Impact of Tau Polarization for the determination of high $\tan\beta$ and A_τ[§]

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Abstract

In order to determine the fundamental MSSM parameters M_1, M_2, μ and $\tan\beta$ the tau polarization from $\tilde{\tau}$ decays can be explored as a 'bridge' between the gaugino/higgsino and the stau sector in particular in the high $\tan\beta$ range. Even in the case of high $\tan\beta$ an accuracy of $\delta(\tan\beta) \approx 5\%$ with an simultanous determination of A_τ is possible without assuming a specific SUSY breaking scheme.

1 Introduction

The Minimal Supersymmetric extension of the Standard Model (MSSM) is one of the most promising extensions of the Standard Model (SM). However, SUSY has to be broken and in the unconstrained version of the MSSM a parameterization of all possible soft SUSY breaking terms leads to 105 new parameters in addition to the ones of the SM.

A linear collider (LC) is, due to its clear signatures, the most promising tool for revealing the underlying structure of the physics beyond the SM as e.g. the precise determination of these parameters.

Strategies to determine the fundamental parameters in the chargino/neutralino sector have already been worked out in [1, 2] and references therein. However, for $\tan\beta > 10$, the gaugino/higgsino sector is weak dependent on $\tan\beta$ so that its determination becomes rather inaccurate. We therefore concentrate in this paper on the production of the τ SUSY partners $\tilde{\tau}_{1,2}$ and their decays into $\tilde{\chi}^0_1$ and using the polarization of the τ's for the accurate determination of high $\tan\beta$. Since the τ polarization involves simultanously mixing parameters from both neutralino and stau sectors it plays the role of a 'bridge' between these two sectors.

The importance of the τ polarization has already been pointed out in [3, 4]. We derive the compact formula for the polarization including the general mixing in the neutralino sector and show in which regions of MSSM parameter space the polarization will be a

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suitable observable. Contrary to the former studies we assume no specific character of the LSP $\tilde{\chi}_1^0$ or specific GUT relations between the underlying gaugino parameters. We show with an numerical example that the polarization on τ's is well suited for a rather accurate determination even of high $\tan\beta$ as well as for a simultanous determination of A_τ.

2 The Stau Sector

2.1 Masses and Mixing

Since the τ lepton has the largest Yukawa coupling of the three lepton families the weak eigenstates $\tilde{\tau}_{L,R}$ mix to the mass eigenstates $\tilde{\tau}_{1,2}$, where the mass matrix is given by:

$$\mathcal{M}^2_{\tilde{\tau}} = \begin{pmatrix} M_L^2 + m_\tau^2 + D_L & m_\tau(A_\tau - \mu\tan\beta) \\ m_\tau(A_\tau - \mu\tan\beta) & M_E^2 + m_\tau^2 + D_R \end{pmatrix} = \begin{pmatrix} m_{LL}^2 & m_{LR}^2 \\ m_{LR}^2 & m_{RR}^2 \end{pmatrix} \tag{1}$$

with the D-terms $D_L = (-\frac{1}{2} + \sin^2\theta_W)\cos(2\beta)m_Z^2$ and $D_R = -\sin^2\theta_W\cos(2\beta)m_Z^2$, the trilinear slepton–Higgs $\tilde{\ell}_R^*$-$\tilde{\ell}_L$-H_1 coupling A_τ, the higgsino mass parameter μ, the ratio of the Higgs expectation values $\tan\beta = v_2/v_1$ and the SU(2) doublet (singlet) mass parameters M_L (M_E). The mass parameters m_{LL}^2, m_{RR}^2 have to be positiv for $\tan\beta > 1$, whereas the sign of the off–diagonal terms m_{LR}^2 depends on A_τ and μ.

The mass eigenvalues are given by

$$m^2_{\tilde{\tau}_{1,2}} = \frac{1}{2}[m_{LL}^2 + m_{RR}^2 \mp \sqrt{(m_{LL}^2 - m_{RR}^2)^2 + 4(m_{LR}^2)^2}], \tag{2}$$

and a large mass difference may be quite natural. In many SUSY scenarios $\tilde{\tau}_1$ is similar light as light charginos/neutralinos. A future high $\mathcal{L}$ LC will be well suited to measure the masses with an high accuracy of e.g. about $\delta(m_{\tilde{\tau}_1}) \sim 0.6$ GeV [5].

The stau mixing angle $\theta_{\tilde{\tau}}$, $[0,\pi]$ is given by:

$$\tan(2\theta_{\tilde{\tau}}) = \frac{-2m_{LR}^2}{m_{RR}^2 - m_{LL}^2} =: \xi \tag{3}$$

so that the mass parameters m_{LL}^2, m_{RR}^2, m_{LR}^2 can also be expressed via the measurable observables $m^2_{\tilde{\tau}_{1,2}}$:

$$m_{LL}^2 = \frac{m^2_{\tilde{\tau}_1} + m^2_{\tilde{\tau}_2}}{2} - \frac{m^2_{\tilde{\tau}_2} - m^2_{\tilde{\tau}_1}}{2}\cos(2\theta_{\tilde{\tau}}), \tag{4}$$

$$m_{RR}^2 = \frac{m^2_{\tilde{\tau}_1} + m^2_{\tilde{\tau}_2}}{2} + \frac{m^2_{\tilde{\tau}_2} - m^2_{\tilde{\tau}_1}}{2}\cos(2\theta_{\tilde{\tau}}), \tag{5}$$

$$m_{LR}^2 = \frac{1}{2}(m^2_{\tilde{\tau}_1} - m^2_{\tilde{\tau}_2})\sin(2\theta_{\tilde{\tau}}). \tag{6}$$

One sees from (4) and (5) that one can distinguish the two cases

1. if $m_{LL}^2 > m_{RR}^2$ then $\cos(2\theta_{\tilde{\tau}}) = -\frac{1}{\sqrt{1+\xi^2}} < 0$

2. if $m^2_{LL} < m^2_{RR}$ then $\cos(2\theta_{\tilde\tau}) = +\frac{1}{\sqrt{1+\xi^2}} > 0$

The cross section for $\tilde\tau_i\tilde\tau_i$ production depends on the mixing angle $\cos 2\theta_{\tilde\tau}$ ([6] and references therein) and $\cos\theta_{\tilde\tau}$ can be accurately determined, with a two–fold ambiguity, via the cross section for $\tilde\tau_i$ production with polarized beams or via a polarization asymmetry $A_{Pol} = (\sigma_L - \sigma_R)/(\sigma_L + \sigma_R)$.

In the following we discuss the τ polarization from $\tilde\tau_i$ decays $P_{\tilde\tau_i\to\tau}$. The tau polarization P_τ can be derived from the energy distribution of the decay products of the tau lepton.

2.2 τ polarization from $\tilde\tau_i$ decays

In this section we study the polarization P_τ from the decays

$$\tilde\tau_1 \to \tilde\chi^0_i\tau, \quad \text{and} \quad \tilde\tau_2 \to \tilde\chi^0_i\tau, \quad i = 1,\ldots,4 \tag{7}$$

taking into account a general neutralino mixing in the MSSM [7].

The tau polarization for (7) is given by (using the narrow width approximation) [3]:

$$P_{\tilde\tau_1\to\tau} = \frac{(a^R_{1i})^2 - (a^L_{1i})^2}{(a^R_{1i})^2 + (a^L_{1i})^2}, \quad \text{and} \quad P_{\tilde\tau_2\to\tau} = \frac{(a^R_{2i})^2 - (a^L_{2i})^2}{(a^R_{2i})^2 + (a^L_{2i})^2}. \tag{8}$$

with

$$a^{L,R}_{1i} = \cos\theta_{\tilde\tau} a^{L,R}_{Li} + \sin\theta_{\tilde\tau} a^{L,R}_{Ri}, \quad \text{and} \quad a^{L,R}_{2i} = -\sin\theta_{\tilde\tau} a^{L,R}_{Li} + \cos\theta_{\tilde\tau} a^{L,R}_{Ri}, \tag{9}$$

where the coefficients a^k_{ij} are defined by the Lagrangian

$$\mathcal{L} = \sum_{\substack{i=1,2\\ j=1,\ldots,4}} \tilde\tau_i \bar\tau (P_L a^R_{ij} + P_R a^L_{ij})\tilde\chi^0_j. \tag{10}$$

in the neutralino basis $(\tilde B, \tilde W_3, \tilde H_1, \tilde H_2)$:

$$a^R_{Lj} = -\frac{g}{\sqrt 2}\frac{m_\tau}{m_W\cos\beta}N_{j3}, \qquad a^L_{Rj} = a^R_{Lj} \tag{11}$$

$$a^L_{Lj} = +\frac{g}{\sqrt 2}[N_{j2} + N_{j1}\tan\theta_W], \quad a^R_{Rj} = -\frac{2g}{\sqrt 2}N_{j1}\tan\theta_W. \tag{12}$$

Taking into account a general mixing in the neutralino sector we derive the τ polarization:

$$P_{\tilde\tau_1\to\tau} = \frac{(4 - x^2_W) - (4 + x^2_W - 2y^2_h)\cos 2\theta_{\tilde\tau} + 2(2 + x_W)y_h \sin 2\theta_{\tilde\tau}}{(4 + x^2_W + 2y^2_h) - (4 - x^2_W)\cos 2\theta_{\tilde\tau} + 2(2 - x_W)y_h \sin 2\theta_{\tilde\tau}}. \tag{13}$$

And for the case $m^2_{LL} > m^2_{RR}$ the formula takes the form:

$$= \frac{(4 - x^2_W)\sqrt{1+\xi^2} + (4 + x^2_W - 2y^2_h) - 2(2 + x_W)y_h\xi}{(4 + x^2_W + 2y^2_h)\sqrt{1+\xi^2} + (4 - x^2_W) - 2(2 - x_W)y_h\xi}, \tag{14}$$

where the mixing angle $\theta_{\tilde\tau}$ i.e. ξ is given by (3) and $P_{\tilde\tau_2\to\tau}$ can be obtained from eq. (13) by changing the sign of $\cos 2\theta_{\tilde\tau}$ and $\sin 2\theta_{\tilde\tau}$.

The coefficient x_W contains the complete contribution from the gaugino components, y_h is a combination of the factors of the Yukawa coupling and the complete contribution from the higgsino components x_h.

$$x_W = \frac{\tan\theta_W N_{11} + N_{12}}{\tan\theta_W N_{11}} \tag{15}$$

$$y_h = \frac{\frac{1}{\cos\beta}\frac{m_\tau}{m_W}\ N_{13}}{\tan\theta_W\ N_{11}} =: \frac{1}{\cos\beta}\frac{m_\tau}{m_W} x_h. \tag{16}$$

One sees from eq. (13) that the transformation between the two cases $m^2_{LL} > m^2_{RR}$, whose hierarchy is motivated by the MSSM as far as no addional D–terms have to be included, and $m^2_{LL} < m^2_{RR}$ lead only to an exchange of $P_{\tilde\tau_1\to\tau} \leftrightarrow P_{\tilde\tau_2\to\tau}$.

2.2.1 $\tan\beta$ dependence of the τ polarization

With the coefficients x_W and x_h in eqn. (13)–(14) the *complete* $\tan\beta$ dependence from the neutralino sector has been separated and the coefficient y_h shows the interplay between the Yukawa coupling and the $\tilde\chi^0_i$ higgsino admixture. All components of (15), (16) are given explicitly as function of the fundamental MSSM parameters in [2] and in an approximation, which is valid for high values of $\tan\beta$ in [7].

One can summarize that the dependence on $\tan\beta$ of $P_{\tilde\tau_{1,2}\to\tau}$ is given in a three-fold way:

1. by the $\tan\beta$ dependence of the mixing angle $\theta_{\tilde\tau}$ (3) and the off-diagonal term in (1);

2. by the coefficients x_W and x_h, which corresponds to the $\tan\beta$ dependence of the neutralino sector,

3. by the coefficient y_h which corresponds to the $\tilde\tau$ Yukawa coupling $\frac{1}{\cos\beta}\frac{m_\tau}{m_W}$.

One sees clearly from (16) that for a given mixing angle θ_τ or ξ only the coefficient y_h contains a strong dependence on $\tan\beta$ of $P_{\tilde\tau_1\to\tau}$ from the Yukawa coupling. Obviously a sufficient large higgsino admixture part x_h has to be for that.

The $\tan\beta$ dependence of x_W and x_h is weak since the neutralino mass eigenvalues m^2_i as well as the components of the eigenvectors N_{ij} become in the high $\tan\beta$ approximation $f(1 + const \times \frac{1}{\tan\beta})$. Therefore it is a good approximation to take all neutralino mixing contributions in the high $\tan\beta$ approximation.

$\tilde{\chi}_1^0$	$P_{\tilde{\tau}_1\to\tau}$	$P^{\xi\to\pm\infty}_{\tilde{\tau}_1\to\tau}$	$P^{\xi\to 0}_{\tilde{\tau}_1\to\tau}$	$P_{\tilde{\tau}_2\to\tau}$	$P^{\xi\to\pm\infty}_{\tilde{\tau}_2\to\tau}$	$P^{\xi\to 0}_{\tilde{\tau}_2\to\tau}$
a) $\approx N_{11}$	$\frac{3\sqrt{1+\xi^2}+5}{5\sqrt{1+\xi^2}+3}$	$\frac{3}{5}$	$+1$	$\frac{3\sqrt{1+\xi^2}-5}{5\sqrt{1+\xi^2}-3}$	$\frac{3}{5}$	-1
b) $\approx c_{11}N_{11}$ $+c_{12}N_{12}$	$\frac{(4-x_W^2)\sqrt{1+\xi^2}+(4+x_W^2)}{(4+x_W^2)\sqrt{1+\xi^2}+(4-x_W^2)}$	$\frac{4-x_W^2}{4+x_W^2}$	$+1$	$\frac{(4-x_W^2)\sqrt{1+\xi^2}-(4+x_W^2)}{(4+x_W^2)\sqrt{1+\xi^2}-(4-x_W^2)}$	$\frac{4-x_W^2}{4+x_W^2}$	-1
c) $\approx N_{13}$	$\frac{-1}{\sqrt{1+\xi^2}}$	0	-1	$\frac{+1}{\sqrt{1+\xi^2}}$	0	$+1$

Table 1: τ polarization for extrema for the mixing angle and specific neutralino mixing.

2.3 Limiting cases: extrema for the mixing angle

Assuming no constraints for the τ mass parameters one can vary the mixing angle $\theta_{\tilde{\tau}}$ from 0 to π, i.e. ξ from $0 \to \pm\infty$, and we get for these extrema the following expressions:

$$P^{\xi\to\pm\infty}_{\tilde{\tau}_1\to\tau} = \frac{4 - x_W^2 + 2\,(2 + x_W)\,y_h}{4 + x_W^2 - 2\,(+2 - y_h - x_W)\,y_h} \tag{17}$$

$$P^{\xi\to 0}_{\tilde{\tau}_1\to\tau} = \frac{4 - y_h^2}{4 + y_h^2} \tag{18}$$

$$P^{\xi\to\pm\infty}_{\tilde{\tau}_2\to\tau} = \frac{4 - x_W^2 - 2\,(2 + x_W)\,y_h}{4 + x_W^2 + 2\,(+2 - y_h - x_W)\,y_h} \tag{19}$$

$$P^{\xi\to 0}_{\tilde{\tau}_2\to\tau} = \frac{-x_W^2 + y_h^2}{x_W^2 + y_h^2} \tag{20}$$

The formulae for the tau polarization even simplifies for specific neutralino mixing cases. We give the corresponding expressions in Table 1 for the processes $\tilde{\tau}_1 \to \tau\tilde{\chi}_1^0$ and $\tilde{\tau}_2 \to \tau\tilde{\chi}_1^0$. The conventions are chosen so that in a no–mixing case, i.e. $\tilde{\tau}_1 \to \tilde{\tau}_R < \tilde{\tau}_2 \to \tilde{\tau}_L$, and when only a pure U(1) gauge coupling interacts between $\ell\tilde{\ell}\tilde{\chi}^0$, the polarization $P_{\tilde{\tau}_1^- \to \tau_R^-} = +1$ and $P_{\tilde{\tau}_2^- \to \tau_L^-} = -1$. Studying the polarization of the antiparticle τ^+ one has to take into account that the SUSY partner of $\tau^+_{L,R}$ is $\tilde{\tau}^+_{R,L}$, so that $P_{\tilde{\tau}_R^+ \to \tau_L^+} = -1$ and $P_{\tilde{\tau}_L^+ \to \tau_R^+} = +1$. We show in Fig. 1 representative examples for illustration.

The corresponding plot for case a), Table 1, is given in Fig. 1a. The asymptotic limit $\frac{3}{5}$ for $\xi \to \pm\infty$ can clearly be seen.

The corresponding plot for case b), Table 1, is given in Fig. 1b. The asymptotic limit depends now on the gaugino parameters and since the wino fraction x_W^2 is less than 1 the polarization $P_{\tilde{\tau}_1\to\tau}$ gets closer to 1 and will always be higher than the asyptotic value $\frac{3}{5}$ of the pure bino case a).

In the pure higgsino case c), Table 1, the no–stau–mixing case shows a helicity flipping behaviour, $P_{\tilde{\tau}_1\to\tau} \to -1$ and $P_{\tilde{\tau}_2\to\tau} \to +1$, which is the typical feature of the Yukawa couplings.

To give a feeling even for the neutralino mixing effects we plot in Fig. 2a the polarizations $P_{\tilde{\tau}_1\to\tau}$, $P_{\tilde{\tau}_2\to\tau}$ for the mixed case $x_W = 0.8$ and $y_h = 0.6$. The polarizations even interchange for a specific mixing angle.

So if the fraction of the higgsino mixing part is small compared to the gaugino mixing one expects $P_{\tilde{\tau}_1\to\tau}$ close to unity and $P_{\tilde{\tau}_2\to\tau}$ variable in a large range, eqn. (17)–(20). This is the case for all the SPS scenarios [8] as shown in the Table 2.

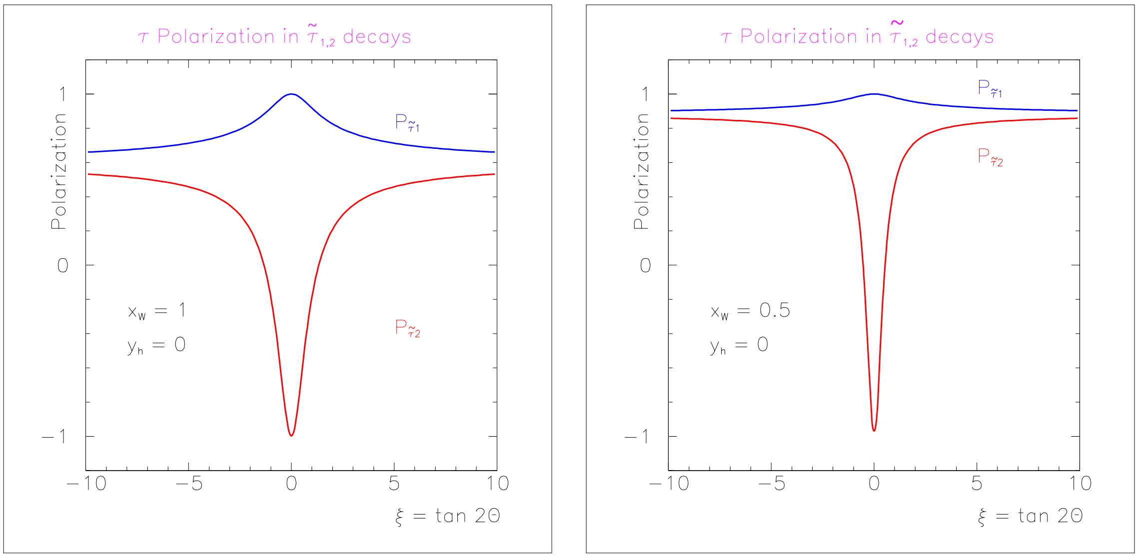

Figure 1: The dependence of the tau polarizations $P_{\tilde{\tau}_1 \to \tau}$ and $P_{\tilde{\tau}_2 \to \tau}$ from the mixing angles ξ for a pure bino case (left) with the neutralino mixing variables $x_W = 1$, $y_h = 0$ and for a pure gaugino case (right) with $x_W = 0.5$, $y_h = 0$.

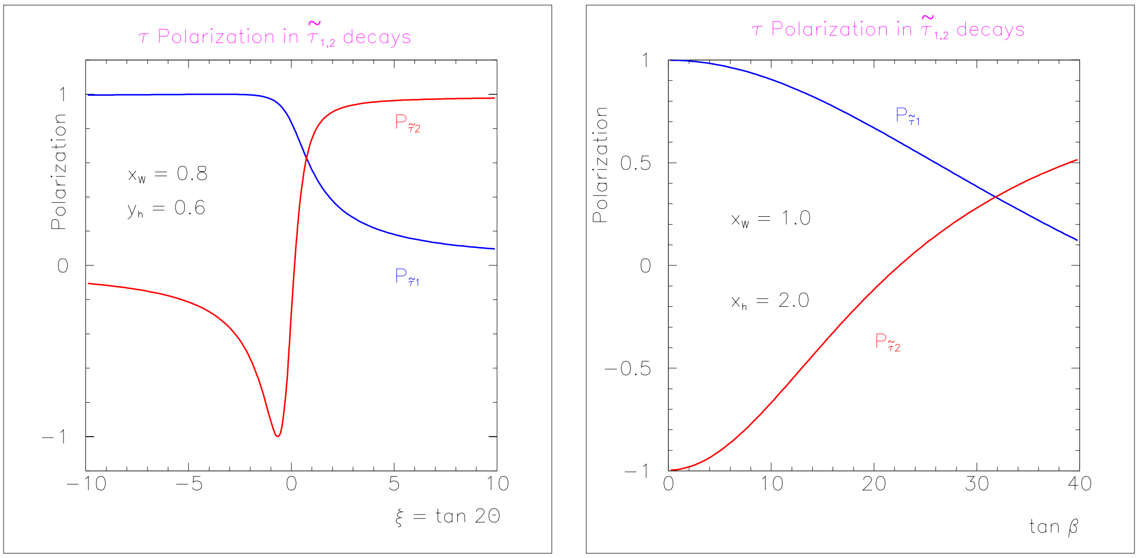

Figure 2: The dependence of the tau polarizations $P_{\tilde{\tau}_1 \to \tau}$ and $P_{\tilde{\tau}_2 \to \tau}$ from the mixing angles ξ (left) for a mixed case with the neutralino mixing variables $x_W = 0.8$, $y_h = 0.6$ and as function of $\tan\beta$ (right) where the mixing angle is chosen to be small $\xi \to 0$. In this case $\tilde{\tau}_1 \to \tilde{\tau}_R$, $\tilde{\tau}_2 \to \tilde{\tau}_L$.

In order to demonstrate the $\tan\beta$ dependence coming from the interplay between the Yukawa coupling and the higgsino admixture of the LSP we show in Fig. 2b the polarizations $P_{\tilde{\tau}_1 \to \tau}$ and $P_{\tilde{\tau}_2 \to \tau}$ for the case with a rather large higgsino admixture and set

$\xi \to 0$, i.e. $\tilde{\tau}_1 \to \tilde{\tau}_R$ and $\tilde{\tau}_2 \to \tilde{\tau}_L$.

Parameter Point	$\tan\beta$	τ Polarization		slopes	
		$P_{\tilde{\tau}_1\to\tau}$	$P_{\tilde{\tau}_2\to\tau}$	$d(P_{\tilde{\tau}_1\to\tau})/d(\tan\beta)$	$d(P_{\tilde{\tau}_2\to\tau})/d(\tan\beta)$
SPS 1a	10	98.1 %	-50 %	-0.3%	5.0%
SPS 1b	30	97.0 %	-40 %	-0.1%	1.6%
SPS 3	10	99.2 %	-80 %	-0.1%	2.4%
SPS 4	50	99.6 %	-62 %	$+0.1\%$	-2.0%
SPS 5	5	97.8 %	-60 %	-0.6%	7.0%
SPS 6	10	99.0 %	-65 %	-0.2%	4.0%

Table 2: Tau polarization and $\tan\beta$ slopes for the SPS scenarios [8]. The program ISAJET 7.58 [9] has been used for the parameter evaluation. The SPS 2 focuspoint point scenario does not have a stable slope.

In order to get an impression in which ranges of the parameter space the tau polarizations from $\tilde{\tau}_{1,2}$ decays may have large variations, we show in Fig. 3 the polarizations $P_{\tilde{\tau}_{1,2}\to\tau}$ as functions of M_2 and μ. The other relevant parameters have been chosen to $\tan\beta = 40$, $M_L = 300$ GeV, $M_E = 150$ GeV, $A_\tau = -254.2$ GeV. Both plots show that one could get high values for the polarization for a large region of the $M_2 - \mu$ parameter space. In particular in the higgsino–like region with $\mu < M_2$ the polarization $P_{\tilde{\tau}_1\to\tau}$ is very variable.

3 MSSM parameter determination for high $\tan\beta$

3.1 Parameter determination in the $\tilde{\tau}_{1,2}$ sector

We assume that one can measure at a high $\mathcal{L}$ LC the masses with rather high accuracy of about % level via mass threshold scans.

We study the light system $e^+e^- \to \tilde{\tau}_1\tilde{\tau}_1$ and determine the mixing angle $\cos\theta_{\tilde{\tau}}$ via polarized cross sections $\sigma(e^+_{L,R}e^-_{R,L} \to \tilde{\tau}_1\tilde{\tau}_1) \sim \cos(2\theta_{\tilde{\tau}})$ or via the asymmetry $A_{pol} = (\sigma_L - \sigma_R)/(\sigma_L + \sigma_R)$. If one wants to derive $\theta_{\tilde{\tau}}$ unambigously one would also need $\sigma(e^+e^- \to \tilde{\tau}_1\tilde{\tau}_2) \sim \sin(2\theta_{\tilde{\tau}})$, which is more tricky because of the difficult reconstruction of the ρ, π decay products from the $\tilde{\tau}_2$ decays.

We determine the mixing angle via measuring the production $e^-e^+ \to \tilde{\tau}_1^-\tilde{\tau}_1^+$ in the configuration (RL), $P(e^-) = +80\%$, $P(e^+) = -60\%$, since in this case the worse background from W^+W^- is strongly suppressed and in contrary the signal is enhanced. From the rates σ_{RL} we could derive the mixing angle rather accurately, $\cos\theta_\tau = 0.15 \pm 0.01$, Fig.4*, if we measure $\sigma_{RL}(\tilde{\tau}_1^-\tilde{\tau}_1^+) = 112$ fb at $\sqrt{s} = 500$ GeV and assume that a statistical error of $\pm 1\sigma$ was taken into account. In Table 3 we list the corresponding cross sections $\sigma(\tilde{\tau}_i\tilde{\tau}_j)$ for $\sqrt{s} = 500$ GeV and $\sqrt{s} = 800$ GeV.

In principle one can alternatively also derive the mixing angle via measuring the polarization asymmetry A_{pol} of the production process. Taking rates with left polarized

*Generically the cross section is a quadratic polynom in $\cos(2\theta_{\tilde{\tau}})$. With a suitably high degree of beam polarization, however, the quadratic terms are suppressed and only one solution survives.

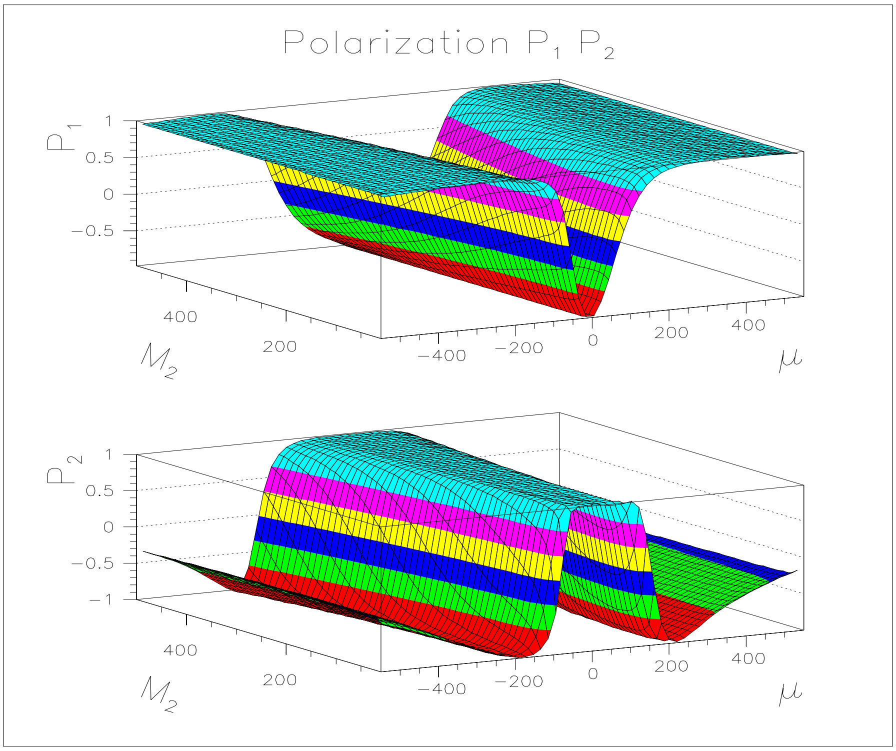

Figure 3: Contourplots in the M_2-μ plane for $P_{\tilde{\tau}_1 \to \tau}$ and $P_{\tilde{\tau}_2 \to \tau}$.

electrons leads, however, to an enhancement of the strong WW background and a tricky analysis would be needed to get the wanted experimental information.

3.2 Measurement of the τ polarization

We have computed the cross sections and disctributions for the 4 final state particle processes with all spin correlations taken into account:

$$e^+e^- \to \tilde{\tau}_1^+\tilde{\tau}_1^- \quad \text{with} \quad \tilde{\tau}_1^- \to \tau_{L,R}^-\tilde{\chi}_1^0 \quad \text{and} \quad \tau_{L,R}^- \to \nu_\tau \pi^- \tag{21}$$

In order to measure the polarization of the τ's one has to study their decays into π or ρ–mesons and to fit kinematically the energy distributions of the decay products [3]. The

	$\sqrt{s} = 500$ GeV	$\sqrt{s} = 800$ GeV		
$(P(e^-), P(e^+))$	$\sigma(e^-e^+ \to \tilde{\tau}_1\tilde{\tau}_1)$	$\sigma(e^-e^+ \to \tilde{\tau}_1\tilde{\tau}_1)$	$\sigma(e^-e^+ \to \tilde{\tau}_1\tilde{\tau}_2)$	$\sigma(e^-e^+ \to \tilde{\tau}_2\tilde{\tau}_2)$
unpolarized	48.6 fb	29.7 fb	0.2 fb	12.0 fb
$(-0.8, 0)$	25.6 fb	15.9 fb	0.3 fb	18.3 fb
$(+0.8, 0)$	71.6 fb	43.5 fb	0.2 fb	5.7 fb
$(-0.8, 0.6)$	31.6 fb	19.8 fb	0.4 fb	28.8 fb
$(+0.8, -0.6)$	112.1 fb	68.1 fb	0.3 fb	6.7 fb

Table 3: Cross sections for the reference scenario with polarized beams. The pairs $\tilde{\tau}_1\tilde{\tau}_2$, although kinematically accessible at $\sqrt{s} = 500$ GeV, lead to rates less than 0.1 fb.

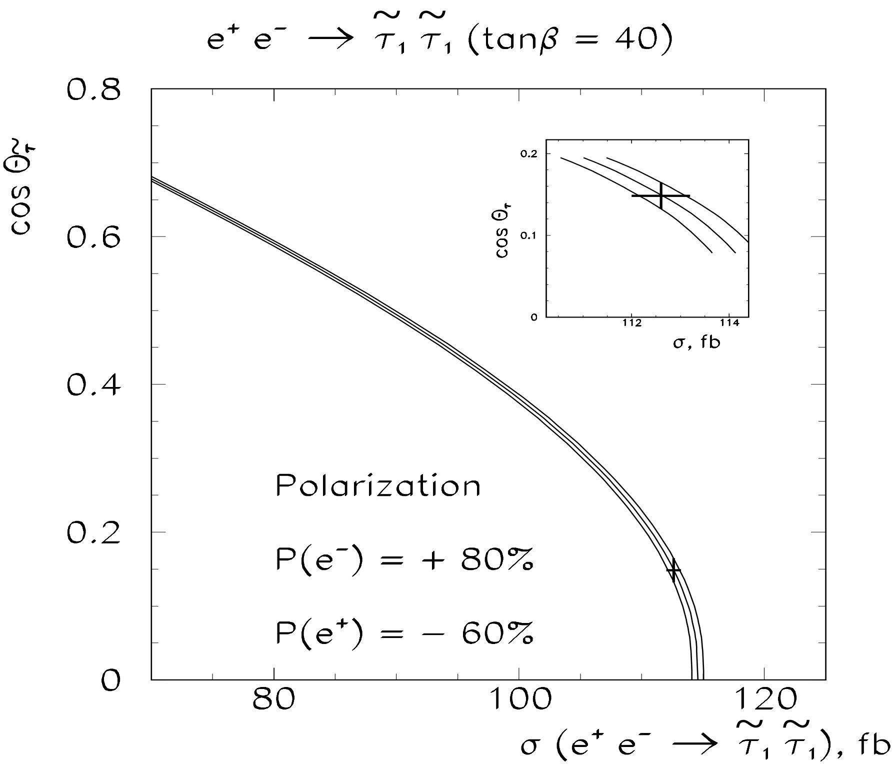

Figure 4: Mixing angle $\cos\theta_{\tilde{\tau}} = 0.15 \pm 0.01$ via measurement of the polarized cross section $\sigma(e^+e^- \to \tilde{\tau}_1\tilde{\tau}_1)$ at $\sqrt{s} = 500$ GeV and $P(e^-) = +80\%$, $P(e^+) = -60\%$ and on the assumption that 1 standard deviation as statistical error was taken into account.

decays of polarized τ to π- and ρ-mesons are implemented into CompHEP and are cross checked with TAUOLA [10].

The calculations are performed by means of the CompHEPV41 [11] with implemented MSSM and mSUGRA models. In order to estimate an accuracy of polarization measurements we have generated a number of unweighted events which correspond to the production cross section with the corresponding decay branching fractions and the overall registration efficiency of signal events:

$$N = \sigma(e^+e^- \to \tilde{\tau}_1^- \tilde{\tau}_1^+) * Br(\tilde{\tau}_1^- \to \tau\tilde{\chi}_1^0) * Br(\tau \to \nu_\tau \pi^-) * \mathcal{L} * \mathrm{eff} \qquad (22)$$

Based on the results of MC analysis given in [3] we assumed 30% for the efficiency and we assume $500 fb^{-1}$ for a luminosity. So in total about 3300 umweighted events have been simulated.

The pion energy distribution is shown in the Fig. 5 left. The distribution is described by the formular presented in a slightly different form as that one given in [3].

$$\frac{1}{\sigma}\frac{d\sigma}{dy_\pi}(e^+e^- \to \tilde{\tau}_1^- \tilde{\tau}_1^+ \to \tau\tilde{\chi}_1^0\tilde{\tau}_1^+ \to \nu_\tau\pi^-\tilde{\chi}_1^0\tilde{\tau}_1^+)$$
$$= \frac{1}{x_{max} - x_{min}} \begin{cases} (1 - P_{\tilde{\tau}_1\to\tau}) log\frac{x_{max}}{x_{min}} + 2P_{\tilde{\tau}_1\to\tau} y_\pi (\frac{1}{x_{min}} - \frac{1}{x_{max}}), & 0 < y_\pi < x_{min} \\ (1 - P_{\tilde{\tau}_1\to\tau}) log\frac{x_{max}}{y_\pi} + 2P_{\tilde{\tau}_1\to\tau}(1 - \frac{y_\pi}{x_{max}}), & x_{min} < y_\pi \end{cases} \qquad (23)$$

where $y_\pi = \frac{2E_\pi^{CMS}}{\sqrt{s}}$, $x_{min} = \frac{2E_\tau^{min}}{\sqrt{s}}$, $x_{max} = \frac{2E_\tau^{max}}{\sqrt{s}}$, $E_\tau^{max,min} = \frac{E_\tau^* \pm p_\tau \beta_{\tilde{\tau}}}{\sqrt{1-\beta_{\tilde{\tau}}^2}}$, $\beta_{\tilde{\tau}} = \sqrt{1 - \frac{4M_{\tilde{\tau}}^2}{s}}$, $E_\tau^* = \frac{M_{\tilde{\tau}}^2 - M_{\tilde{\chi}_1^0}^2 + M_\tau^2}{2M_{\tilde{\tau}}}$, $p_\tau = \sqrt{E_\tau^{*2} - M_\tau^2}$.

The fit of the energy distribution by the above formula (23) gives for the tau polarization $P_{\tilde{\tau}_1 \to \tau}$ about $57\% \pm 3\%$, compared to the theoretical value of polarization for the reference point of 56%.

The presented example is meant as an illustration only. Measurements of the τ lepton decay mode to ρ with its subsequent decays may help to get even a better accuracy.

3.3 Determination of $\tan\beta$ and the A_τ parameter

In the last section we have shown that $P_{\tilde{\tau}_1 \to \tau}$ could be measured within about 5% accuracy. We derive the value of $\tan\beta$ via inversion of (13) for the calculated values of x_W and x_h and the measured mixing angle $\cos\theta_{\tilde{\tau}}$, see Fig. 5 right, and we determine in our scenario the high $\tan\beta$ with an error of about 5%:

$$\tan\beta = 40 \pm 2 \tag{24}$$

In the case that also the heavier mass of $\tilde{\tau}_2$ can be determined in the experiment, we can even determine the parameter A_τ without assuming anything about the SUSY breaking scheme and GUT relations:

$$A_\tau = \frac{1}{m_\tau}(\frac{1}{2}(m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_2}^2)\sin(2\theta_\tau) + m_\tau \mu \tan\beta). \tag{25}$$

With an error of about 5% in $\tan\beta$ and of about 5% in $m_{\tilde{\tau}_2}$ one could derive A_τ with an accuracy about 8%. For the application of this method on the $\tilde{b}$ and $\tilde{t}$ sector see [7].

4 Conclusions

Tau polarization from stau decays might be an important variable for the determination of the fundamental MSSM parameters. It serves as a 'bridge' between chargino/neutralino and stau sectors.

The stau mixing angles can be precisely determined via polarized rates and in case that the decay neutralino has a suitable higgsino admixture, the study of $P_{\tilde{\tau}_1 \to \tau}$ leads to an accurate determination of $\tan\beta$ even in the case of high $\tan\beta$. For these procedure it is enough to measure only the light system: $m_{\tilde{\tau}_1}$ and $\sigma(e^+e^- \to \tilde{\tau}_1\tilde{\tau}_1)$. In case that also $m_{\tilde{\tau}_2}$ can be measured, even the parameter A_τ can be derived without an assumption about the underlying SUSY breaking scheme.

For a given example we explored $P_{\tilde{\tau}_1 \to \tau}$ and showed that one can determine simultanously A_τ and $\tan\beta$, e.g. in the case of high $\tan\beta$, within 5% accuracy.

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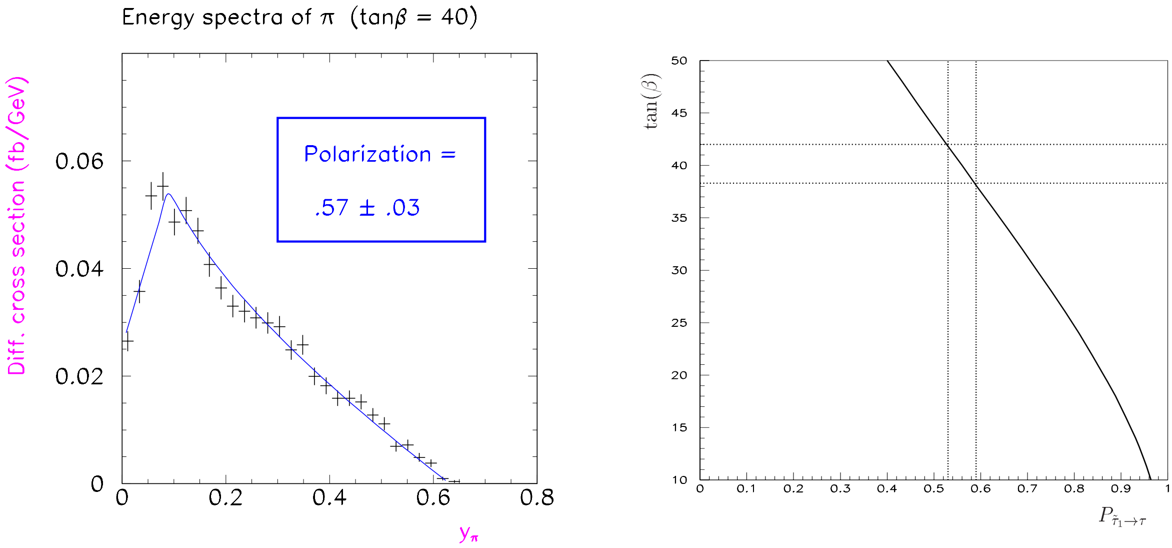

Figure 5: Left: The pion energy histogram for the unweighted events and the 1 σ fit by the distribution formula (23). The total event number is 3300 events which corresponds to an integrated luminosity of $\mathcal{L} = 500$ fb^{-1}. For beam polarization we choose $P_{e^-} = +80\%$, $P_{e^+} = -60\%$. Right: The determination of $\tan\beta = 40 \pm 2$ as function of the measured polarization $P_{\tilde{\tau}_1\to\tau} = 57 \pm 3\%$.

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