmetric standard model

As soon as one goes beyond the most minimal supersymmetric model, the constraints of perturbativity become very significant again, just as they were in our SM analysis [20–25]. The reason is because non-minimal supersymmetric theories add additional Yukawa couplings that contribute directly to the mass of the lightest Higgs, but are not usefully constrained by any known measurement.

The most important example of non-minimal supersymmetry is the next-to-minimal MSSM (NMSSM), which adds another singlet S to the theory. This approach has been used by many authors to make the μ term more natural within supersymmetry. That is, in the MSSM there exists a term in the superpotential $\mu H_u H_d$ which might be best explained by an NMSSM term, $\lambda_s S H_u H_d$, where $\mu = \lambda_s \langle S \rangle$.



(a)



(b)

FIG. 3: (a)The lines plot the lightest Higgs boson mass in the MSSM as a function of the supersymmetry scale Δ_S, whose leading log value is $\Delta_S^2 = m_{\tilde{t}}^2$. The four lines from bottom to top represent $\tan\beta = 2, 3, 5, 30$. (b)The relationship between Δ_S defined by Eq. (5) and $m_{\tilde{t}} \equiv \sqrt{(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)/2}$ for various top squark mixing $X_t = A_t - \mu\cot\beta$ in the limit $m_A \gg m_Z$ (no Higgs mixing effects). Since $\Delta_S > m_{\tilde{t}}$ for much of parameter space, superpartners are expected to be below the value of Δ_S that corresponds to the Higgs boson mass limit $m_h > 114.1\,\mathrm{GeV}$. For the reader's convenience a thick dashed line is plotted for the line $\Delta_S = m_{\tilde{t}}$.

We can write the mass of the lightest scalar of the NMSSM in a very similar way as we did for the MSSM:

$$m_h^2 = m_Z^2 \cos^2 2\beta + \frac{\lambda_s^2}{2\sqrt{2}G_F}\sin^2 2\beta + \eta\frac{3G_F m_t^4}{\sqrt{2}\pi^2}\ln\frac{\Delta_S^2}{m_t^2}, \tag{6}$$

where we take $\eta = 0.78$. Since the Higgs coupling λ_s as well as top (y_t) and bottom (y_b) Yukawa couplings go to non-perturbative values as the scale goes higher, we must insure that all remain perturbative in order not to spoil perturbative gauge coupling unification. To determine what values of λ_s, y_t, and y_b are perturbative, we use the three-loop β functions [26]:

$$\beta_{y_t}^{(3\,\mathrm{loop})} = \frac{6}{16\pi^2}y_t^3 - \frac{22}{(16\pi^2)^2}y_t^5 + \frac{(102+36\zeta(3))}{(16\pi^2)^3}y_t^7 \tag{7}$$

$$\beta_{\lambda_s}^{(3\,\mathrm{loop})} = \frac{4}{16\pi^2}\lambda_s^3 - \frac{10}{(16\pi^2)^2}\lambda_s^5 + \frac{(32+24\zeta(3))}{(16\pi^2)^3}\lambda_s^7. \tag{8}$$

β_{y_b} is the same as β_{y_t} after replacing $y_t \to y_b$. Applying the perturbativity conditions of Eq. (4) we find that perturbative couplings must satisfy $y_t < 4.9$, $y_b < 4.9$, and $\lambda_s < 5.1$.

In Fig. 4, we show the mass of the lightest Higgs boson as a function of the unification scale Λ, requiring that all couplings remain perturbative below Λ. Here we fix $\Delta_S = 500$

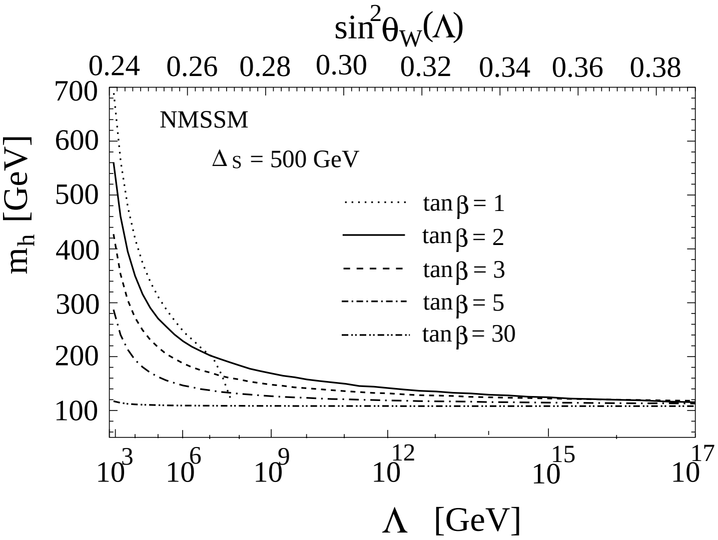


FIG. 4: The five lines are for the same values of $\tan\beta$ in the NMSSM. The λ_s coupling of the superpotential $\lambda_s S H_u H_d$ term is assumed to be at its maximum allowed value without blowing up before the scale Λ ($\lambda_s < 5.1$). Since $\sin^2\theta_W(\Lambda)$ correlates directly with Λ we provide the $\sin^2\theta_W(\Lambda)$ values on the upper axis.

GeV. In the MSSM (without GUT), $\tan\beta < 1$ is excluded by the Higgs search. However, an interesting point in the NMSSM is that such low values of $\tan\beta$ are still allowed if the unification scale Λ is low. The result with $\Delta_S = 500\,\mathrm{GeV}$ for two interesting scenarios $\sin^2\theta_W = 1/4$ and $\sin^2\theta_W = 3/8$, which are realized in $SU(3)$ electroweak unification model and $SU(5)/SO(10)$ grand unification model, respectively, is summarized,

$$\sin^2\theta_W = 1/4 \implies \Lambda = 37\,\mathrm{TeV}, m_h < 350\,\mathrm{GeV},$$
$$\sin^2\theta_W = 3/8 \implies \Lambda \simeq 2\times 10^{16}\,\mathrm{GeV}, m_h < 120\,\mathrm{GeV}.$$

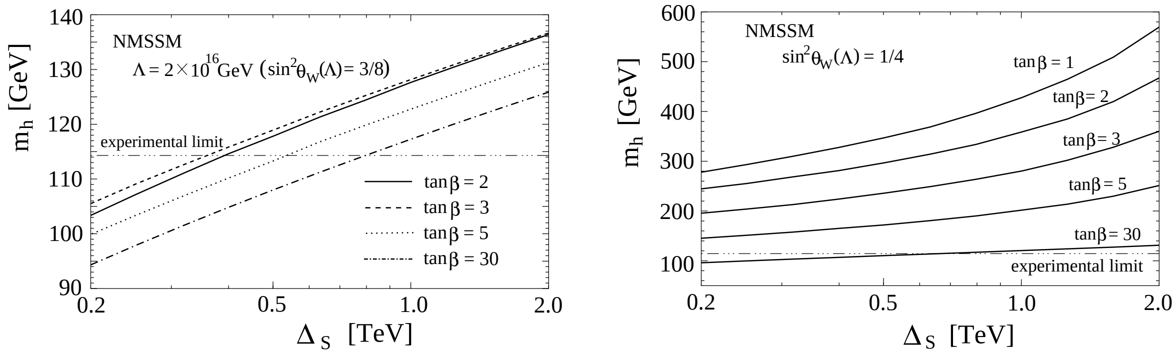
In Fig. 5, we plot the lightest Higgs boson mass in the NMSSM as a function of Δ_S in the two scenarios $\sin^2\theta_W = 1/4$ ($\Lambda \sim 8-110\,\mathrm{TeV}$) and $\sin^2\theta_W = 3/8$ ($\Lambda = 2\times 10^{16}\,\mathrm{GeV}$).

We should stress that the Higgs boson as heavy as 500 GeV is possible even in this framework of supersymmetric theory if the unification scale is low.

IV. CONCLUSIONS

In this talk we have examined Higgs mass upper limits in theories that are perturbative up to a scale Λ. In the several unification scenarios we studies, we found that it is not expected to have a Higgs boson above about 500 GeV and still remain perturbative.

Fortunately, the CERN Large Hadron Collider (LHC) will be able to see all SM-like Higgs bosons easily up to 500 GeV. After discovery of a Higgs boson, our results can



(a) (b)

FIG. 5: (a)The lines plot the lightest Higgs boson mass in the NMSSM as a function of the supersymmetry scale Δ_S. The leading log value for $\Delta_S = m_{\tilde{t}}$. The value of λ_s used in Eq. (6) is at its maximum consistent with $\lambda_s(\Lambda) < 5.1$ (perturbative). Here $\Lambda = 2 \times 10^{16}$ GeV, which corresponds to the simple grand unification scenario of $\sin^2\theta_W(\Lambda) = 3/8$. (b)The lines plot the lightest Higgs boson mass in the NMSSM as a function of the supersymmetry scale Δ_S. The leading log value for $\Delta_S = m_{\tilde{t}}$. The value of λ_s used in Eq. (6) is such that $\lambda_s(\Lambda) < 5.1$ (perturbative). Here $8\,\text{TeV} < \Lambda < 110\,\text{TeV}$ (precise value depends on Δ_S), which corresponds to the $SU(3)$ electroweak unification scenario of $\sin^2\theta_W(\Lambda) = 1/4$.

tell us what kind of unification scenario ($\sin^2\theta_W(\Lambda) = 1/4, \sin^2\theta_W(\Lambda) = 3/8$, or else) is consistent with the perturbativity of the theory.

Acknowledgments

K.T. was supported in part by the U.S. Department of Energy.

[1] K. Tobe and J. D. Wells, Phys. Rev. D **66**, 013010 (2002) [hep-ph/0204196].
[2] E. Gross, talk at this conference.
[3] D. Abbaneo *et al.* [LEP Collaborations], hep-ex/0112021v2.
[4] LEP Higgs Working Group for Higgs boson searches Collaboration, hep-ex/0107029.
[5] M. E. Peskin and J. D. Wells, Phys. Rev. D **64**, 093003 (2001) [hep-ph/0101342].
[6] H. J. He, N. Polonsky, and S. f. Su, Phys. Rev. D **64**, 053004 (2001) [arXiv:hep-ph/0102144].
[7] V. A. Novikov, L. B. Okun, A. N. Rozanov, and M. I. Vysotsky, [hep-ph/0203132].

[8] S. Dimopoulos and D. E. Kaplan, hep-ph/0201148; S. Dimopoulos, D. E. Kaplan, and N. Weiner, hep-ph/0202136; S. Dimopoulos and D. E. Kaplan, hep-ph/0203001.

[9] T. j. Li and W. Liao, hep-ph/0202090.

[10] L. J. Hall and Y. Nomura, hep-ph/0202107.

[11] I. Antoniadis and K. Benakli, Phys. Lett. B **326**, 69 (1994) [hep-th/9310151]; I. Antoniadis, E. Kiritsis, and T. N. Tomaras, Phys. Lett. B **486**, 186 (2000) [hep-ph/0004214]; I. Antoniadis, K. Benakli, and M. Quiros, hep-th/0108005.

[12] S. Weinberg, Phys. Rev. D **5**, 1962 (1972).

[13] F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992) [hep-ph/9206242].

[14] P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).

[15] I. Jack and H. Osborn, J. Phys. A **16**, 1101 (1983).

[16] D. I. Kazakov, O. V. Tarasov, and A. A. Vladimirov, Sov. Phys. JETP **50**, 521 (1979) [Zh. Eksp. Teor. Fiz. **77**, 1035 (1979)].

[17] U. Nierste and K. Riesselmann, Phys. Rev. D **53**, 6638 (1996) [hep-ph/9511407]; T. Hambye and K. Riesselmann, Phys. Rev. D **55**, 7255 (1997) [hep-ph/9610272].

[18] Y. Okada, M. Yamaguchi, and T. Yanagida, Prog. Theor. Phys. **85**, 1 (1991); H. E. Haber and R. Hempfling, Phys. Rev. Lett. **66**, 1815 (1991); J. R. Ellis, G. Ridolfi, and F. Zwirner, Phys. Lett. B **257**, 83 (1991).

[19] M. Carena, H. E. Haber, S. Heinemeyer, W. Hollik, C. E. Wagner, and G. Weiglein, Nucl. Phys. B **580**, 29 (2000) [hep-ph/0001002]. See also J. R. Espinosa and R. J. Zhang, JHEP **0003**, 026 (2000) [hep-ph/9912236].

[20] J. R. Espinosa and M. Quiros, Phys. Lett. B **279**, 92 (1992).

[21] G. L. Kane, C. Kolda, and J. D. Wells, Phys. Rev. Lett. **70**, 2686 (1993) [hep-ph/9210242].

[22] J. R. Espinosa and M. Quiros, Phys. Rev. Lett. **81**, 516 (1998) [hep-ph/9804235].

[23] M. Masip, R. Munoz-Tapia, and A. Pomarol, Phys. Rev. D **57**, 5340 (1998) [hep-ph/9801437].

[24] M. Drees, Phys. Rev. D **35**, 2910 (1987).

[25] T. Moroi and Y. Okada, Phys. Lett. B **295**, 73 (1992);

[26] I. Jack, D. R. Jones, and C. G. North, Nucl. Phys. B **473**, 308 (1996) [hep-ph/9603386]; P. M. Ferreira, I. Jack, and D. R. Jones, Phys. Lett. B **387**, 80 (1996) [hep-ph/9605440].

[27] Although within the framework of non-SUSY theories, it will not be easy to construct realistic GUT models, we keep this idea within the space of possibilities.