Full one-loop corrections to sfermion decays

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Abstract

We analyze the partial decay widths of sfermions decaying into charginos and neutralinos $\Gamma(\tilde{f} \to f'\chi)$ at the one-loop level. We present the renormalization framework, and discuss the value of the corrections for top- and bottom-squark decays.

1 Introduction

One of the basic predictions of Supersymmetry (SUSY) is the equality between the couplings of SM particles and that of their superpartners. The simplest processes in which this prediction could be tested, is the partial decay widths of sfermions into Standard Model (SM) fermions and charginos/neutralinos:

$$\Gamma(\tilde{f} \to f'\chi) \ . \tag{1}$$

By measuring these partial decay widths (or the corresponding branching ratios) one could measure the fermion-sfermion-chargino/neutralino Yukawa couplings and compare them with the SM fermion gauge couplings.

We have computed the full one-loop electroweak corrections to the partial decay widths (1). As we will show, the radiative corrections induce finite shifts in the couplings which are non-decoupling.

The QCD corrections to the process (1) were computed in [1], and the Yukawa corrections to bottom-squarks decaying into charginos was given in [2]. Here we present the last step, namely, the full electroweak corrections in the framework of the Minimal Supersymmetric Standard Model (MSSM). Full details of the present work can be found in [3].

2 Renormalization and radiative corrections

The computation to one-loop level of the partial decay width (1) requires the renormalization of the full MSSM Lagrangian, taking into account the relations among the different sectors and the mixing parameters. We choose to work in an on-shell renormalization scheme, in which the renormalized parameters are the measured quantities. The SM sector is renormalized according to the standard on-shell SM α-scheme [4], and the MSSM Higgs sector (in particular the renormalization of $\tan\beta$) is treated as in [5].

As far as the sfermion sector is concerned, we follow the procedure described in [2]. However, in the present analysis we treat simultaneoulsy top-squarks and bottom-squarks. Due to $SU(2)_L$ invariance the parameters in these two sectors are not independent, and we can not supply with independent on-shell conditions for both sectors. We choose as input parameters the on-shell masses of both bottom-squarks, the lightest top-squark mass, and the mixing angles in both sectors[1]:

$$(m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_b, m_{\tilde{t}_2}, \theta_t), \qquad m_{\tilde{f}_1} > m_{\tilde{f}_2}. \tag{2}$$

The remaining parameters are computed as a function of those in (2). In particular, the trilinear soft-SUSY-breaking couplings read:

$$A_{\{b,t\}} = \mu\{\tan\beta, \cot\beta\} + \frac{m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2}{2\,m_f} \sin 2\,\theta_f\,, \tag{3}$$

with $\tan\beta = v_2/v_1$, the ratio of the vacuum expectation values of the two Higgs boson doublets. The approximate (necessary) condition to avoid colour-breaking minima in the MSSM Higgs potential [6],

$$A_q^2 < 3\,(m_{\tilde{t}}^2 + m_{\tilde{b}}^2 + M_H^2 + \mu^2)\,, \tag{4}$$

imposes a tight correlation between the sfermion mass splitting and the mixing angle at large $\tan\beta$. Since the heaviest top-squark mass $(m_{\tilde{t}_1})$ is not an input parameter, it receives finite radiative corrections:

$$\Delta m_{\tilde{t}_1}^2 = \delta m_{\tilde{t}_1}^2 + \Sigma_{\tilde{t}_1}(m_{\tilde{t}_1}^2)\,, \tag{5}$$

where $\delta m_{\tilde{t}_1}^2$ is a combination of the counterterms of the parameters in (2), and the counterterms of the gauge and Higgs sectors.

The chargino/neutralino sector contains six particles, but only three independent input parameters: the soft-SUSY-breaking $SU(2)_L$ and $U(1)_Y$ gaugino masses (M and M'), and the higgsino mass parameter (μ). The situation in this sector is quite different from the sfermion case, since in this case no independent counterterms for the mixing matrix elements can be introduced. We stick to the following procedure: First, we introduce a set of *renormalized* parameters (M, M', μ) in the expression of the chargino and neutralino matrices ($\mathcal{M}$ and $\mathcal{M}^0$), and diagonalize them by means of unitary matrices $M_D = U^*\mathcal{M}V^\dagger$, $M_D^0 = N^*\mathcal{M}^0 N^\dagger$. Now U, V and N must be regarded as *renormalized mixing matrices*. The counterterm mass matrices are then $\delta M_D = U^*\delta\mathcal{M}V^\dagger$, $\delta M_D^0 = N^*\delta\mathcal{M}^0 N^\dagger$, which are non-diagonal. At this point, we introduce renormalization conditions for certain elements of δM_D and δM_D^0. In particular, we use on-shell renormalization conditions for the two chargino masses (M_1 and M_2), which allows to compute the counterterms δM and $\delta\mu$. This information, together with the on-shell condition for the lightest neutralino mass (M_1^0) allows to derive the expression for the counterterm $\delta M'$. The other neutralino masses ($M_{2,3,4}^0$) receive radiative corrections. In this framework the renormalized one-loop chargino/neutralino 2-point functions are *non*-diagonal. Therefore one must

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[1]Throughout this work we make use of third generation notation. The notation is as in [2, 3].

take into account this mixing either by including explicitly the reducible $\chi_r - \chi_s$ mixing diagrams, or by means of external mixing wave-function terms ($\mathcal{Z}^{0\beta\alpha}_{\{L,R\}}$, $\mathcal{Z}^{-ij}_{\{L,R\}}$). See Refs. [7] for different (but one-loop equivalent) approaches to the renormalization of the chargino/neutralino sector.[2]

The complete one-loop computation consists of:

- renormalization constants for the parameters and wave functions in the bare Lagrangian,

- one-loop one-particle irreducible three-point functions,

- mixing terms among the external charginos and neutralinos,

- soft- and hard- photon bremsstrahlung.

All kind of MSSM particles are taken into account in the loops: SM fermions, sfermions, electroweak gauge bosons, Higgs bosons, Goldstone bosons, Fadeev-Popov ghosts, charginos, neutralinos. The computation is performed in the 't Hooft-Feynman gauge, using dimensional reduction for the regularization of divergent integrals. The loop computation itself is done using the computer algebra packages *FeynArts 3.0* and *FormCalc 2.2* [9, 10]. The numerical evaluation of one-loop integrals makes use of *LoopTools 1.2* [10].[3]

3 Results

The results show the very interesting property that none of the particles of the MSSM decouples from the corrections to the observables (1). This can be well understood in terms of renormalization group (RG) running of the parameters and SUSY breaking. Take, e.g., the effects of squarks in the electron-selectron-photino coupling. Above the squark mass scale ($Q > m_{\tilde{q}}$) the electron electromagnetic coupling ($\alpha(Q)$) is equal (by SUSY) to the electron-selectron-photino coupling ($\tilde{\alpha}(Q)$), and both couplings run according to the same RG equations. At $Q = m_{\tilde{q}}$ the squarks *decouple* from the RG running of the couplings. At $Q < m_{\tilde{q}}$, $\alpha(Q)$ runs due to the contributions from pure quark loops, but $\tilde{\alpha}(Q)$ does not run anymore, and it is *frozen* at the squark scale, that is: $\tilde{\alpha}(Q < m_{\tilde{q}}) = \alpha(m_{\tilde{q}})$. Therefore, when comparing these two couplings at a scale $Q < m_{\tilde{q}}$, they differ by the logarithmic running of $\alpha(Q)$ from the squark scale to Q: $\tilde{\alpha}(Q)/\alpha(Q) - 1 = \beta \log(m_{\tilde{q}}/Q)$.

The above discussion has two important consequences:

1. The non-decoupling can be used to extract information of the high-energy part of the SUSY spectrum: one can envisage a SUSY model in which a significant splitting among the different SUSY masses exists, e.g. $m_{\tilde{q}} \gg m_{\tilde{l}}$, where the sleptons lie below the production threshold in an e^+e^- linear collider, but the squarks are above it. By means of high precision measurements of the lepton-slepton-chargino/neutralino couplings one might be able to extract information of the squark sector of the model, to be checked with the available data from the LHC.

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[2]See Ref. [8] for a review of radiative corrections to SUSY processes.

[3]The resulting FORTRAN code can be obtained from
http://www-itp.physik.uni-karlsruhe.de/~guasch/progs/.

2. By the same token, it means that the value of the radiative corrections depends on all parameters of the model, and we can not make precise quantitative statements unless the full SUSY spectrum is known. This drawback can be partially overcome by the introduction of *effective coupling matrices*, which can be defined as follows. The subset of fermion-sfermion one-loop contributions to the self-energies of gauge-boson, Higgs-bosons, Goldstone-bosons, charginos and neutralinos form a gauge invariant finite subset of the corrections. Therefore these contributions can be absorbed into a finite shift of the chargino/neutralino mixing matrices U, V and N appearing in the couplings: $U^{eff} = U + \Delta U^{(f)}$, $V^{eff} = V + \Delta V^{(f)}$, $N^{eff} = N + \Delta N^{(f)}$. In this way we can *decouple* the computation of the *universal* (or *super-oblique* [11]) corrections. These corrections contain the non-decoupling logarithms from sfermion masses.

As an example of the *universal* corrections we have computed the electron-selectron contributions to the $\Delta U^{(f)}$ and $\Delta V^{(f)}$ matrices, assuming zero mixing angle in the selectron sector ($\theta_e = 0$), we have identified the leading terms in the approximation $m_{\tilde{e}_i}, m_{\tilde{\nu}} \gg (M_W, M_i) \gg m_e$, and analytically cancelled the divergences and the renormalization scale dependent terms; finally, we have kept only the terms logarithmic in the slepton masses. The result for $\Delta U^{(f)}$ reads as follows:

$$\Delta U_{i1}^{(f)} = \frac{\alpha}{4\pi s_W^2} \log\left(\frac{M_{\tilde{e}_L}^2}{M_X^2}\right) \left[\frac{U_{i1}^3}{6} - U_{i2} \frac{\sqrt{2}\, M_W\, (M\, c_\beta + \mu\, s_\beta)}{3\, (M^2 - \mu^2)\, (M_1^2 - M_2^2)^2} \left(M^4 - M^2\, \mu^2 + \right.\right.$$
$$\left.\left. + 3\, M^2\, M_W^2 + \mu^2\, M_W^2 + M_W^4 + M_W^4\, c_{4\beta} + (\mu^2 - M^2)\, M_i^2 + 4\, M\, \mu\, M_W^2\, s_{2\beta}\right)\right],$$

$$\Delta U_{i2}^{(f)} = \frac{\alpha}{4\pi s_W^2} \log\left(\frac{M_{\tilde{e}_L}^2}{M_X^2}\right) U_{i1} \frac{M_W\, (M\, c_\beta + \mu\, s_\beta)}{3\sqrt{2}\, (M^2 - \mu^2)\, (M_1^2 - M_2^2)^2} \times$$
$$\times \left((M^2 - \mu^2)^2 + 4\, M^2\, M_W^2 + 4\, \mu^2\, M_W^2 + 2\, M_W^4 + 2\, M_W^4\, c_{4\beta} + 8\, M\, \mu\, M_W^2\, s_{2\beta}\right)(6)$$

$M_{\tilde{e}_L}^2$ being the soft-SUSY-breaking mass of the $(\tilde{e}_L, \tilde{\nu})$ doublet, whereas M_X is a SM mass. In the on-shell scheme for the SM electroweak theory we define parameters at very different scales, basically $M_X = M_W$ and $M_X = m_e$. These wide-ranging scales enter the structure of the counterterms and so must appear in eq.(6) too. As a result the leading log in the various terms of this equation will vary accordingly. For simplicity in the notation we have factorized $\log M_{\tilde{e}_L}^2/M_X^2$ as an overall factor. In some cases this factor can be very big, $\log M_{\tilde{e}_L}^2/m_e^2$; it comes from the electron-selectron contribution to the chargino-neutralino self-energies.

In Fig. 1 we show the relative correction to the matrix elements of U for a sfermion spectrum around 1 TeV. The thick black lines in Fig. 1 correspond to spurious divergences in the relative corrections due to the renormalization prescriptions. Corrections as large as $\pm 10\%$ can only be found in the vicinity of these divergence lines. However, there exist large regions of the $\mu - M$ plane where the corrections are larger than 2%, 3%, or even 4%.

The effects of the universal corrections to the partial decay widths (1) are shown in Fig. 2 for top- and bottom-squark decays as a function of a common slepton mass. Here (and in most of the discussion below) we show the corrections to the total decay widths
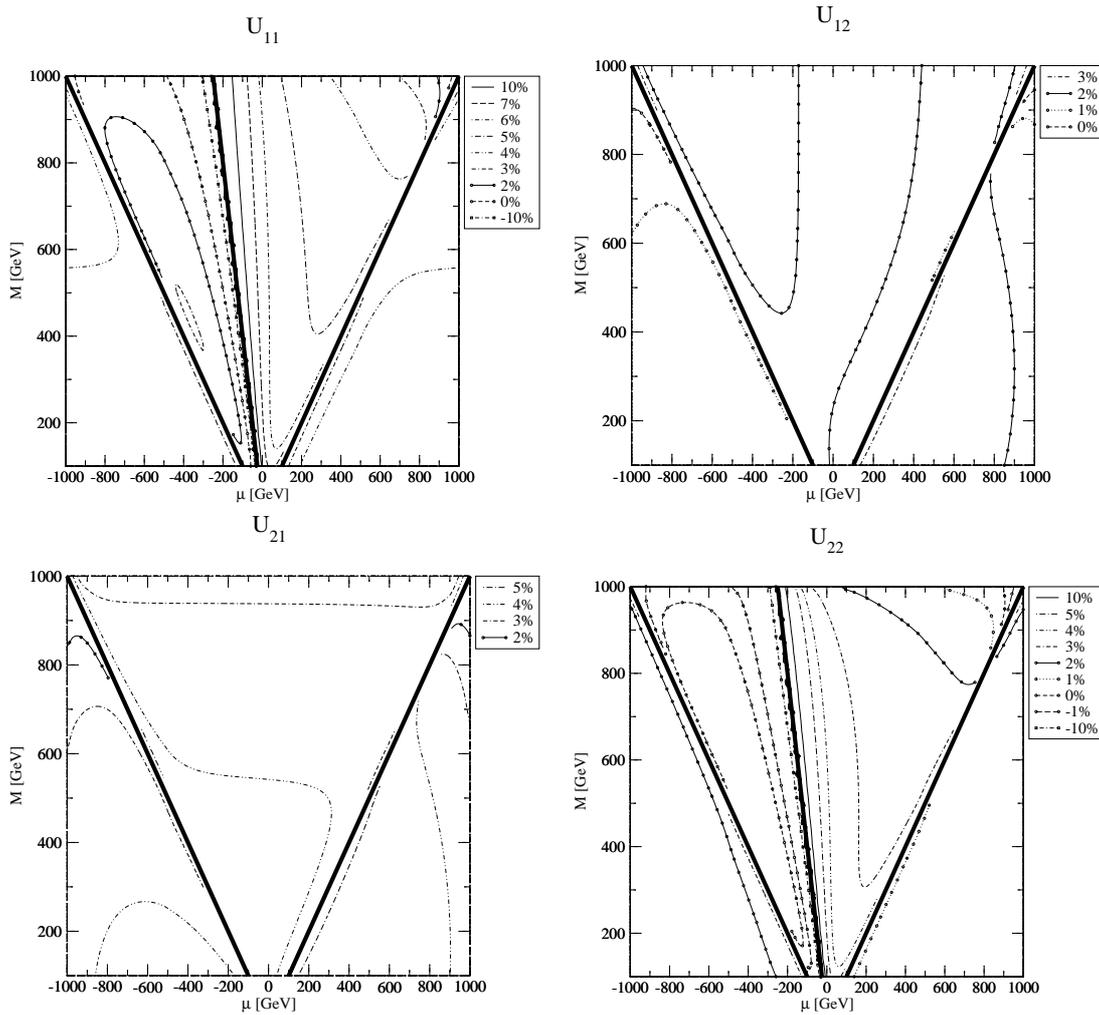
Figure 1: Relative correction to the effective chargino coupling matrix $\Delta U^{(f)}/U$ in the $M - \mu$ plane, for $\tan\beta = 4$ and a sfermion spectrum around 1 TeV ($m_{\tilde{l}_2} = m_{\tilde{d}_2} = m_{\tilde{u}_2} = 1$ TeV, $m_{\tilde{l}_1} = m_{\tilde{d}_1} = m_{\tilde{l}_2} + 5$ GeV, $\theta_l = \theta_q = \theta_b = 0$, $\theta_t = -\pi/5$).

of sfermions into charginos and neutralinos, that is

$$\delta(\tilde{f}_a \to f'\chi) = \frac{\sum_r \left(\Gamma(\tilde{f}_a \to f'\chi_r) - \Gamma^0(\tilde{f}_a \to f'\chi_r) \right)}{\sum_r \Gamma^0(\tilde{f}_a \to f'\chi_r)} \ , \tag{7}$$

with $\chi = \chi^\pm$ or $\chi = \chi^0$. We will not show results for processes whose branching ratio are less that 10% in all of the explored parameter space. The default parameter set used is:

$$\tan\beta = 4 \ , m_t = 175 \text{ GeV} \ , m_b = 5 \text{ GeV} \ , m_{\tilde{b}_2} = m_{\tilde{d}_2} = m_{\tilde{u}_2} = m_{\tilde{e}_2} = 300 \text{ GeV} \ ,$$
$$m_{\tilde{b}_1} = m_{\tilde{d}_1} = m_{\tilde{e}_1} = m_{\tilde{b}_2} + 5 \text{ GeV} \ , m_{\tilde{u}_2} = 290 \text{ GeV} \ , m_{\tilde{t}_2} = 300 \text{ GeV} \ ,$$
$$\theta_b = \theta_d = \theta_u = \theta_e = 0 \ , \theta_t = -\pi/5 \ , \mu = 150 \text{ GeV} \ , M = 250 \text{ GeV} \ , M_{H^\pm} = 120 \text{ GeV} \ ,$$
$$\tag{8}$$

The logarithmic behaviour from eq. (6) is evident in this figure. The logarithmic regime is attained already for slepton masses of order 1 TeV. The universal corrections are seen
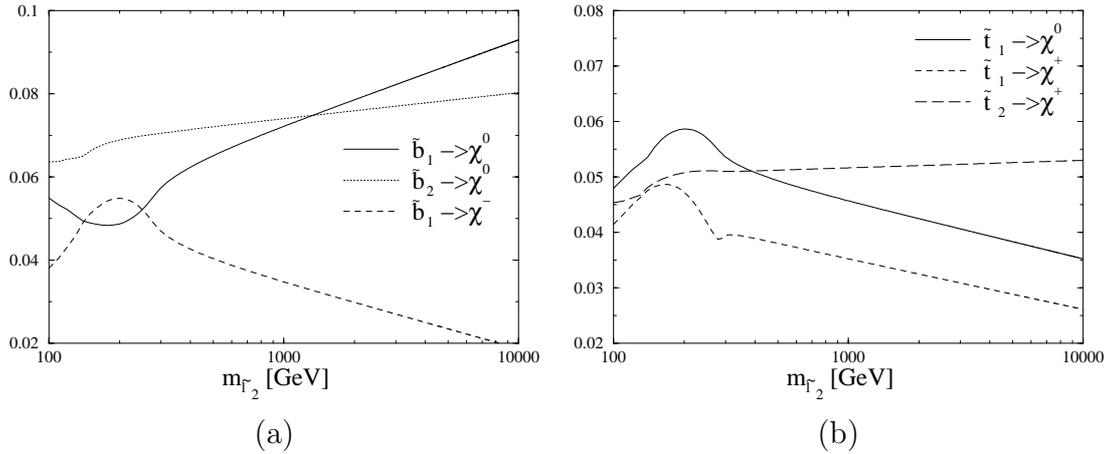
Figure 2: Universal relative corrections (7) to third generation squark partial decay widths as a function of a common slepton mass using the input parameter set (8).

to be positive for all squark decays, ranging between 4% and 7% for slepton masses below 1 TeV.

Although above we have singled out the non-decoupling properties of sfermions, we would like to stress that the whole spectrum shows non-decoupling properties. By numerical analysis we have been able to show the existence of logarithms of the gaugino mass parameters (M/M_X and M'/M_X), and the Higgs mass ($M_{H^\pm}/M_X$). However, due to the complicated mixing structure of the model, we were not able to derive simple analytic expressions containing these non-decoupling logarithms. Note that in *any* observable which includes the fermion-sfermion-chargino/neutralino Yukawa couplings at leading order we will have this kind of corrections, therefore the full MSSM spectrum must be taken into account when computing radiative corrections, since otherwise one could be missing large logarithmic contributions of the heavy masses.

As for the *non-universal* part of the contributions, they show a rich structure, as can be seen in Fig. 3. There we show the evolution of the corrections as a function of the μ parameter for top- and bottom-squark decays. A number of divergences are seen in the figure, ones related to the mass renormalization framework (at $|\mu| = M$), and others due to threshold singularities in the external wave function renormalization constants. It is clear that the precise value of the corrections is very much dependent on the correlation among the different SUSY masses.

An important contribution to the corrections of third-generation sfermion decays is the *threshold correction* to the bottom-quark (τ-lepton) Yukawa coupling ($\Delta m_{\{b,\tau\}}$) [12]. In the processes under study (1) two kind of contributions appear: first, the genuine corrections $\Delta m_{\{b,\tau\}}$ from SUSY loops in the fermion self-energy; and second in the loops of sfermion self-energies mixing different chiral states $\tilde{f}_L \leftrightarrow \tilde{f}_R$. This kind of corrections grow with the sfermion mass splitting, the sfermion mixing angle, and $\tan\beta$.

A complementary set of corrections corresponds to the genuine three-point vertex functions including Higgs bosons in the loops. These contributions are proportional to the soft SUSY-breaking trilinear couplings (3), and therefore potentially large. Concretely, if
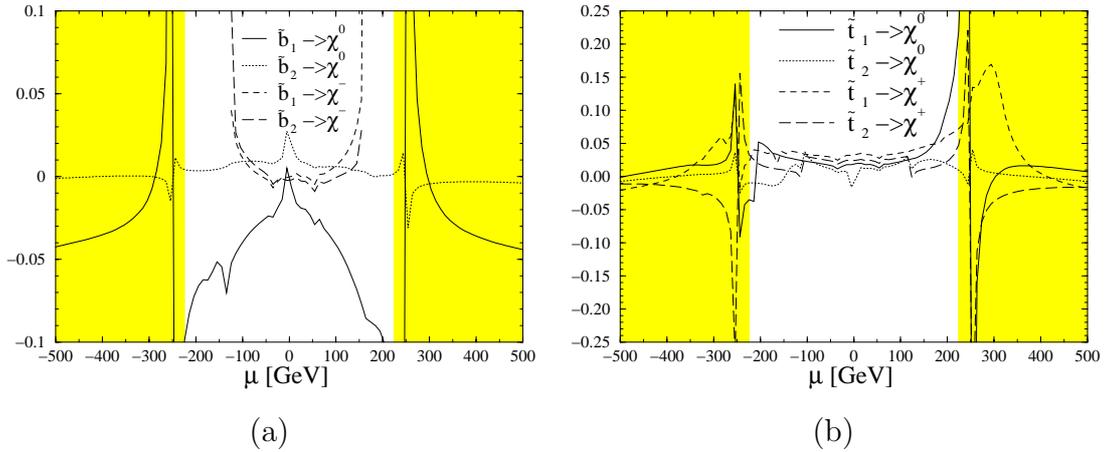
Figure 3: Non-universal corrections to the partial decay width of top- and bottom-squarks as a function of the higgsino mass parameter μ. The shaded regions correspond to the violation of the condition (4).

$\tan\beta$ is large, and the bottom-squark mass splitting (or the mixing angle) is small, the bottom-squark trilinear coupling grows with $\tan\beta$ ($A_b \simeq \mu\tan\beta$), eventually inducing corrections larger than 100%, spoiling the validity of perturbation theory. In Fig. 4a we show the evolution of the corrections to the lightest bottom-squark decay into neutralinos as a function of $\tan\beta$ using the parameter set (8). We see the fast growing of the corrections, reaching -100% at $\tan\beta \simeq 30$. Fortunately, applying the (necessary) restriction (4) keeps the A_q parameter small. In Fig. 4b we show again the evolution of the corrections as a function of $\tan\beta$, but this time keeping a fixed value for the trilinear couplings $A_b = 600$ GeV, $A_t = -78$ GeV. The figure shows that the corrections stay well below 10% all over the $\tan\beta$ range for this channel.

The complementarity between the $\Delta m_{\{b,\tau\}}$-like and the A_f-like corrections is as follows: at large $\tan\beta$, if the bottom-squark mass splitting is large, there will be large corrections of type $\Delta m_{\{b,\tau\}}$; on the other hand, if the bottom-squark mass splitting is small, there will be large corrections of the type A_f. Note that the QCD corrections contain Δm_b terms but not A_f terms. When analyzing QCD corrections alone, one could choose a small splitting, obtaining small corrections, however we have seen that this is inconsistent, so one is forced to a large Δm_b^{QCD} contribution, which can reinforce (or screen) the negative corrections from the standard running of the QCD coupling constant[4].

It is known that the electroweak corrections to any process grow as the logarithm squared of the process energy scale due to the Sudakov double-logs [13]. We have observed this behaviour in the process under study.

At the end of the day, we want to analyze the branching ratios, which are the true observables. For this analysis we have to add the QCD corrections to the EW corrections. Due to the large value of the QCD corrections, we made use of the enhanced resummed expression for the bottom-quark Yukawa coupling [14]. In Table 1 we show the tree-level

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[4]Though it is not possible to separate between standard gluon corrections and gluino corrections, one can talk qualitatively about the contributions of the different sectors.
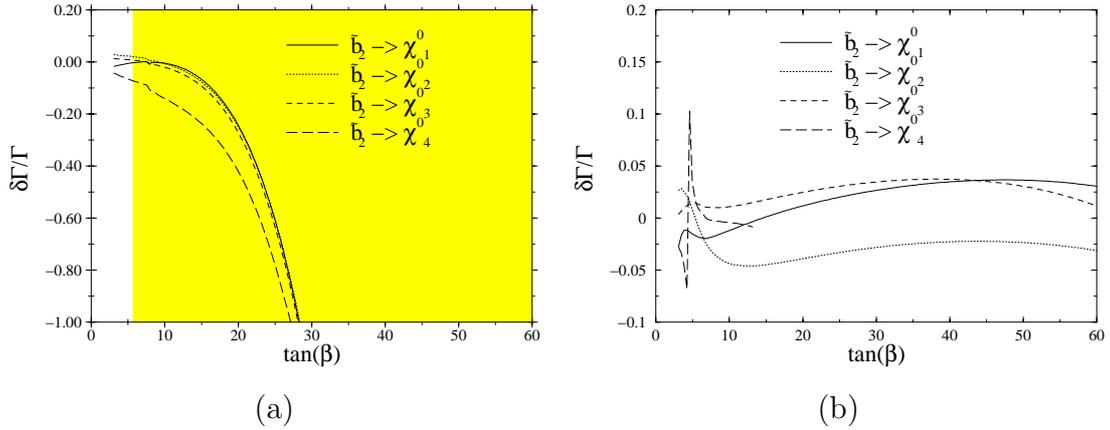
Figure 4: Non-universal relative corrections to the lightest bottom-squark partial decays widths into neutralinos as a function of $\tan\beta$. **a)** Keeping fixed the splitting between the bottom-squarks $m_{\tilde{b}_1} - m_{\tilde{b}_2} = 5$ GeV. **b)** Keeping $A_b = 600$ GeV, $A_t = -78$ GeV. The shaded region corresponds to the violation of the condition (4).

	χ_1^0	χ_2^0	χ_3^0	χ_4^0	χ_1^+	χ_2^+
$BR^{tree}(\tilde{t}_1 \to q\chi)$	0.169	0.249	0.145	-	0.159	0.278
$BR^{QCD}(\tilde{t}_1 \to q\chi)$	0.164	0.257	0.144	-	0.099	0.335
$BR^{total}(\tilde{t}_1 \to q\chi)$	0.177	0.242	0.143	-	0.122	0.316
$BR^{tree}(\tilde{t}_2 \to q\chi)$	0.058	-	-	-	0.942	-
$BR^{QCD}(\tilde{t}_2 \to q\chi)$	0.063	-	-	-	0.937	-
$BR^{total}(\tilde{t}_2 \to q\chi)$	0.065	-	-	-	0.935	-
$BR^{tree}(\tilde{b}_1 \to q\chi)$	0.272	0.092	0.047	0.014	0.575	-
$BR^{QCD}(\tilde{b}_1 \to q\chi)$	0.308	0.104	0.031	0.018	0.538	-
$BR^{total}(\tilde{b}_1 \to q\chi)$	0.291	0.092	0.031	0.018	0.568	-
$BR^{tree}(\tilde{b}_2 \to q\chi)$	0.502	0.332	0.123	-	0.042	-
$BR^{QCD}(\tilde{b}_2 \to q\chi)$	0.541	0.386	0.054	-	0.019	-
$BR^{total}(\tilde{b}_2 \to q\chi)$	0.528	0.395	0.056	-	0.020	-

Table 1: Tree-level and corrected branching ratios of top- and bottom-squark decays into charginos and neutralinos for the parameter set (8) and $m_{\tilde{g}} = 500$ GeV. Branching ratios below 10^{-3} are not shown.

and corrected branching ratios for top- and bottom-squarks using the input parameter set (8) and $m_{\tilde{g}} = 500$ GeV. From inspection of Table 1 we see that the EW corrections can induce a change on the branching ratios of the leading decay channels of squarks comparable to the QCD corrections. Therefore both contributions must be taken into account on equal footing in the analysis of the phenomenology of sfermions.

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