Full electroweak one–loop corrections to $A^0 \to \tilde{f}_i \, \bar{\tilde{f}}_j$

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Abstract

We discuss the full electroweak one–loop corrections to the decay of the pseudoscalar Higgs boson A^0 into two sfermions within the Minimal Supersymmetric Standard Model. In particular, we consider the sfermions of the third generation, $\tilde{t}_i$, $\tilde{b}_i$ and $\tilde{\tau}_i$, including the left–right mixing. The electroweak corrections can go up to $\sim 15\%$ and can therefore not be neglected.

The Minimal Supersymmetric Standard Model (MSSM) [1] requires five physical Higgs bosons: two neutral CP–even (h^0 and H^0), one heavy neutral CP–odd (A^0), and two charged ones ($H^\pm$) [2, 3]. The existence of heavy neutral Higgs bosons would provide a conclusive evidence of physics beyond the SM. Therefore, searching for Higgs bosons is one of the main goals of future collider projects like TEVATRON, LHC or an e^+e^- Linear Collider.

In this talk, we consider the decay of the CP–odd Higgs boson A^0 into two sfermions, $A^0 \to \tilde{f}_i \, \bar{\tilde{f}}_j$. The decays into sfermions can be the dominant decay modes of Higgs bosons in a large parameter region if the sfermions are relatively light [4, 5]. In particular, third generation sfermions $\tilde{t}_i$, $\tilde{b}_i$ and $\tilde{\tau}_i$ can be much lighter than the other sfermions due to their large Yukawa couplings and their large left–right mixing. We have calculated the full electroweak corrections in the on–shell scheme and have implemented the SUSY–QCD corrections from [6]. We will show that the electroweak corrections are significant and need to be included.

At tree–level the Higgs sector depends on two parameters, for instance m_{A^0} and $\tan\beta$. m_{A^0} is the mass of the pseudoscalar Higgs boson A^0, and $\tan\beta = \frac{v_2}{v_1}$ is the ratio of the vacuum expectation values of the two neutral Higgs doublet states [2, 3]. In the chargino and neutralino systems there are the higgsino mass parameter μ, the $U(1)$ and $SU(2)$ gaugino mass parameters M' and M, respectively. We assume that the gluino mass $m_{\tilde{g}}$ is related to M by $m_{\tilde{g}} = (\alpha_s(m_{\tilde{g}})/\alpha_2) \sin^2\theta_W M$.

The decay width for $A^0 \to \tilde{f}_i \, \bar{\tilde{f}}_j$ depends on the left–right mixing. This mixing is described by the sfermion mass matrix in the left–right basis $(\tilde{f}_L, \tilde{f}_R)$, and in the mass basis $(\tilde{f}_1, \tilde{f}_2)$, $\tilde{f} = \tilde{t}, \tilde{b}$ or $\tilde{\tau}$,

$$\mathcal{M}^2_{\tilde{f}} = \begin{pmatrix} m^2_{\tilde{f}_L} & a_f \, m_f \\ a_f \, m_f & m^2_{\tilde{f}_R} \end{pmatrix} = \left(R^{\tilde{f}}\right)^\dagger \begin{pmatrix} m^2_{\tilde{f}_1} & 0 \\ 0 & m^2_{\tilde{f}_2} \end{pmatrix} R^{\tilde{f}} \,, \qquad (1)$$

where $R^{\tilde{f}}_{i\alpha}$ is a 2 x 2 rotation matrix with rotation angle $\theta_{\tilde{f}}$, which relates the mass eigenstates $\tilde{f}_i$, $i = 1, 2$, $(m_{\tilde{f}_1} < m_{\tilde{f}_2})$ to the gauge eigenstates $\tilde{f}_\alpha$, $\alpha = L, R$, by $\tilde{f}_i = R^{\tilde{f}}_{i\alpha} \tilde{f}_\alpha$ and

$$m^2_{\tilde{f}_L} = M^2_{\{\tilde{Q},\tilde{L}\}} + (I^{3L}_f - e_f \sin^2\theta_W) \cos 2\beta \, m^2_Z + m^2_f \,, \tag{2}$$

$$m^2_{\tilde{f}_R} = M^2_{\{\tilde{U},\tilde{D},\tilde{E}\}} - e_f \sin^2\theta_W \cos 2\beta \, m^2_Z + m^2_f \,, \tag{3}$$

$$a_f = A_f - \mu \, (\tan\beta)^{-2I^{3L}_f} \,. \tag{4}$$

$M_{\tilde{Q}}$, $M_{\tilde{L}}$, $M_{\tilde{U}}$, $M_{\tilde{D}}$ and $M_{\tilde{E}}$ are soft SUSY breaking masses, A_f is the trilinear scalar coupling parameter, I^{3L}_f and e_f are the third component of the weak isospin and the electric charge of the sfermion f, and θ_W is the Weinberg angle.
The mass eigenvalues and the mixing angle in terms of primary parameters are

$$m^2_{\tilde{f}_{1,2}} = \frac{1}{2}\left(m^2_{\tilde{f}_L} + m^2_{\tilde{f}_R} \mp \sqrt{(m^2_{\tilde{f}_L} - m^2_{\tilde{f}_R})^2 + 4a^2_f m^2_f}\right) \tag{5}$$

$$\cos\theta_{\tilde{f}} = \frac{-a_f \, m_f}{\sqrt{(m^2_{\tilde{f}_L} - m^2_{\tilde{f}_1})^2 + a^2_f m^2_f}} \qquad (0 \le \theta_{\tilde{f}} < \pi) \,, \tag{6}$$

and the trilinear breaking parameter A_f can be written as

$$m_f A_f = \frac{1}{2}\left(m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}\right) \sin 2\theta_{\tilde{f}} \; + \; m_f \, \mu \, (\tan\beta)^{-2I^{3L}_f} \,. \tag{7}$$

At tree–level the decay width of $A^0 \to \tilde{f}_i \, \bar{\tilde{f}}_j$ is given by

$$\Gamma^{\rm tree}(A^0 \to \tilde{f}_i \, \bar{\tilde{f}}_j) = \frac{N^f_C \, \kappa(m^2_{A^0}, m^2_{\tilde{f}_i}, m^2_{\tilde{f}_j})}{16\,\pi\, m^3_{A^0}} \; |G^{\tilde{f}}_{ij}|^2 \tag{8}$$

with $\kappa(x, y, z) = \sqrt{(x - y - z)^2 - 4yz}$ and the colour factor $N^f_C = 3$ for squarks and $N^f_C = 1$ for sleptons respectively. The Higgs–Sfermion–Sfermion couplings for the pseudoscalar Higgs boson A^0 are given by

$$G^{\tilde{f}}_{ij} = \frac{i}{\sqrt{2}} \, h_f \left(A_f \begin{Bmatrix} \cos\beta \\ \sin\beta \end{Bmatrix} + \mu \begin{Bmatrix} \sin\beta \\ \cos\beta \end{Bmatrix} \right) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}_{ij} \tag{9}$$

for $\begin{Bmatrix} \text{up} \\ \text{down} \end{Bmatrix}$–type sfermions respectively. h_f denotes the Yukawa couplings $h_t = g\, m_t/(\sqrt{2} m_W \sin\beta), h_b = g\, m_b/(\sqrt{2} m_W \cos\beta)$ and $h_\tau = g\, m_\tau/(\sqrt{2} m_W \cos\beta)$ for top, bottom and tau, respectively.

The one–loop corrected (renormalized) amplitude $G^{\tilde{f}\,\rm ren}_{ij}$ can be expressed as

$$G^{\tilde{f}\,\rm ren}_{ij} = G^{\tilde{f}}_{ij} + \delta G^{\tilde{f}}_{ij} = G^{\tilde{f}}_{ij} + \delta G^{\tilde{f}(v)}_{ij} + \delta G^{\tilde{f}(w)}_{ij} + \delta G^{\tilde{f}(c)}_{ij} \,, \tag{10}$$

where $G^{\tilde{f}}_{ij}$ denotes the tree–level A^0–$\tilde{f}_i$–$\tilde{f}_j$ coupling in terms of the on–shell parameters, $\delta G^{\tilde{f}(v)}_{ij}$ and $\delta G^{\tilde{f}(w)}_{ij}$ are the vertex and wave–function corrections, respectively. Here we only show the diagrams of the vertex graphs (Fig. 1).

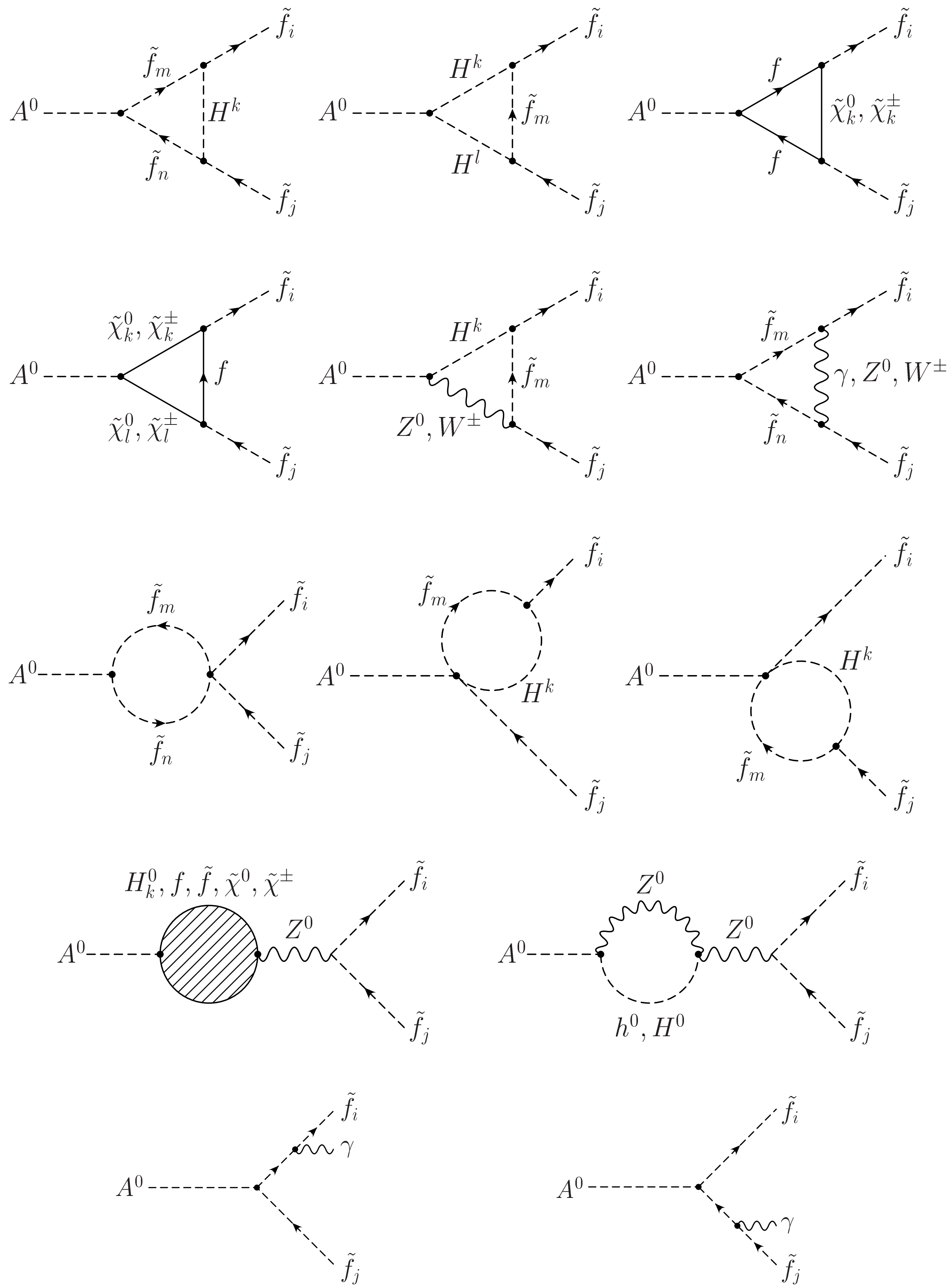

Figure 1: Vertex and photon emission diagrams relevant to the calculation of the virtual electroweak corrections to the decay width $A^0 \to \tilde{f}_i \, \bar{\tilde{f}}_j$.

Note that in addition to the one–particle irreducible vertex graphs also one–loop induced reducible graphs with A^0–Z^0 mixing have to be considered. Since all parameters in the coupling $G^{\tilde{f}}_{ij}$ have to be renormalized, the counter term correction reads

$$\delta G^{\tilde{f}(c)}_{ij} = \frac{\delta h_f}{h_f} G^{\tilde{f}}_{ij} + \frac{i}{\sqrt{2}} h_f \, \delta\left(A_f \begin{Bmatrix} \cos\beta \\ \sin\beta \end{Bmatrix} + \mu \begin{Bmatrix} \sin\beta \\ \cos\beta \end{Bmatrix}\right) . \tag{11}$$

The Yukawa coupling counter term can be decomposed into corrections to the electroweak coupling g, the masses of the fermion f and the gauge boson W and the mixing angle β,

$$\frac{\delta h_f}{h_f} = \frac{\delta g}{g} + \frac{\delta m_f}{m_f} - \frac{\delta m_W}{m_W} + \begin{Bmatrix} -\cos^2\beta \\ \sin^2\beta \end{Bmatrix} \frac{\delta \tan\beta}{\tan\beta} . \tag{12}$$

For the trilinear coupling we get with eq. (7)

$$\frac{\delta A_f}{A_f} = \frac{\delta(m_f A_f)}{m_f A_f} - \frac{\delta m_f}{m_f} , \tag{13}$$

$$\begin{aligned} \delta(m_f A_f) = {} & \delta\left(m_f \mu \begin{Bmatrix} \cot\beta \\ \tan\beta \end{Bmatrix}\right) + \frac{1}{2}\left(\delta m^2_{\tilde{f}_1} - \delta m^2_{\tilde{f}_2}\right) \sin 2\theta_{\tilde{f}} \\ & + \left(m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}\right) \cos 2\theta_{\tilde{f}} \, \delta\theta_{\tilde{f}} . \end{aligned} \tag{14}$$

In the on–shell scheme the renormalization condition for the electroweak gauge boson sector reads

$$\frac{\delta g}{g} = \frac{\delta e}{e} + \frac{1}{\tan^2\theta_W}\left(\frac{\delta m_W}{m_W} - \frac{\delta m_Z}{m_Z}\right) \tag{15}$$

with m_W and m_Z fixed as well as the fermion and sfermion masses as the physical (pole) masses. For $\tan\beta$ we use the condition [7] $\mathrm{Im}\hat{\Pi}_{A^0 Z^0}(m^2_A) = 0$ which gives the counter term

$$\frac{\delta \tan\beta}{\tan\beta} = \frac{1}{m_Z \sin 2\beta} \mathrm{Im}\Pi_{A^0 Z^0}(m^2_{A^0}). \tag{16}$$

The higgsino mass parameter μ is renormalized in the chargino sector [8] where it enters in the 22–element of the chargino mass matrix X,

$$X = \begin{pmatrix} M & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix} \qquad \to \delta\mu = (\delta X)_{22} , \tag{17}$$

and the counter term of the sfermion mixing angle, $\delta\theta_{\tilde{f}}$, is fixed such that it cancels the anti–hermitian part of the sfermion wave–function corrections [9, 10],

$$\delta\theta_{\tilde{f}} = \frac{1}{4}\left(\delta Z^{\tilde{f}}_{12} - \delta Z^{\tilde{f}}_{21}\right) = \frac{1}{2(m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2})} \mathrm{Re}\left(\Pi^{\tilde{f}}_{12}(m^2_{\tilde{f}_2}) + \Pi^{\tilde{f}}_{21}(m^2_{\tilde{f}_1})\right) . \tag{18}$$

The one–loop corrected decay width is then given by

$$\Gamma(A^0 \to \tilde{f}_i\,\bar{\tilde{f}}_j) \;=\; \frac{N_C^f\,\kappa(m_{A^0}^2, m_{\tilde{f}_i}^2, m_{\tilde{f}_j}^2)}{16\,\pi\,m_{A^0}^3}\left[|G_{ij}^{\tilde{f}}|^2 + 2\mathrm{Re}\left(G_{ij}^{\tilde{f}}\cdot\delta G_{ij}^{\tilde{f}}\right)\right], \qquad (19)$$

The infrared divergences in eq. (19) are cancelled by the inclusion of real photon emission, see the last two Feynman diagrams of Fig. 1. The decay width of $A^0(p) \to \tilde{f}_i(k_1)+\bar{\tilde{f}}_j(k_2)+\gamma(k_3)$ can be written as

$$\Gamma(A^0 \to \tilde{f}_i\,\bar{\tilde{f}}_j\,\gamma) \;=\; \frac{(e\,e_f)^2\,N_C^f\,|G_{ij}^{\tilde{f}}|^2}{16\,\pi^3\,m_{A^0}}\left[\left(m_{A^0}^2 - m_{\tilde{f}_i}^2 - m_{\tilde{f}_j}^2\right)I_{12} - m_{\tilde{f}_i}^2 I_{11} - m_{\tilde{f}_j}^2 I_{22} - I_1 - I_2\right] \qquad (20)$$

with the phase–space integrals I_n and I_{mn} defined as [11]

$$I_{i_1\dots i_n} = \frac{1}{\pi^2}\int\frac{d^3k_1}{2E_1}\frac{d^3k_2}{2E_2}\frac{d^3k_3}{2E_3}\delta^4(p-k_1-k_2-k_3)\frac{1}{(2k_3k_{i_1}+\lambda^2)\dots(2k_3k_{i_n}+\lambda^2)}. \qquad (21)$$

The corrected (UV– and IR–convergent) decay width is then given by

$$\Gamma^{\rm corr}(A^0 \to \tilde{f}_i\,\bar{\tilde{f}}_j) \;\equiv\; \Gamma(A^0 \to \tilde{f}_i\,\bar{\tilde{f}}_j) \,+\, \Gamma(A^0 \to \tilde{f}_i\,\bar{\tilde{f}}_j\,\gamma)\,. \qquad (22)$$

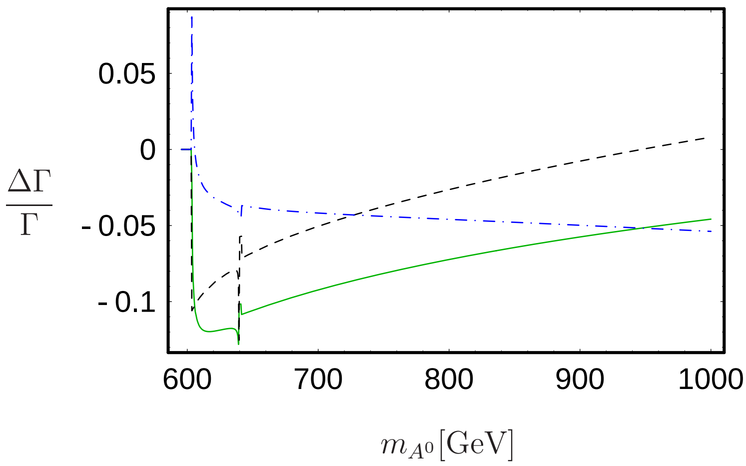

Figure 2: Relative corrections to $A^0 \to \tilde{t}_1\bar{\tilde{t}}_2$, separated into leading Yukawa (black dashed line) and the remaining electroweak (blue dash-dotted line) corrections. The green solid line corresponds to the full electroweak corrections.

In the following numerical examples, we assume $M_{\tilde{Q}_{1,2}} = M_{\tilde{U}_{1,2}} = M_{\tilde{D}_{1,2}} = M_{\tilde{L}_{1,2}} = M_{\tilde{E}_{1,2}}$, $M_{\tilde{Q}} \equiv M_{\tilde{Q}_3} = \frac{9}{8}M_{\tilde{U}_3} = \frac{9}{10}M_{\tilde{D}_3} = M_{\tilde{L}_3} = M_{\tilde{E}_3}$ for the first, second and third generation soft SUSY breaking masses and $A \equiv A_t = A_b = A_\tau$. We take $m_t = 175$ GeV, $m_b = 5$ GeV, $m_Z = 91.2$ GeV, $m_W = 80$ GeV and $\sin^2\theta_W = 0.23$ for Standard Model values and the gaugino unification relation $M' = \dfrac{5}{3}\tan^2\theta_W M$.

In Fig. 2 we show the m_{A^0}–dependence of the relative correction to $A^0 \to \tilde{t}_1\bar{\tilde{t}}_2$, separated into leading Yukawa and the remaining electroweak corrections using $\tan\beta = 7$ and $\{M_{\tilde{Q}_1}, M_{\tilde{Q}}, A, M, \mu\} = \{1500, 300, -500, 120, -260\}$ GeV as input parameters. As can be seen for larger values of m_{A^0}, the remaining electroweak corrections can become bigger than the leading Yukawa corrections and need to be included.

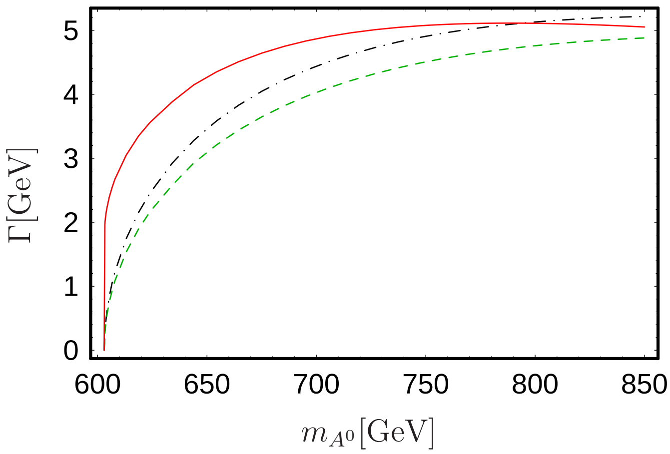

Figure 3: Tree–level (black dash-dotted line), full electroweak corrected (green dashed line) and full one–loop (electroweak and SUSY–QCD) corrected (red solid line) decay width of $A^0 \to \tilde{t}_1\bar{\tilde{t}}_2$.

In Fig. 3, in addition to the tree–level and electroweak corrected decay width for $A^0 \to \tilde{t}_1\bar{\tilde{t}}_2$ we have also included SUSY–QCD corrections from [6]. As input set we have taken the same parameters as in Fig. 2. Note that the electroweak corrections can be of the same size as the QCD corrections.

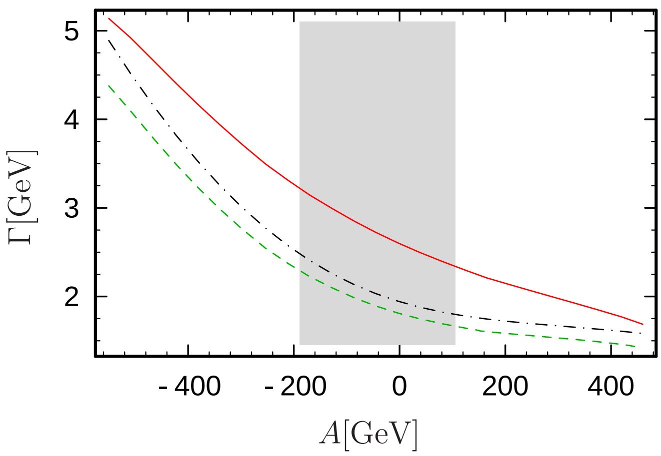

Figure 4: A–dependence of tree–level (black dash-dotted line), full electroweak corrected (green dashed line) and full one–loop (electroweak and SUSY–QCD) corrected (red solid line) decay width of $A^0 \to \tilde{t}_1\bar{\tilde{t}}_2$. The gray area is excluded by phenomenology.

In Fig. 4 we show the tree–level (black dash-dotted line), the full electroweak (green dashed line) and the full one–loop corrected (electroweak and SUSY–QCD, red solid line) decay width of $A^0 \to \tilde{t}_1\bar{\tilde{t}}_2$ as a function of A. As can be seen electroweak corrections do not strongly depend on the parameter A and are almost constant about 8%. As input parameters we have chosen the values given above and $m_{A^0} = 700$ GeV.

In conclusion, we have calculated the full electroweak one–loop corrections to $A^0 \to \tilde{t}_1\bar{\tilde{t}}_2$. We found that in a wide region of parameter space electroweak corrections can go beyond 10% and therefore have to be included.

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