On the Renormalization and Regularization of the MSSM

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Motivation

Precision of a future linear collider extremely high!

⇒ Requires adequate theoretical precision for SM and MSSM processes:

$e^+e^- \to 2\,f$: full $\mathcal{O}(\alpha^2)$ and leading higher-order corrections
$e^+e^- \to 4\,f$: full $\mathcal{O}(\alpha)$ and leading higher-order corrections

But: Consistent invariant regularization scheme for supersymmetric theories missing!

Dimensional regularization:

't Hooft,Veltman'72
Breitenlohner,Maison'77

- fully consistent
- breaks supersymmetry, chiral symmetry
  ⇒ symmetry identities can be consistently restored!

Dimensional reduction:

Siegel'79

- inconsistent (Siegel'80)
- "practically working" and easy scheme at 1-loop level!
- no deeper understanding of symmetry violation!
  When is SUSY or gauge-invariance violated?!

Remarks:

- symmetry violation by non-logarithmic contributions
  ⇒ breaking terms are usually non-leading!
- symmetry violation at 1-loop disturb renormalizability at 2-loop

Classical theory

Symmetries formulated in BRS form

$$\mathrm{s}V_\mu = \underbrace{\mathrm{D}_\mu c}_{\text{local gauge transf.}} + \underbrace{\epsilon^\alpha \sigma_{\mu\alpha\dot\alpha}\bar\lambda^{\dot\alpha} + \lambda^\alpha \sigma_{\mu\alpha\dot\alpha}\bar\epsilon^{\dot\alpha}}_{\text{susy transformation}} - \underbrace{\mathrm{i}\xi^\nu \partial_\nu V_\mu}_{\text{translation}}$$

with ghost fields c, ϵ_α, ξ^μ.

Important property: nilpotency $\mathbf{s}^2 V_\mu = 0$

Classical action,vertex functional

$$\Gamma_{\mathrm{cl}} = \Gamma_{\mathrm{sym}} + \Gamma_{\mathrm{soft}} + \Gamma_{\mathrm{g.f.}} + Y_i \mathbf{s}\phi_i$$

- Γ_{sym}: symmetric action, no ghosts, constructed from superfields
- Γ_{soft}: soft susy breaking, introduced via external superfields, required for spont. symmetry breaking
- $\Gamma_{\mathrm{g.f.}}$: gauge fixing and ghost term, has to respect rigid gauge inv., $= \mathbf{s}\int \mathrm{d}^4x(\bar c F)$
- $Y_i \mathbf{s}\phi_i$: external sources Y for all non-linear transforming BRS transformations

Slavnov-Taylor (ST) operator:

$$\mathcal{S}(\Gamma) = \sum_{\text{linear}} \mathrm{s}\phi_k \frac{\delta\Gamma}{\delta\phi_k} + \sum_{\text{non linear}} \underbrace{\frac{\delta\Gamma}{\delta Y_i}}_{= \mathrm{s}\phi_i + \mathcal{O}(\hbar)} \frac{\delta\Gamma}{\delta\phi_i} = 0$$

$$\mathcal{S}(\Gamma + \gamma) = \mathcal{S}(\Gamma) + \underbrace{\mathcal{S}_\Gamma}_{\text{=linearized ST operator}} \gamma + \mathcal{O}(\gamma^2)$$

Renormalization using a non-invariant regularization scheme

General method:

- Calculate radiative corrections of order $\mathcal{O}(\hbar^n)$:

$$\Gamma = \Gamma_{\rm cl} + \sum_n \Gamma^{(n)}$$

  $\Gamma^{(n)}$ involve loops and invariant counterterms

- ST identity will be in general violated:

$$\mathcal{S}(\Gamma) = \hbar^n \Delta + \mathcal{O}(\hbar^{n+1}).$$

  Quantum action principle: Δ is a local field polynomial

  Example: $\Delta = 0$ for dimensional regularization in QED
  $\Delta =$ susy breaking terms for dimensional regularization in MSSM

- Search for a $\hat{\Delta}$ with

$$\Delta = \mathcal{S}_{\Gamma_{\rm cl}} \hat{\Delta} + \hbar^n r_i \mathcal{A}_i$$

  and replace $\Gamma \to \Gamma - \hat{\Delta}$

$$\mathcal{S}(\Gamma) = \hbar^n r_i \mathcal{A}_i + \mathcal{O}(\hbar^{n+1}),$$

  $\mathcal{A}_i$: possible anomalies
  MSSM: susy Adler-Bardeen anomaly with $r_i = 0$

Nilpotency relations:

$$\begin{array}{llll} \mathcal{S}_\gamma \mathcal{S}(\gamma) & = 0 \text{ for all } \gamma, & \rightarrow & \mathcal{S}_{\Gamma_{\rm cl}}\Delta = 0 \\ \mathcal{S}_{\Gamma_{\rm cl}}\mathcal{S}_{\Gamma_{\rm cl}} & = 0, & \rightarrow & \mathcal{S}_{\Gamma_{\rm cl}}\hat{\Delta} = 0 \end{array}$$

$\Rightarrow$Nilpotency relations restricts number of symmetry restoring counterterms $\hat{\Delta}$ and breaking terms Δ!

MSSM: Nilpotency condition only fulfilled with additional requirement $\Gamma_{\rm ct} = \Gamma_{\rm ct}(\epsilon y'_\lambda - \bar{\epsilon}\bar{y}'_\lambda)$

with λ' =U(1)$_Y$ photino, y'_λ = external source of λ'

In practise:

- Add to Feynman rules symmetry-restoring counterterms $\hat{\Delta}$ which
  - respect all symmetries respected by the regularization scheme
  - do not respect symmetries violated by the reg. scheme

  MSSM: symmetry-restoring counterterms are all non-supersymmetric terms for dim. reg.
- Determine symmetry-restoring counterterms by ST identities:

$$\left.\frac{\delta^N \mathcal{S}(\Gamma)}{\delta\phi_{i_1}\cdots\delta\phi_{i_N}}\right|_{\phi\to 0} = 0$$

  for all field monomials $(\phi_{i_1}\cdots\phi_{i_N})$ out of Δ.
  - $(\phi_{i_1}\cdots\phi_{i_N})$ can be a over-complete set including Δ but need not respect $\mathcal{S}_{\Gamma_{\rm cl}}(\phi_{i_1}\cdots\phi_{i_N}) = 0$
  - Set of linear equations can break down in sub-blocks
  - Approximations can be useful

In practise extremely complicated!

Symmetry requirements of the MSSM

- ST identity: $\mathcal{S}(\Gamma) = 0$
  - involves local gauge invariance, rigid susy, and rigid translations in BRS form
- Ward operators: $\mathcal{W}\Gamma = \int \mathrm{d}^4x \delta\phi_i \frac{\delta\Gamma}{\delta\phi_i} = 0$
  - rigid SU(2)$_I$ gauge invariance
  - local U(1)$_Y$ gauge invariance

Bandelloni,Becchi,Blasi,Collina'78, Kraus'98,Grassi'98

$$\mathcal{W}'_{\mathrm{local}}\Gamma = \Box(B' + \mathrm{i}\xi^\mu\partial_\mu\bar{c}')$$

$\Rightarrow$ required to fix hypercharge in higher orders.

  - lepton and quark number conservation
  - continuous R symmetry
- Minimum requirement of Higgs potential

$$\int \mathrm{d}^4x \frac{\delta\Gamma}{\delta H_i(x)}\bigg|_{\phi\to 0} = 0.$$

- Nilpotency requirement $\Gamma_{\mathrm{ct}} = \Gamma_{\mathrm{ct}}(\epsilon^\alpha y'_{\lambda\alpha} - \bar{\epsilon}_{\dot\alpha}\bar{y}'_\lambda{}^{\dot\alpha})$
- Linear gauge-fixing condition $\frac{\delta\Gamma}{\delta B} =$ linear
- Translation-ghost equation $\frac{\delta\Gamma}{\delta\xi^\mu} =$ linear
- CP invariance

Soft-supersymmetry breaking

Soft-supersymmetry breaking terms:

Girardello,Grisaru'82

- Mass terms for scalar fields: $-M^2_{ij}\bar{\phi}_i\phi_j$
- Holomorphic bilinear and trilinear terms in scalar fields:
  $-B_{ij}\phi_i\phi_j - A_{ijk}\phi_i\phi_j\phi_k$
- Mass terms for gauginos: $M_\lambda\bar{\lambda}^\alpha\lambda_\alpha + \mathrm{c.c.}$

Soft-susy breaking via dimensionless superfield (spurion):

$$\hat{A} = e^{-\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu}[A + \sqrt{2}\theta^\alpha a_\alpha + \theta^2(F_A + v_A)]$$

$\Rightarrow$ Insert $\hat{A}$ everywhere possible in Γ_{cl} with new couplings!

$\Rightarrow$ Generates all soft-supersymmetry breaking terms!

Example: gaugino mass term

$$\int \mathrm{d}^6 z \hat{A}\hat{F}^\alpha\hat{F}_\alpha \stackrel{\hat{A}=\theta^2 v_A}{\longrightarrow} \int \mathrm{d}^4 x \lambda^\alpha\lambda_\alpha$$

$\Rightarrow$ Spurion can appear infinite many times yielding an infinite number of coupling constants!

Soft-supersymmetry breaking

Only couplings which survive for A, $a_\alpha \to 0$ are physically relevant:

Hollik,Kraus,Stöckinger'00

Two solutions of $\mathcal{S}(\Gamma_{1,2})|_{A,a_\alpha=0} = 0$ are physical equivalent, i.e. $(\Gamma_1 - \Gamma_2)|_{A,a_\alpha,Y=0} = 0$.

$\Rightarrow$ Physical observables are not affected by renormalization conditions for couplings to A, a_α!

Choosing most easy solution:

Piguet,Maggiore,Wolf'96

$$A = 0, \qquad a_\alpha = \sqrt{2}\epsilon_\alpha A_1, \qquad F_A = A_2$$

$$\hat{A} = e^{-\mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu}[2\theta^\alpha \epsilon_\alpha A_1 + \theta^2(A_2 + v_A)]$$

- $\hat{A}$ appears only finite times since $\hat{A}^2 = 0$.
- $u = A_1, v = A_2 + v_A - \mathrm{i}\xi^\mu\partial_\mu A_1$ is BRS doublet:

$$\mathbf{s}u = v, \mathbf{s}v = 0 \qquad \Rightarrow \qquad \Gamma = \Gamma|_{u,v=0} + \mathcal{S}_\Gamma \tilde{\Delta}(u, v)$$

  $\Rightarrow$ Symmetry-restoring counterterms do not depend on $A_{1,2}$!

- A_1 can appear without $\epsilon_\alpha \Rightarrow$ require $\Gamma = \Gamma(\epsilon_\alpha A_1)$
- MSSM becomes invariant under continuous R symmetry

New approach through non-renormalization theorems

$\Rightarrow$ Further restriction on invariant counterterms!

Kraus,Stöckinger'01,'02

Gauge fixing

Gauge-fixing has to maintain all symmetries!

Ansatz: $(\zeta^+ = \zeta^- = \zeta^3)$

$$\Gamma_{\text{g.f.}} = \int \mathrm{d}^4x\, \mathbf{s}\left(\bar{c}^a F^a + \frac{1}{2}\zeta^a \bar{c}^a B^a\right), \quad a = +, -, 3, \prime$$

with

$$F^a = \partial^\mu V_\mu^a - [\mathrm{i}(\bar{\Phi}_1^a + \bar{\mathbf{v}}_1^a)H_1 + \mathrm{i}(\bar{\Phi}_2^a + \bar{\mathbf{v}}_2^a)H_2 + \text{h.c.}].$$

For R_ξ gauge, choose $\mathbf{v}_i^a$ such that

$$F^\pm = \partial^\mu W_\mu^\pm \pm \mathrm{i} M_W \zeta^{G^\pm} G^\pm,$$
$$\begin{pmatrix} F' \\ F^3 \end{pmatrix} = R_V \begin{pmatrix} \partial^\mu A_\mu \\ \partial^\mu Z_\mu + M_Z \zeta^{ZG^0} G^0 \end{pmatrix}.$$

Remarks:

- Introduction of doublets $\mathbf{v}_i^a$ to exclude physical Higgs from $\Gamma_{\text{g.f.}}$
  Reason: different field renormalization of H_i
- Rigid gauge invariance by proper choice of $\delta^{\text{gauge}}\Phi_i^a$
- Introduce external fields Ψ_i^a which form BRS doublet with Φ_i^a
- $\Phi_i' + \bar{\mathbf{v}}_i'$, $\hat{\Phi} = 2T^a(\Phi_i^a + \bar{\mathbf{v}}_i^a)$ mix with the physical Higgs fields

$\Rightarrow$ Φ_i^a, Ψ_i^a can be ignored for rigid gauge-invariant regularization schemes!

Invariant counterterms

Genuine invariant counterterms:

$$\Gamma_{\rm ct,inv} = \underbrace{\Gamma_{\rm ct,inv,1}}_{\swarrow} + \underbrace{\Gamma_{\rm ct,inv,2}}_{\searrow}$$

$\Gamma_{\rm ct,inv,1} = (\delta g \frac{\delta}{\delta g} + \ldots)\Gamma_{\rm cl}$

renormalization of:

- gauge couplings g, g'
- Yukawa couplings f_R, f_U, f_D
- μ-parameter
- Fayet-Iliopoulos term v'
- renormalization of linear transf. field V'_μ

$\Gamma_{\rm ct,inv,2} = \mathcal{S}_{\Gamma_{\rm cl}} \hat{\Gamma}_{\rm ct,inv,2}$

- couplings to BRS doublets:
  - soft susy breaking parameters
  - gauge-fixing parameters
  - Φ_i^a parameters
- field renormalization of non-linear transf. fields $\frac{1}{2}\delta z_\phi \mathcal{S}_{\Gamma_{\rm cl}} \int {\rm d}^4 x Y_\phi \phi$

Remarks:

- Genuine invariant counterterms do not appear in Ward and ST operators
- Renormalization of V'_μ, g', λ', ... connected due to local U(1)$_Y$ Ward identity

Finite field reparametrizations:

$$\Gamma(\phi_i^{\text{sym}}) = \Gamma(R_{ij}\phi_j^{\text{phys}}).$$

- only fields with same quantum numbers mix
- appear in Ward and ST operators explicitely
- important to fulfil on-shell renormalization conditions
- partially redundant to genuine invariant counterterms
- required for infrared power-counting renormalizability

Example:

$$\begin{pmatrix} V'_\mu \\ V^3_\mu \end{pmatrix} = \underbrace{\begin{pmatrix} \sqrt{z_{V'}} & 0 \\ 0 & \sqrt{z_V} \end{pmatrix}}_{\substack{\text{genuine invariant} \\ \text{counterterms} \\ \text{divergent}}} \underbrace{\begin{pmatrix} (R_V)_{11} & (R_V)_{12} \\ (R_V)_{21} & (R_V)_{22} \end{pmatrix}}_{\substack{\text{field reparametrization} \\ \text{finite}}} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$$

Infrared finiteness

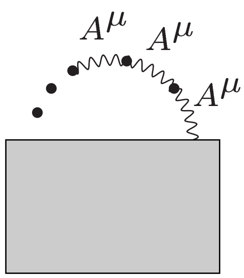
Power-counting renormalizability explicitely proven!

(Proof done in BPHZL scheme)

Requirements by IR power-counting:

$$\Gamma_{A^\mu Z^\nu}|_{p^2=0} = \Gamma_{c_A \bar{c}_Z}|_{p^2=0} = \Gamma_{c_Z \bar{c}_A}|_{p^2=0} = 0 \qquad (1)$$

$$\Gamma_{A^\mu A^\nu}|_{p^2=0} = \Gamma_{c_A \bar{c}_A}|_{p^2=0} = 0 \qquad (2)$$



$\Rightarrow$ vertices $\bullet = \Gamma_{A^\mu A^\nu}|_{p^2=0} \neq 0$
yield infrared problem!

$\Rightarrow$ Non-invariant counterterms $A^\mu A_\mu$, $A^\mu Z_\mu$, $c_Z \bar{c}_A$, $c_A \bar{c}_Z$, $c_A \bar{c}_A$ are already fixed by symmetries!

BPHZL scheme: forbidden counterterms

- Demixing requirements (1) established by field reparametrization:

$$\left[(\delta R_V)_{ZA} A_\mu \frac{\delta}{\delta Z_\mu} + (\delta R_V)_{ZZ} Z_\mu \frac{\delta}{\delta Z_\mu} \right] \frac{1}{2} \left(M_Z^2 Z^\mu Z_\mu \right)$$

$$= (\delta R_V)_{ZA} M_Z^2 A^\mu Z_\mu + (\delta R_V)_{ZZ} M_Z^2 Z^\mu Z_\mu .$$

Kraus'98

$\Rightarrow$ δR_V appears in ST and Ward operators explicitely

- Requirements (2) fulfilled as a consequence of the ST identity and the demixing requirements (1).

On-shell renormalization scheme

On-shell renormalization scheme (schematically):

- Mass conditions: $\mathrm{Re}\Gamma_{\phi_i\phi_i}|_{p^2=m_i^2} = 0$
- Demixing conditions: $\mathrm{Re}\Gamma_{\phi_i\phi_j}|_{p^2=m_i^2} = 0, \quad i \neq j$
- Residuum conditions: $\mathrm{Re}\partial_{p^2}\Gamma_{\phi_i\phi_i}|_{p^2=m_i^2} = 1$

$\Rightarrow$ On-shell conditions very similar to the once of the Standard Model.

Remarks:

- Several masses determined by mass relation and not by renormalization conditions!

  E.g. masses of $H^0, h^0, H^\pm, \chi^0_{2,3,4}, \tilde{\mathrm{d}}_2, \tilde{\mathrm{e}}_2$ are not free parameters.

  Masses in propagators fulfil only lowest-order mass relations!
- Demixing possible only up to finite decay widths!

  Known problems related to unstable particles.

  Unstable particles do not appear in asymptotic states.

Summary and Outlook

Summary:

- Renormalization of the MSSM in a scheme independent way
- Soft-supersymmetry breaking parameter via external fields
- Rigid gauge-invariant gauge-fixing term
- Complete and consistent set of symmetry identities
- Complete set of invariant counterterms
- Infrared finiteness of the theory explicitely proven
- Consistent set of renormalization conditions

Outlook:

- Restoration of symmetry identities in practice
  $\Rightarrow$ Introduction of symmetry-restoring counterterms
  I. Fischer,D. Stöckinger, M. R.
- Apply non-renormalization theorems to MSSM
  $\Rightarrow$ Deeper understanding of invariant counterterms in MSSM
  E. Kraus, M. R.