

Electroweak Sudakov corrections in the MSSM

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Abstract

For superpartner masses not much heavier than the weak scale $M = M_W$, large logarithmic corrections of the Sudakov type arise at TeV energies. In this paper we summarize recent results of supersymmetric (susy) electroweak radiative corrections for sfermion and charged Higgs production at e^+e^- colliders in the MSSM. The results are given to sub-leading logarithmic (SL) accuracy to all orders in perturbation theory in the “light” susy-mass scenario. Prospects for the determination of $\tan\beta \geq 10$ are discussed which is independent of soft susy-breaking terms to SL accuracy.

Recently, there has been considerable interest in the high energy limit of the electroweak Standard Model (SM) [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. Those studies have concluded that at TeV energies virtual corrections of the Sudakov type are very large and higher order resummations are necessary to reach the desired percentile accuracy of future TeV linear colliders. Presently, a full subleading logarithmic (SL) approach exists in terms of the infrared evolution equation method [15] for arbitrary processes to all orders [1, 16, 17]. Further studies for massless fermions indicate that also sub-subleading angular terms can be large and need to be included at least through the two loop level [8].

In general, new physics responsible for electroweak symmetry breaking is expected in the TeV regime and the minimal supersymmetric SM (MSSM) remains an attractive candidate. If supersymmetry is relevant to the so called hierarchy problem, then the masses of the new superpartners cannot be much heavier than the weak scale $M \equiv M_W \sim M_Z$. In such a “light” susy mass scenario, similarly large radiative corrections can be expected as in the SM at TeV energies. At one loop this was confirmed by several works of the last few years [18, 19].

As in the SM, large corrections at the one loop level indicate that higher order contributions need to be included, both for the consistency of perturbation theory as well as the precision goals at a future TeV linear collider. In Ref. [20] we have presented results of higher order electroweak corrections to scalar production in the context of the MSSM.

The leading double (DL) and subleading angular dependent corrections originate only from the exchange of spin 1 gauge bosons and are therefore in principle identical to those

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of the SM. This is due to the assumption of softly broken supersymmetry and the identity of the gauge couplings in those theories between the gauge bosons and the SM particles with those of the gauge bosons and the accompanying sparticles. Novel contributions compared to the SM arise on the SL level, however, both from the gauge as well as the Yukawa sector through the novel particle content.

As in the SM, the SL corrections were found to exponentiate in operator form on the n -particle space with rotated Born-matrix elements [9, 16]. This conclusion, obtained in the effective theories beforehand (in Refs. [7] and [8] at first for massless fermion production processes and in Ref. [16] for arbitrary electroweak processes) has recently been confirmed by explicit calculation in terms of the physical fields for the angle dependent SL corrections [21]. The universal, process independent SL corrections exponentiate due to Ward identities in both the gauge as well as the Yukawa sector [5, 20].

A slight complication is given by the so-called SL-RG dependent terms [6, 22], where anomalous dimension terms proportional to the MSSM β -functions arise. In the following we give these corrections for the case that the full particle content contributes which, however, in a real TeV collider environment might not be the case. Since the scale of these terms is m_s , the mass-scale of the superpartners, and since we assume $m_s \sim M$, the sub-subleading corrections of the type $\mathcal{O}\left(\alpha^2 \log \frac{m_s^2}{M^2} \log^2 \frac{s}{M^2}\right)$, which originate from this problem, are negligible. Similar sub-subleading terms could also modify the SL gauge contributions if some susy particles are heavier. Nevertheless for clarity we distinguish these two scales below and also give the results only for the case of a heavy photon with $\lambda = M$. The omitted QED type corrections must be included via matching as in the SM [1, 17].

Under the above assumptions we find for the production of sfermions (\tilde{f}) in e^+e^- collisions the following all orders MSSM corrections to SL accuracy relative to the Born cross section:

$$\begin{aligned}
d\sigma_{e_\alpha^+ e_\alpha^- \rightarrow \tilde{f}_\beta \tilde{f}_\beta}^{\text{SL}} &= d\sigma_{e_\alpha^+ e_\alpha^- \rightarrow \tilde{f}_\beta \tilde{f}_\beta}^{\text{Born}} \times \\
&\exp \left\{ -\frac{g^2(m_s^2) I_{e_\alpha^-} (I_{e_\alpha^-} + 1)}{8\pi^2} \left[\log^2 \frac{q^2}{M^2} - \frac{1}{3} \tilde{\beta}_0 \frac{g^2(m_s^2)}{4\pi^2} \log^3 \frac{q^2}{m_s^2} \right] \right. \\
&\quad - \frac{g'^2(m_s^2) Y_{e_\alpha^-}^2}{32\pi^2} \left[\log^2 \frac{q^2}{M^2} - \frac{1}{3} \tilde{\beta}'_0 \frac{g'^2(m_s^2)}{4\pi^2} \log^3 \frac{q^2}{m_s^2} \right] \\
&\quad + \left(\frac{g^2(m_s^2)}{8\pi^2} I_{e_\alpha^-} (I_{e_\alpha^-} + 1) + \frac{g'^2(m_s^2) Y_{e_\alpha^-}^2}{8\pi^2 \cdot 4} \right) 2 \log \frac{q^2}{M^2} \\
&\quad - \frac{g^2(m_s^2) I_{\tilde{f}_\beta} (I_{\tilde{f}_\beta} + 1)}{8\pi^2} \left[\log^2 \frac{q^2}{M^2} - \frac{1}{3} \tilde{\beta}_0 \frac{g^2(m_s^2)}{4\pi^2} \log^3 \frac{q^2}{m_s^2} \right] \\
&\quad - \frac{g'^2(m_s^2) Y_{\tilde{f}_\beta}^2}{32\pi^2} \left[\log^2 \frac{q^2}{M^2} - \frac{1}{3} \tilde{\beta}'_0 \frac{g'^2(m_s^2)}{4\pi^2} \log^3 \frac{q^2}{m_s^2} \right] \\
&\quad \left. + \left(\frac{g^2(m_s^2)}{8\pi^2} I_{\tilde{f}_\beta} (I_{\tilde{f}_\beta} + 1) + \frac{g'^2(m_s^2) Y_{\tilde{f}_\beta}^2}{8\pi^2 \cdot 4} \right) 2 \log \frac{q^2}{M^2} \right\}
\end{aligned}$$

$$\begin{aligned}
& -\frac{g^2(m_s^2)}{8\pi^2} \left(\frac{1 + \delta_{\beta,R}}{2} \frac{\hat{m}_{\tilde{f}}^2}{M^2} + \delta_{\beta,L} \frac{\hat{m}_{\tilde{f}'}^2}{2M^2} \right) \log \frac{q^2}{m_s^2} \\
& -\frac{g^2(m_s^2)}{8\pi^2} \log \frac{q^2}{M^2} \left[\left(\tan^2 \theta_w Y_{e_\alpha^-} Y_{\tilde{f}} + 4I_{e_\alpha^-}^3 I_{\tilde{f}}^3 \right) \log \frac{t}{u} \right. \\
& \left. + \frac{\delta_{\alpha,L} \delta_{\beta,L}}{\tan^2 \theta_w Y_{e_\alpha^-} Y_{\tilde{f}}/4 + I_{e_\alpha^-}^3 I_{\tilde{f}}^3} \left(\delta_{d,\tilde{f}} \log \frac{-t}{q^2} - \delta_{u,\tilde{f}} \log \frac{-u}{q^2} \right) \right] \Big\} \quad (1)
\end{aligned}$$

where I_j denotes the total weak isospin of the particle j , Y_j its weak hypercharge and at high q^2 the invariants are given by $t = -\frac{q^2}{2}(1 - \cos \theta)$ and $u = -\frac{q^2}{2}(1 + \cos \theta)$. The helicities are those of the fermions (f) whose superpartner is produced. In addition we denote $\hat{m}_{\tilde{f}} = m_t/\sin \beta$ if $\tilde{f} = \tilde{t}$ and $\hat{m}_{\tilde{f}} = m_b/\cos \beta$ if $\tilde{f} = \tilde{b}$. \tilde{f}' denotes the corresponding isopartner of \tilde{f} . For particles other than those belonging to the third family of quarks/squarks, the Yukawa terms are negligible. Eq. (1) depends on the important parameter $\tan \beta = \frac{v_u}{v_d}$, the ratio of the two vacuum expectation values, and displays an exact supersymmetry in the sense that the same corrections are obtained for the fermionic sector in the regime above the electroweak scale M .

Here we assume that the asymptotic MSSM β -functions can be used with

$$\tilde{\beta}_0 = \frac{3}{4}C_A - \frac{n_g}{2} - \frac{n_h}{8}, \quad \tilde{\beta}'_0 = -\frac{5}{6}n_g - \frac{n_h}{8} \quad (2)$$

$$g^2(q^2) = \frac{g^2(m_s^2)}{1 + \tilde{\beta}_0 \frac{g^2(m_s^2)}{4\pi^2} \ln \frac{q^2}{m_s^2}}, \quad g'^2(q^2) = \frac{g'^2(m_s^2)}{1 + \tilde{\beta}'_0 \frac{g'^2(m_s^2)}{4\pi^2} \ln \frac{q^2}{m_s^2}} \quad (3)$$

where $C_A = 2$, $n_g = 3$ and $n_h = 2$. In practice, one has to use the relevant numbers of active particles in the loops. These terms correspond to the RG-SL corrections just as in the case of the SM as discussed in Ref. [6] but now with the MSSM particle spectrum contributing. They originate only from RG terms within loops which without the RG contribution would give a DL correction. It should be noted that the one-loop RG corrections do not exponentiate and are omitted in the above expressions. They are, however, completely determined by the renormalization group in softly broken supersymmetric theories such as the MSSM and sub-subleading at higher than one loop order.

In the case of charged Higgs production we have analogously:

$$\begin{aligned}
& d\sigma_{e_\alpha^+ e_\alpha^- \rightarrow H^+ H^-}^{\text{SL}} = d\sigma_{e_\alpha^+ e_\alpha^- \rightarrow H^+ H^-}^{\text{Born}} \times \\
& \exp \left\{ -\frac{g^2(m_s^2)}{8\pi^2} I_{e_\alpha^-} (I_{e_\alpha^-} + 1) \left[\log^2 \frac{q^2}{M^2} - \frac{1}{3} \tilde{\beta}_0 \frac{g^2(m_s^2)}{4\pi^2} \log^3 \frac{q^2}{m_s^2} \right] \right. \\
& - \frac{g'^2(m_s^2) Y_{e_\alpha^-}^2}{32\pi^2} \left[\log^2 \frac{q^2}{M^2} - \frac{1}{3} \tilde{\beta}'_0 \frac{g'^2(m_s^2)}{4\pi^2} \log^3 \frac{q^2}{m_s^2} \right] \\
& \left. + \left(\frac{g^2(m_s^2)}{8\pi^2} I_{e_\alpha^-} (I_{e_\alpha^-} + 1) + \frac{g'^2(m_s^2) Y_{e_\alpha^-}^2}{8\pi^2 \cdot 4} \right) 2 \log \frac{q^2}{M^2} \right. \\
& \left. - \frac{g^2(m_s^2)}{8\pi^2} I_H (I_H + 1) \left[\log^2 \frac{q^2}{M^2} - \frac{1}{3} \tilde{\beta}_0 \frac{g^2(m_s^2)}{4\pi^2} \log^3 \frac{q^2}{m_s^2} \right] \right\}
\end{aligned}$$

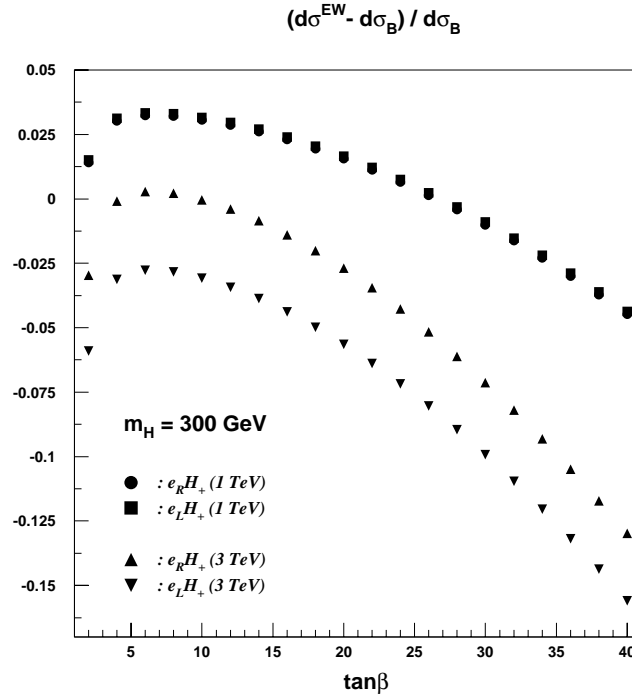


Figure 1: The dependence of the relative corrections to the Born cross section for charged Higgs production ($m_H = 300$ GeV) as a function of $\tan\beta$ at 90° scattering angle. It can be seen that for large values of $\tan\beta$, the Sudakov suppression is enhanced.

$$\begin{aligned}
& -\frac{g'^2(m_s^2)Y_H^2}{32\pi^2} \left[\log^2 \frac{q^2}{M^2} - \frac{1}{3}\tilde{\beta}'_0 \frac{g'^2(m_s^2)}{4\pi^2} \log^3 \frac{q^2}{m_s^2} \right] \\
& + \left(\frac{g^2(m_s^2)}{8\pi^2} I_H (I_H + 1) + \frac{g^2(m_s^2) Y_H^2}{8\pi^2} \frac{1}{4} \right) 2 \log \frac{q^2}{M^2} \\
& - 3 \frac{g^2(m_s^2)}{16\pi^2} \left[\frac{m_t^2}{M^2} \cot^2 \beta + \frac{m_b^2}{M^2} \tan^2 \beta \right] \log \frac{q^2}{m_s^2} \\
& - \frac{g^2(m_s^2)}{4\pi^2} \log \frac{q^2}{M^2} \left[\delta_{\alpha,L} \left(\frac{1}{2c_w^2} \log \frac{t}{u} + 2c_w^2 \log \frac{-t}{q^2} \right) + \delta_{\alpha,R} \tan^2 \theta_w \log \frac{t}{u} \right] \} \quad (4)
\end{aligned}$$

It should be noted here that the Yukawa terms proportional to $\tan\beta$ are quite large due to the additional factor of $3 = N_C$ from the quark loops [20]. Overall, both Yukawa contributions in Eq. (1) and (4) reinforce the Sudakov suppression factor of the leading double logarithmic terms. In addition, the universal positive SL gauge terms are identical (and compared to the SM smaller) for spin $\frac{1}{2}$ and spin 0 particles due to the exact supersymmetry present at high energies. All supersymmetry breaking terms are constants and thus beyond our level of approximation.

In Fig. 1 we display the $\tan\beta$ dependence of the relative corrections to charged Higgs production for $m_H = 300$ GeV. It demonstrates that the relative $\tan\beta$ dependence of the resummed corrections alone accounts for over 10 % at 3 TeV and over 5 % at 1 TeV in the window $10 \leq \tan\beta \leq 40$.

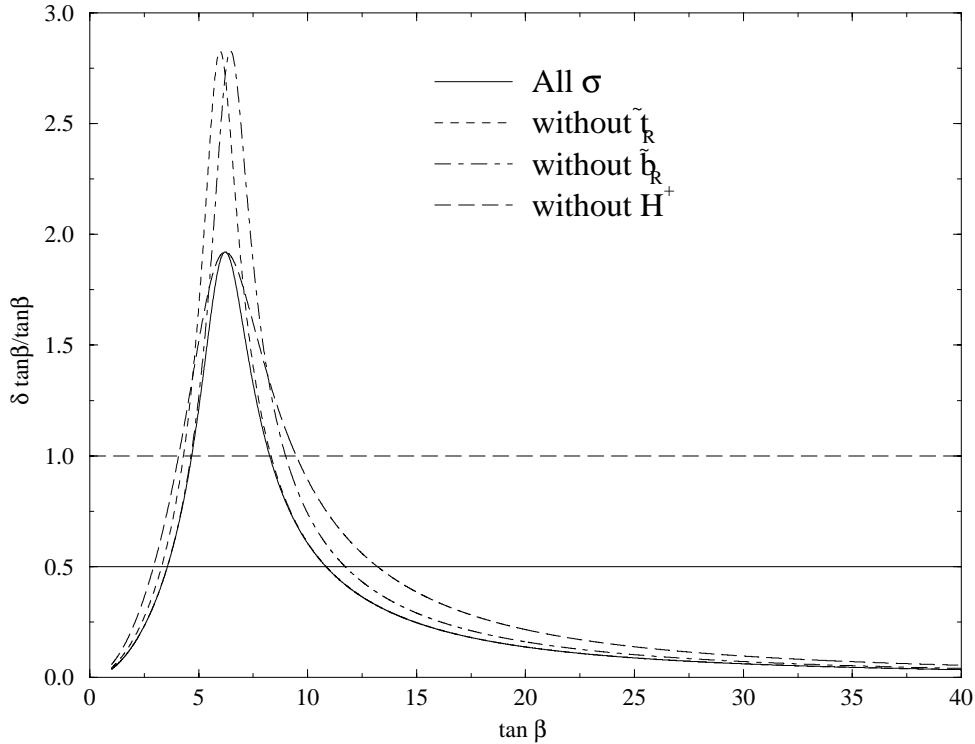


Figure 2: The relative error in $\tan\beta$ after performing 10 measurements with a relative accuracy of 1 %. The curves can be improved if a higher precision could be reached.

Since the precise measurement of large values of $\tan\beta$ at the LHC is not straightforward [23] or strongly model dependent [24, 25, 26], it is interesting to investigate if this significant dependence on $\tan\beta$ through virtual corrections can be used for an independent measurement. In this context we envision a series of N precise measurements at $\sqrt{q_1^2}, \sqrt{q_2^2}, \dots, \sqrt{q_N^2}$ of various cross sections and introduce the one loop quantity $\epsilon(q^2)$ as the difference between measurements and the theoretical asymptotic DL and SL logarithms of *gauge* origin. Then its asymptotic expansion is given by

$$\epsilon(q^2) \equiv \frac{\alpha}{4\pi} F(\tan\beta) \ln \frac{q^2}{m_s^2} + G + \mathcal{O}\left(\frac{M^2}{q^2}\right) \quad (5)$$

Here it is crucial to note that G is a constant which depends on mass ratios and that the function $F(\tan\beta)$ does *not* depend on soft breaking terms. Then we can eliminate the influence of the mass terms by subtraction via

$$\delta_i \equiv \epsilon(q_i^2) - \epsilon(q_1^2) = F(\tan\beta^*) \ln \frac{q_i^2}{q_1^2} \quad (6)$$

where $\tan\beta^*$ is the *true* unknown value that describes the experimental measurements.

Performing a minimizing χ^2 analysis leads to the results for the accuracy of the $\tan\beta$ determination depicted in Fig. 2 as a function of the size of $\tan\beta$ and assuming a 1 % accuracy of 10 measurements of the various scalar on-shell production cross sections in the range between 0.8 and 3 GeV [20].

It is clearly visible that a 50 % measurement is possible for $\tan\beta \geq 10$, a 25 % determination for $\tan\beta \geq 15$ and a few percent measurement for $\tan\beta \geq 25$. Thus, the virtual radiative MSSM corrections are not only crucial for precision measurements at a TeV linear collider at one and two loop order, they also contain valuable information about the important Higgs sector parameter $\tan\beta$.

In particular we emphasize again that this gauge invariant determination to SL accuracy is independent of both the soft breaking terms and obviously of the renormalization scheme. Ambiguities with respect to the renormalization of $\tan\beta$ [27] would enter, however, at the single logarithmic level at two loops. It must also be noted that a one loop treatment is insufficient for energies above 1 TeV in the MSSM and the SM and further investigations toward an all orders SL analysis of the full MSSM particle production at a future linear collider is ongoing [28].

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